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Approximate reasoning with fuzzy soft set

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Abstract

In this work, we develop a reasoning mechanism under uncertainty for a typical class of decision making problems based on the theory of fuzzy soft set. Accordingly, we consider a logic of fuzzy soft sets. In the sequel, implication operations dealing with fuzzy soft sets over different universal sets and different sets of parameters have been studied extensively. An algorithm for rule based reasoning has been developed for a modus ponens type rule of inference using the proposed logical structure. The entire proposal is illustrated with an example. Results of application of our proposed inference are compared with those obtained by Mondal and Raha on the problem of management of hypertension using a medical diagnostic support system.

Keywords: Soft set, fuzzy set, fuzzy soft set, inexact reasoning.

1 Introduction

Soft set theory is found to be an efficient tool in dealing with linguistic statements approximating human perceptions. In this theory, there are provisions for use of substantial parameters to describe an object of concern so that it is free from those difficulties that are associated with other contemporary theories dealing with uncertainty. Molodtsov [16] introduced the concept of a soft set. Structures of soft sets have been studied by many [15, 21]. Ali et al. [21] have shown that a collection of soft sets with reference to so-called new operations gives rise to many algebraic structures and proved that it forms a complete modular lattice. Das et al. [5] proposed an algorithmic approach for multiple attribute group decision making problems using interval-valued intuitionistic fuzzy soft matrix. They developed an algorithm to find the desired alternatives based on product interval-valued intuitionistic fuzzy soft matrix. In the process, the authors combined choice matrix and score value of the set of alternatives. They also proposed the correlation co-efficient for interval-valued hesitant fuzzy soft set [6], as also correlation efficiency to show the importance of hesitant fuzzy soft set. They then developed an algorithm to apply correlation co-efficient in decision making problem, where information is presented in hesitant fuzzy environment. In [4], the authors introduced intuitionistic trapezoidal fuzzy soft set [4] as well as weighted intuitionistic fuzzy soft set. They developed an algorithm [3] based on intuitionistic fuzzy soft set and applied it to a particular disease. Authors in [20] defined similarity measure for hesitant fuzzy soft sets which can be used for constructing similarity based approximate reasoning with fuzzy soft set. Recently in [11], authors redefined spherical fuzzy soft sets, studied some of its properties and constructed an aggregation operator.

In 2001, Maji et al. [12] proposed the concept of fuzzy soft set. They studied some properties regarding fuzzy soft union, intersection and complement. Ahmad and Kharal [1] defined arbitrary fuzzy soft union, intersection, complement etc. Cagman et al. [8] defined fuzzy parameterized fuzzy soft set and formulated a procedure for decision making using fuzzy soft set. Majumdar and Samanta [14] defined generalized fuzzy soft set and applied it to medical diagnosis. In 2007, Roy and Maji [13] used fuzzy soft set theory to decision making problems. Ali and Shabir [2] defined logic connectives for fuzzy soft sets and introduced logical foundation to fuzzy soft sets, which is crucial for the development of a theory of approximate reasoning. However, there has been no significant contributions to the study of approximate

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reasoning based on fuzzy soft set. Being a natural extension of fuzzy set, fuzzy soft set requires a proper logical foundation like fuzzy set for the purpose of an adequate theory of approximate reasoning.

In fuzzy set theory, degree of membership of an element of a set ranges over $[0, 1]$ which can be replaced with a bounded distributive lattice [9]. In [10], the author proposed that logic connectives should be defined on certain types of lattices. Therefore, in [18] and [24], logic connectives such as conjunction, disjunction, negation and implications as well as generating functions like t-norms and t-conorms are studied in a more general setting for a bounded distributive lattice.

The theory of soft set has the ability of hybridization. In this regard, fuzzy soft sets and their applications have been explored by many authors [12, 13]. It has been shown that a collection of fuzzy soft sets forms a complete modular lattice structure with respect to certain binary operations to be defined on them. Flexibility in the choice of operations on the set of parameters may increase the number of connectives for fuzzy soft sets rather than those for a lattice.

Any logical system dealing with fuzzy soft set is more or less synonymous with fuzzy soft logic. In this research, we consider operations on fuzzy soft sets so as to develop a logic of fuzzy soft set. Subsequently, we try to develop a reasoning mechanism with fuzzy soft sets and fuzzy soft relations. Accordingly, we have proposed a logic of fuzzy soft set. Using logical operations we try to establish different rules for inference. We have designed a powerful algorithm for one such rule of inference for deduction of new results from existing ones. This, we hope, will definitely help us in finding a solution to different decision making problems.

Interpreting fuzzy implications is one of the main operations in fuzzy logic. As fuzzy implication is a generalization of the classical one to fuzzy logic, likewise a fuzzy soft implication is a generalization of a fuzzy implication suitable for parametrized collection of fuzzy subsets of the universal set. As there exist many families of fuzzy implications, the same is true for implications on fuzzy soft sets. We have examined the properties of such implications. It is found that most of the definitions are straight-forward generalizations of their generic counterparts.

Combining fuzzy soft set with fuzzy soft relation, we attempt to develop a reasoning mechanism and arrive at a logic of fuzzy soft set. Different rules of inference are sought to be established on the basis of logical operations. In order to arrive at a solution for different decision making problems, the design of a powerful algorithm for one such rule of inference is attempted. The reasoning proposal is illustrated with an example from the literature [17]. It is compared with the result reported therein for a small class of rules and is found to be significant.

This paper is organized into six sections. The work is introduced in Section 1. We then defined and subsequently illustrated some terms in order to communicate with fuzzy soft sets in Section 2. The logical aspects of reasoning have been considered for fuzzy soft sets in Section 3. A rule of inference has been developed/formulated. The computational procedure is presented in a subsection. In Section 4, an applicable form of fuzzy soft reasoning based on well-known generalized modus ponens rule of inference has been considered. A case study on management of hypertension is presented to illustrate the efficiency of the proposal. In the sequel, a comparative study has also been reported. The work is briefly concluded in Section 5. This is followed by a list of references in Section 6.

2 Preliminaries

Definition 2.1. [16] *A soft set on the universal set U with respect to the parameter set A is a mapping denoted by M_A and is given by, $M_A : A \longrightarrow 2^U$. Let M_A and N_A be two soft sets over a common universe U. Their union is a soft set denoted by $(M \cup N)_A$ and is defined by $(M \cup N)_A(e) = M_A(e) \cup N_A(e)$, for any $e \in A$. Their intersection is a soft set denoted by $(M \cap N)_A$ and is defined by $(M \cap N)_A(e) = M_A(e) \cap N_A(e)$ $\forall e \in A$. The complement of M_A is denoted by M_A^c and is defined by a mapping*

$$M_A^c : \; A \to 2^U \text{ such that } M_A^c(e) = U - M_A(e), \; \forall e \in A.$$

M_A is said to be a soft subset of N_A if $\forall e \in A$, $M_A(e) \subseteq N_A(e)$ and is denoted by $M_A \subseteq N_A$.

Definition 2.2. [23] *Let U be a universal set. The collection $\widetilde{M}$ of ordered pairs of the form $\{(u, \widetilde{M}(u)) \mid u \in U\}$ is called a fuzzy set where, $\widetilde{M} : \; U \to [0, 1]$. Let $\widetilde{M}$ and $\widetilde{N}$ be two fuzzy sets over the universal set U. Their union is a fuzzy set denoted by $\widetilde{M} \cup \widetilde{N}$ and is defined by*

$$\{\widetilde{M} \cup \widetilde{N}\}(u) = \max(\widetilde{M}(u), \widetilde{N}(u)), \; \forall u \in U.$$

Their intersection is denoted by $\{\widetilde{M} \cap \widetilde{N}\}$ and is defined by $\{\widetilde{M} \cap \widetilde{N}\}(u) = \min\{\widetilde{M}(u), \widetilde{N}(u)\}$, for any $u \in U$. The complement of $\widetilde{M}$ is denoted by $\sim \widetilde{M}$ and is defined by $\sim \widetilde{M}(u) = 1 - \widetilde{M}(u) \, \forall u \in U$. $\widetilde{M}$ is a fuzzy subset of $\widetilde{N}$ denoted by $\widetilde{M} \subseteq \widetilde{N}$ if

$$\forall u \in U, \, \widetilde{M}(u) \leq \widetilde{N}(u).$$

A specificity measure [7] Sp_f *is such that*

(i) $\forall$ *fuzzy subset* $\widetilde{M}$ *of* $U, Sp_f(\widetilde{M}) \in [0,1]$.

(ii) $Sp_f(\widetilde{M}) = 1 \iff \widetilde{M}$ *is singleton.*

(iii) $\widetilde{M} \subseteq \widetilde{N} \Rightarrow Sp_f(\widetilde{M}) \geq Sp_f(\widetilde{N})$.

Yager [22] *proposed one of such measure as*

$$Sp_f(\widetilde{M}) = \int_0^{\bar{\alpha}} \frac{d\alpha}{|\widetilde{M}_\alpha|},$$

where $\bar{\alpha} = \max_u \widetilde{M}(u)$, $\widetilde{M}_\alpha$ *is the* $\alpha-$*cut and* $|\bullet|$ *denotes the cardinality. The third axiom says that the wider a fuzzy set is, the less specific it becomes.*

Definition 2.3. [8] *A fuzzy soft set* $\widetilde{M}_A$ *over a universal set* U *with respect to a parameter set* A *is a parametrized collection of fuzzy sets from the collection of all fuzzy sets* $\mathcal{F}(U)$ *by the given mapping*

$$\widetilde{M}_A : A \rightarrow \mathcal{F}(U).$$

Let $\widetilde{M}_A$ *and* $\widetilde{N}_A$ *be two fuzzy soft sets over a common universe* U *and common parameter set* A. *The union of these two fuzzy soft sets is denoted by* $(\widetilde{M} \cup \widetilde{N})_A$ *and is given by*

$$(\widetilde{M} \cup \widetilde{N})_A(e) = \widetilde{M}_A(e) \cup \widetilde{N}_A(e), \quad \forall e \in A,$$

and we write, $\widetilde{M}_A \cup \widetilde{N}_A = (\widetilde{M} \cup \widetilde{N})_A$. *The* $\cup$ *on right hand side is a* $t-$*conorm between two fuzzy sets* $\widetilde{M}_A(e)$ *and* $\widetilde{N}_A(e)$. *Intersection of the two fuzzy soft sets is denoted by* $(\widetilde{M} \cap \widetilde{N})_A$ *and is defined by*

$$(\widetilde{M} \cap \widetilde{N})_A(e) = \widetilde{M}_A(e) \cap \widetilde{N}_A(e), \quad \forall e \in A;$$

and we write $\widetilde{M}_A \cap \widetilde{N}_A = (\widetilde{M} \cap \widetilde{N})_A$. *The* $\cap$ *on right hand side is a* $t-$*norm between two fuzzy sets* $\widetilde{M}_A(e)$ *and* $\widetilde{N}_A(e)$. *A fuzzy soft set* $\widetilde{M}_A$ *is called null fuzzy soft set if and only if*

$$\widetilde{M}_A(e) = \phi \ \forall e \in A,$$

i.e., a fuzzy soft set of U *mapping every element of* U *onto* 0 *for every parameter. A null fuzzy soft set is denoted by* $\widetilde{0}_A$. *A fuzzy soft set* $\widetilde{N}_A$ *is said to be a universal fuzzy soft set if and only if*

$$\widetilde{N}_A(e) = U \ \forall e \in A,$$

i.e., a fuzzy soft set of U *mapping every element of* U *onto the real number* 1 *for every parameter. A universal fuzzy soft set is denoted by* $\widetilde{U}_A$ *or* $\widetilde{1}_A$. *The complement of* $\widetilde{M}_A$ *is denoted by* $\widetilde{M}^c_A$ *where*

$$\widetilde{M}^c_A : A \rightarrow \mathcal{F}(U),$$

is a mapping given by

$$\widetilde{M}^c_A(e) = \left\{ \frac{(\widetilde{1}_A(e) - \widetilde{M}_A(e))(u)}{u} : u \in U \right\}.$$

Two fuzzy soft sets $\widetilde{M}_A$ *and* $\widetilde{N}_A$ *are said to be equal if and only if,* $\widetilde{M}_A(e) = \widetilde{N}_A(e), \ \forall e \in A$ *or else,* $\widetilde{M}_A \subseteq \widetilde{N}_A$ *and* $\widetilde{N}_A \subseteq \widetilde{M}_A$. *A fuzzy soft set* $\widetilde{M}_A$ *is said to be a fuzzy soft subset of* $\widetilde{N}_A$ *if and only if,* $\widetilde{M}_A(e) \subseteq \widetilde{N}_A(e)$, *for any* $e \in A$.

Definition 2.4. *We define a specificity measure for fuzzy soft set* $\widetilde{M}_A$, *where considering* $A = \{e_1, e_2, \ldots, e_n\}$ *is an ordered set, as a n-tuple vector specificity measure for fuzzy set* $\widetilde{M}_A(e)$ *for every* $e \in A$.

$$Sp_{fs}(\widetilde{M}_A) = (Sp_f(\widetilde{M}_A(e_1)), Sp_f(\widetilde{M}_A(e_2)), .., Sp_f(\widetilde{M}_A(e_n))).$$

Note that

(i) *for every fuzzy soft subset* $\widetilde{M}_A$ *of* U, $Sp_{fs}(\widetilde{M}_A) \in [0,1]^{|A|}$, *where* $|A|$ *is the number of parameter.*

(ii) $Sp_{fs}(\widetilde{M}_A) = (1, 1, ..., 1)$ *if* $\widetilde{M}_A$ *is singleton.*

(iii) $\widetilde{M}_A \subseteq \widetilde{N}_A \Rightarrow Sp_f(\widetilde{M}_A(e)) \geq Sp_f(\widetilde{N}_A(e))$ *for each* e.

Definition 2.5. *Let $\widetilde{M}$ be a fuzzy set over the universe $U=\{u_1,u_2,\ldots,u_n\}$. Let $V=\{v_1,v_2,\ldots,v_m\}$ be another universal set. Then the cylindrical extension of $\widetilde{M}$ over the universal set $U\times V$ is denoted by $ce(\widetilde{M})$ and is defined by*

$$ce(\widetilde{M})=\left\{\frac{\widetilde{M}(u_i)}{u_i\times v_j}:u_i\in U,v_j\in V\right\}.$$

Definition 2.6. *Let $\widetilde{M}$ be a fuzzy relation over the universal set $U\times V$. The projection of the fuzzy relation over the universal set U is denoted by $proj_U(\widetilde{M})$ and is defined by*

$$proj_U(\widetilde{M})=\left\{\frac{\max\limits_{v\in V}\{\widetilde{M}(u,v)\}}{u}:u\in U\right\}.$$

Example 2.7. *Let $U=\{u_1,u_2,u_3\}$ and $A=\{e_1,e_2,e_3\}$. We define a mapping $\widetilde{M}_A:A\to\mathcal{F}(U)$ by,*

$$\begin{aligned}\widetilde{M}_A(e_1)&=\{1/u_1,0.2/u_2,0.3/u_3\},\\ \widetilde{M}_A(e_2)&=\{0.2/u_1,0.4/u_2,0.8/u_3\},\\ \widetilde{M}_A(e_3)&=\{1/u_1,0.9/u_2,0.6/u_3\}.\end{aligned}$$

$(\widetilde{M}_A,A)$ is a fuzzy soft set and we denote it by $\widetilde{M}_A$. Similarly, we define $\widetilde{N}_A:A\to\mathcal{F}(U)$ by,

$$\begin{aligned}\widetilde{N}_A(e_1)&=\{0.25/u_2,0.37/u_3\},\\ \widetilde{N}_A(e_2)&=\{0.15/u_1,0.45/u_2,0.75/u_3\},\\ \widetilde{N}_A(e_3)&=\{0.95/u_1,0.25/u_2,0.55/u_3\}.\end{aligned}$$

Clearly, $(\widetilde{N}_A,A)$ is a fuzzy soft set and we denote it by $\widetilde{N}_A$. The union of $\widetilde{M}_A$ and $\widetilde{N}_A$ is denoted by $(\widetilde{M}\cup\widetilde{N})_A$ and is given by (taking t-conorm as max*(a,b))*

$$\begin{aligned}(\widetilde{M}\cup\widetilde{N})_A(e_1)&=\{1/u_1,0.25/u_2,0.37/u_3\},\\ (\widetilde{M}\cup\widetilde{N})_A(e_2)&=\{0.2/u_1,0.45/u_2,0.8/u_3\},\\ (\widetilde{M}\cup\widetilde{N})_A(e_3)&=\{1/u_1,0.9/u_2,0.6/u_3\}.\end{aligned}$$

The intersection of $\widetilde{M}_A$ and $\widetilde{N}_A$ is given by (taking t-norm as min*(a,b))*

$$\begin{aligned}(\widetilde{M}\cap\widetilde{N})_A(e_1)&=\{0.2/u_2,0.3/u_3\},\\ (\widetilde{M}\cap\widetilde{N})_A(e_2)&=\{0.15/u_1,0.4/u_2,0.75/u_3\},\\ (\widetilde{M}\cap\widetilde{N})_A(e_3)&=\{0.95/u_1,0.25/u_2,0.55/u_3\}.\end{aligned}$$

The complement of $\widetilde{M}_A$ is given below:

$$\begin{aligned}&\widetilde{M}^c_A(e_1)=\widetilde{1}_A(e_1)-\widetilde{M}_A(e_1)=\{0.8/u_2,0.7/u_3\},\\ &\widetilde{M}^c_A(e_2)=\{0.8/u_1,0.6/u_2,0.2/u_3\},\\ &\widetilde{M}^c_A(e_3)=\{0.1/u_2,0.4/u_3\}.\end{aligned}$$

Let us now consider a few properties of fuzzy soft sets. First of all, let us denote the collection of all fuzzy soft sets over the common universal set U and with respect to the common parameter set A by $\mathcal{FS}(A,U)$. We define binary operations '$\leq$' on $\mathcal{FS}(A,U)$ by

$$\widetilde{M}_A\leq\widetilde{N}_A\ \Leftrightarrow\ \widetilde{M}_A\subseteq\widetilde{N}_A\ \ \forall\widetilde{M}_A,\ \widetilde{N}_A\in\mathcal{FS}(A,U),$$

$$\widetilde{M}_A\vee\widetilde{N}_A\ \Leftrightarrow\ \widetilde{M}_A\cup\widetilde{N}_A\ \ \forall\widetilde{M}_A,\ \widetilde{N}_A\in\mathcal{FS}(A,U),$$

$$\widetilde{M}_A\wedge\widetilde{N}_A\ \Leftrightarrow\ \widetilde{M}_A\cap\widetilde{N}_A\ \ \forall\widetilde{M}_A,\ \widetilde{N}_A\in\mathcal{FS}(A,U),$$

and a unary operation $\sim$ known as complement operation on $\mathcal{FS}(A,U)$. Then, $\mathcal{FS}(A,U)$ forms a complete, distributive lattice satisfying De-morgan's law, with lowest element $\widetilde{0}_A$ and greatest element $\widetilde{1}_A$.

3 A logic of fuzzy soft sets

In this paper, we restrict our study to a propositional logic. In choosing truth functions for different connectives, we have considered that any such logic is an extension of the classical propositional logic. In the sequel, we note that proper choice of the semantics of the conjunction and implication, determines a whole propositional calculus.

Definition 3.1. *The cylindrical extension of $\widetilde{M}_A$ over $\mathcal{FS}(A \times B, U \times V)$ is denoted by $\widetilde{ce}(\widetilde{M}_A)$ and defined by*

$$\widetilde{ce}(\widetilde{M}_A) = \left\{ \frac{\widetilde{ce}(\widetilde{M}_A(e))}{(e,f)} \ \mid \ (e,f) \in A \times B \right\},$$

where, $\widetilde{ce}(\widetilde{M}_A(e))$ is the cylindrical extension of the fuzzy set $\widetilde{M}_A(e)$ on $U \times V$.

Example 3.2. *The cylindrical extension of $\widetilde{M}_A$ as given in Example 2.7 on $U \times V$ with parameter set $A \times B$ is given by (here, V and B are as defined in Example 3.4)*

$$\widetilde{ce}(\widetilde{M}_A)(e_1, f_1) = \left\{ \frac{1}{(u_1,v_1)}, \frac{1}{(u_1,v_2)}, \frac{1}{(u_1,v_3)}, \frac{0.2}{(u_2,v_1)}, \frac{0.2}{(u_2,v_2)}, \frac{0.2}{(u_2,v_3)}, \frac{0.3}{(u_3,v_1)}, \frac{0.3}{(u_3,v_2)}, \frac{0.3}{(u_3,v_3)} \right\},$$

$$\widetilde{ce}(\widetilde{M}_A)(e_1, f_2) = \left\{ \frac{1}{(u_1,v_1)}, \frac{1}{(u_1,v_2)}, \frac{1}{(u_1,v_3)}, \frac{0.2}{(u_2,v_1)}, \frac{0.2}{(u_2,v_2)}, \frac{0.2}{(u_2,v_3)}, \frac{0.3}{(u_3,v_1)}, \frac{0.3}{(u_3,v_2)}, \frac{0.3}{(u_3,v_3)} \right\},$$

$$\widetilde{ce}(\widetilde{M}_A)(e_1, f_3) = \left\{ \frac{1}{(u_1,v_1)}, \frac{1}{(u_1,v_2)}, \frac{1}{(u_1,v_3)}, \frac{0.2}{(u_2,v_1)}, \frac{0.2}{(u_2,v_2)}, \frac{0.2}{(u_2,v_3)}, \frac{0.3}{(u_3,v_1)}, \frac{0.3}{(u_3,v_2)}, \frac{0.3}{(u_3,v_3)} \right\}.$$

Other elements can be computed in a similar way.

Definition 3.3. *The fuzzy soft implication is interpreted as a fuzzy soft relation over $U \times V$. For given $\widetilde{M}_A \in \mathcal{FS}(A, U)$ and $\widetilde{N}_B \in \mathcal{FS}(B, V)$, we denote the fuzzy soft implication by $\widetilde{M}_A \to \widetilde{N}_B$ and is defined by*

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B) = \left\{ \frac{I(\widetilde{M}_A(e), \widetilde{N}_B(f))}{(e,f)} \ \mid (e,f) \in A \times B \right\},$$

where, 'I' is some fuzzy implication operation.

If we represent $\widetilde{M}_A = (\widetilde{M}_{e_1}, \widetilde{M}_{e_2}, \widetilde{M}_{e_3}, ..., \widetilde{M}_{e_m})$ and $\widetilde{N}_B = (\widetilde{N}_{f_1}, \widetilde{N}_{f_2}, \widetilde{N}_{f_3}, ..., \widetilde{N}_{f_n})$ then we can think $\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)$ as following

$$\begin{bmatrix} \widetilde{M}_{e_1} \to \widetilde{N}_{f_1} & \widetilde{M}_{e_1} \to \widetilde{N}_{f_2} & \cdots & \widetilde{M}_{e_1} \to \widetilde{N}_{f_n} \\ \widetilde{M}_{e_2} \to \widetilde{N}_{f_1} & \widetilde{M}_{e_2} \to \widetilde{N}_{f_2} & \cdots & \widetilde{M}_{e_2} \to \widetilde{N}_{f_n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{M}_{e_m} \to \widetilde{N}_{f_1} & \widetilde{M}_{e_m} \to \widetilde{N}_{f_2} & \cdots & \widetilde{M}_{e_m} \to \widetilde{N}_{f_n} \end{bmatrix},$$

where $\to$ represents an implication. Note here that fuzzy-soft implication basically adds a layer of uncertainty to the collection as compared to fuzzy implication. This is why it is better at handling uncertainty.

Here are some examples of fuzzy-soft implication induced by fuzzy implication.

$I_L(x, y) = min(1, 1 - x + y)$ induces

$$\widetilde{I}_L(\widetilde{M}_A, \widetilde{N}_B) = \left\{ \frac{I_L(\widetilde{M}_A(e), \widetilde{N}_B(f))}{(e,f)} \ \mid (e,f) \in A \times B \right\};$$

$I_P(x, y) = 1 - x + xy$ induces

$$\widetilde{I}_P(\widetilde{M}_A, \widetilde{N}_B) = \left\{ \frac{I_P(\widetilde{M}_A(e), \widetilde{N}_B(f))}{(e,f)} \ \mid (e,f) \in A \times B \right\},$$

and $I_Z(x, y) = max(1 - x, y)$ induces

$$\widetilde{I}_Z(\widetilde{M}_A, \widetilde{N}_B) = \left\{ \frac{I_Z(\widetilde{M}_A(e), \widetilde{N}_B(f))}{(e,f)} \ \mid (e,f) \in A \times B \right\}.$$

We list below some basic properties for $\widetilde{I}$ to satisfy:
(*I*1) *If* $\widetilde{M}_A \leq \widetilde{M}'_A$, *then* $\widetilde{I}(\widetilde{M}'_A, \widetilde{N}_B) \leq \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)$,
(*I*2) *If* $\widetilde{N}_B \leq \widetilde{N}'_B$, *then* $\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B) \leq \widetilde{I}(\widetilde{M}_A, \widetilde{N}'_B)$,
(*I*3) $\widetilde{I}(\widetilde{0}_A, \widetilde{N}_B) = \widetilde{1}_{A\times B}$,
(*I*4) $\widetilde{I}(\widetilde{1}_A, \widetilde{N}_B) = \widetilde{ce}(\widetilde{N}_B)$,
(*I*5) $\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B) \geq \widetilde{ce}(\widetilde{N}_B)$,
(*I*6) $\widetilde{I}(\widetilde{M}_A, \widetilde{M}_A) = \widetilde{1}_{A\times A}$,
(*I*7) $\widetilde{I}(\widetilde{M}_A, \widetilde{I}(\widetilde{N}_B, \widetilde{P}_C)) = \widetilde{I}(\widetilde{N}_B, \widetilde{I}(\widetilde{M}_A, \widetilde{P}_C))$,
(*I*8) $\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B) = \widetilde{1}_{A\times B}$ *if and only if* $\widetilde{M}_A \leq \widetilde{N}_B$,
(*I*9) $\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B) = \widetilde{I}(\widetilde{N}^c_B, \widetilde{M}^c_A)$.

Here, $\widetilde{I}_L$ *satisfies all of these properties, and both* $\widetilde{I}_P$ *and* $\widetilde{I}_Z$ *satisfies I1-I5, I7 and I9. We have seen here that these properties are consistent with the definitions given by the authors in* [2].

Example 3.4. *Let* $\widetilde{M}_A$ *be a fuzzy soft set as given in Example 2.7 and define* $\widetilde{N}_B$ *with the parameter set* $B=\{f_1, f_2, f_3\}$ *over the universe* $V=\{v_1, v_2, v_3\}$ *as given by*

$$\begin{aligned}\widetilde{N}_B(f_1) &= \{1/v_1, 0.25/v_2, 0.5/v_3\},\\ \widetilde{N}_B(f_2) &= \{0.2/v_1, 0.4/v_2, 0.98/v_3\},\\ \widetilde{N}_B(f_3) &= \{0.95/v_1, 0.3/v_2, 0.5/v_3\}.\end{aligned}$$

Then, $\widetilde{M}_A \to \widetilde{N}_B$ *is explicitly given below;*

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_1, f_1) = \left\{\frac{1.00}{(u_1,v_1)}, \frac{0.25}{(u_1,v_2)}, \frac{0.50}{(u_1,v_3)}, \frac{1.00}{(u_2,v_1)}, \frac{1.00}{(u_2,v_2)}, \frac{1.00}{(u_2,v_3)}, \frac{1.00}{(u_3,v_1)}, \frac{0.95}{(u_3,v_2)}, \frac{1.00}{(u_3,v_3)}\right\}.$$

Other elements can be computed in a similar way. Here, $I_L(x, y)=min(1, 1 - x + y)$ *is the fuzzy implication which is taken.*

Definition 3.5. *The projection of* $\widetilde{M}_{A\times B} \in \mathcal{FS}(A \times B, U \times V)$ *on* V *is denoted by* $\widetilde{proj}_V(\widetilde{M}_{A\times B})$ *and is defined by*

$$\widetilde{proj}_V(\widetilde{M}_{A\times B}) = \left\{\frac{\displaystyle\sum_{e\in A} \frac{(\widetilde{proj}_V(\widetilde{M}_{A\times B}(e, f)))}{card(A)}}{f} : f \in B\right\},$$

where, $\widetilde{proj}_V(\widetilde{M}_{A\times B}(e, f))$ *is a projection of the fuzzy set* $\widetilde{M}_{A\times B}(e, f)$ *on V, defined by*

$$\widetilde{proj}_V(\widetilde{M}_{A\times B}(e, f)) = \left\{\frac{\max\limits_{u_i\in U}\{(\widetilde{M}_{A\times B}(e, f))(u_i, v_j) : u_i \in U\}}{v_j} \;\middle|\; v_j \in V\right\}.$$

Example 3.6. *Let* $W=\{u_1, u_2\}$, $G=\{e_1, e_2\}$, $V=\{v_1, v_2, v_3\}$, $B=\{f_1, f_2, f_3\}$ *and a fuzzy soft relation be given by*

$$\begin{aligned}
\widetilde{M}_{G\times B}(e_1, f_1) &= \left\{\frac{1}{(u_1,v_1)}, \frac{0.8}{(u_1,v_2)}, \frac{0.7}{(u_1,v_3)}, \frac{0.9}{(u_2,v_1)}, \frac{0.1}{(u_2,v_2)}, \frac{0.2}{(u_2,v_3)}\right\},\\
\widetilde{M}_{G\times B}(e_1, f_2) &= \left\{\frac{0}{(u_1,v_1)}, \frac{0.3}{(u_1,v_2)}, \frac{0.6}{(u_1,v_3)}, \frac{0.8}{(u_2,v_1)}, \frac{0.9}{(u_1,v_2)}, \frac{0.4}{(u_2,v_3)}\right\},\\
\widetilde{M}_{G\times B}(e_1, f_3) &= \left\{\frac{0.55}{(u_1,v_1)}, \frac{0.63}{(u_1,v_2)}, \frac{0.89}{(u_1,v_3)}, \frac{0.98}{(u_2,v_1)}, \frac{0.15}{(u_2,v_2)}, \frac{0.6}{(u_2,v_3)}\right\},\\
\widetilde{M}_{G\times B}(e_2, f_1) &= \left\{\frac{0.45}{(u_1,v_1)}, \frac{0.76}{(u_1,v_2)}, \frac{0.59}{(u_1,v_3)}, \frac{0.18}{(u_2,v_1)}, \frac{0.23}{(u_2,v_2)}, \frac{0.8}{(u_2,v_3)}\right\},\\
\widetilde{M}_{G\times B}(e_2, f_2) &= \left\{\frac{0.76}{(u_1,v_1)}, \frac{0.83}{(u_1,v_2)}, \frac{0.49}{(u_1,v_3)}, \frac{0.42}{(u_2,v_1)}, \frac{0.67}{(u_2,v_2)}, \frac{0.9}{(u_2,v_3)}\right\},\\
\widetilde{M}_{G\times B}(e_2, f_3) &= \left\{\frac{1}{(u_1,v_1)}, \frac{0}{(u_1,v_2)}, \frac{0.8}{(u_1,v_3)}, \frac{0.6}{(u_2,v_1)}, \frac{0.4}{(u_2,v_2)}, \frac{0.1}{(u_2,v_3)}\right\}.
\end{aligned}$$

The projection of $\widetilde{M}_{A\times B}$ *on* V *will be given by*

$$\widetilde{proj}_V(\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(f_1) = \left\{ \frac{0.725}{v_1}, \frac{0.775}{v_2}, \frac{0.75}{v_3} \right\},$$

$$\widetilde{proj}_V(\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(f_2) = \left\{ \frac{0.78}{v_1}, \frac{0.865}{v_2}, \frac{0.75}{v_3} \right\},$$

$$\widetilde{proj}_V(\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(f_3) = \left\{ \frac{0.99}{v_1}, \frac{0.515}{v_2}, \frac{0.845}{v_3} \right\}.$$

3.1 Rule of inference

We now consider a sentence of the type

'If X is M then Y is N,'

where X and Y are linguistic variables. A rule is a general statement about anything in this world. As for example, 'If a person is **rich** then (s)he can **afford a house at a prime location**' could possibly be considered as a rule. With a specific observation X is M' (**immensely rich**) we can use the rule to conclude the ability of the person to own a house at a given prime location. We are interested in finding a mechanism to derive a specific conclusion from a general knowledge as per the following scheme:

$$\begin{array}{rl} \text{(General knowledge)} & M \implies N \\ \text{(Specific observation)} & M' \\ \text{(Conclusion)} & \qquad\quad N'. \end{array}$$

ALGORITHM MODUS PONENS:

Step 1 (Representation) We represent possibly imprecise concepts M and M' by fuzzy soft sets $\widetilde{M}_A$ and $\widetilde{M}'_A$ over U and with respect to common parameter set A and also represent possibly imprecise concept N by a fuzzy soft set $\widetilde{N}_B$ over V and parameter set B.

Step 2 (Translation) We translate the concept $M \implies N$ by a fuzzy soft implication $\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)$ using some definition of the same.

Step 3 (Cylindrical Extension) We compute $\widetilde{ce}(\widetilde{M}'_A)$, the cylindrical extension of $\widetilde{M}'_A$ over $U \times V$.

Step 4 (Composition) We compose $\widetilde{ce}(\widetilde{M}'_A)$ and $\widetilde{I}$ to form a fuzzy soft relation $\widetilde{S}_{A\times B}$ using some conjunction (, intersection) operation meant for fuzzy soft sets/relations.
Symbolically,

$$\widetilde{S}_{A\times B} = \widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B).$$

Step 5 (Projection) In order to generate a conclusion we take a projection of the fuzzy soft relation $\widetilde{S}_{A\times B}$.

$$\widetilde{N}'_B = \widetilde{proj}(\widetilde{S}_{A\times B}) = \left\{ \frac{\frac{1}{card(B)} \sum_{e\in A} (\widetilde{proj}(\widetilde{S}_{A\times B}(e, f)))}{f} \; : \; f \in B \right\},$$

where, $\widetilde{proj}(\widetilde{S}_{A\times B}(e, f))$ is the projection of fuzzy relation from $(U \times V)$ on U which happens to be a fuzzy soft set over V and parameter set B.

For a better understanding let us represent $\widetilde{S}_{A\times B}$ as

$$\begin{bmatrix} \widetilde{M}'_{e_1} * (\widetilde{M}_{e_1} \to \widetilde{N}_{f_1}) & \cdots & \widetilde{M}'_{e_1} * (\widetilde{M}_{e_1} \to \widetilde{N}_{f_n}) \\ \widetilde{M}'_{e_2} * (\widetilde{M}_{e_2} \to \widetilde{N}_{f_1}) & \cdots & \widetilde{M}'_{e_2} * (\widetilde{M}_{e_2} \to \widetilde{N}_{f_n}) \\ \vdots & \ddots & \vdots \\ \widetilde{M}'_{e_m} * (\widetilde{M}_{e_m} \to \widetilde{N}_{f_1}) & \cdots & \widetilde{M}'_{e_m} * (\widetilde{M}_{e_m} \to \widetilde{N}_{f_n}) \end{bmatrix},$$

where $*$ is the well known fuzzy $sup - min$ composition operation.

Consider a typical element of the above matrix $\widetilde{M}'_e * (\widetilde{M}_e \to \widetilde{N}_f)$ for some $e \in A$ and $f \in B$. Consider $\to$ to be the Lukasiewicz's implication and $*$ to be the $sup - Lukasiewicz\ min$, we have the following equation

$$\widetilde{M}'_e * (\widetilde{M}_e \to \widetilde{N}_f) = \begin{cases} \sup\limits_u(\widetilde{M}'_e) \\ \qquad for\ \{(u,v) : \widetilde{M}_e(u) < \widetilde{N}_f(v)\} \\ 0 \\ \qquad for\ \{(u,v) : \widetilde{M}_e(u) - \widetilde{M}'_e(u) > \widetilde{N}_f(v)\} \\ \\ \sup\limits_u(\widetilde{M}'_e - \widetilde{M}_e) + \widetilde{N}_f \\ \qquad for\ \{(u,v) : \widetilde{M}_e(u) - \widetilde{M}'_e(u) \leq \widetilde{N}_f(v)\}. \end{cases}$$

Observe that the less the difference between antecedent and input$(\widetilde{M}_e - \widetilde{M}'_e)$ at each parameter e the output (N') becomes more similar to the consequent(N) with some flat tail. Otherwise, the output (N') becomes more flat with degree 0 and/or $\sup_u(\widetilde{M}'_e)$. Thus we can see for each $f \in B$, each $e \in A$ contributes $\widetilde{M}'_e * (\widetilde{M}_e \to \widetilde{N}_f)$-much information to $\widetilde{N}'_f$ and for each $f \in B$, $\widetilde{N}'_f$ has $card(A)$ number of such contributing factor. We can take average/minimum of those factors to form $\widetilde{N}'_f$ but in practical cases with some domain knowledge we can assign some weight to each parameters and take a weighted average to get the best $\widetilde{N}'_f$ out of it.

Example 3.7. *Let us consider a simple example to illustrate our reasoning technique. For that, let the fuzzy soft set $\widetilde{M}_A$ be as given in Example 2.7 and that of $\widetilde{N}_B$ as given in Example 3.6. Now, we choose another fuzzy soft set $\widetilde{M}'_A$ over U to perform reasoning with fuzzy soft set. Let $\widetilde{M}'_A$ be as given by*

$$\widetilde{M}'_A(e_1) = \{0.95/u_1, 0.23/u_2, 0.27/u_3\},$$
$$\widetilde{M}'_A(e_2) = \{0.17/u_1, 0.42/u_2, 0.83/u_3\},$$
$$\widetilde{M}'_A(e_3) = \{0.05/u_1, 0.93/u_2, 0.56/u_3\}.$$

The fuzzy soft sets $\widetilde{M}_A$ and $\widetilde{N}_B$ are referred to here again

$$\widetilde{M}_A(e_1) = \{1/u_1, 0.2/u_2, 0.3/u_3\},$$
$$\widetilde{M}_A(e_2) = \{0.2/u_1, 0.4/u_2, 0.8/u_3\},$$
$$\widetilde{M}_A(e_3) = \{0/u_1, 0.9/u_2, 0.6/u_3\};$$

$$\widetilde{N}_B(f_1) = \{1/v_1, 0.25/v_2, 0.5/v_3\},$$
$$\widetilde{N}_B(f_2) = \{0.2/v_1, 0.4/v_2, 0.98/v_3\},$$
$$\widetilde{N}_B(f_3) = \{0.95/v_1, 0.3/v_2, 0.5/v_3\}.$$

Then, $\widetilde{M}_A \to \widetilde{N}_B$ will be given by (taking the fuzzy implication $I_L(x, y)$=min$(1, 1 - x + y)$),

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_1, f_1) = \left\{\frac{1.00}{(u_1, v_1)}, \frac{0.25}{(u_1 \times v_2)}, \frac{0.50}{(u_1, v_3)}, \frac{1.00}{(u_2, v_1)}, \frac{1.00}{(u_2, v_2)}, \frac{1.00}{(u_2, v_3)}, \frac{1.00}{(u_3, v_1)}, \frac{0.95}{(u_3, v_2)}, \frac{1.00}{(u_3, v_3)}\right\},$$

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_1, f_2) = \left\{\frac{0.20}{(u_1, v_1)}, \frac{0.40}{(u_1, v_2)}, \frac{0.98}{(u_1, v_3)}, \frac{1.00}{(u_2, v_1)}, \frac{1.00}{(u_2, v_2)}, \frac{1.00}{(u_2, v_3)}, \frac{0.90}{(u_3, v_1)}, \frac{1.00}{(u_3, v_2)}, \frac{1.00}{(u_3, v_3)}\right\},$$

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_1, f_3) = \left\{\frac{0.95}{(u_1, v_1)}, \frac{0.30}{(u_1, v_2)}, \frac{0.50}{(u_1, v_3)}, \frac{1.00}{(u_2, v_1)}, \frac{1.00}{(u_2, v_2)}, \frac{1.00}{(u_2, v_3)}, \frac{1.00}{(u_3, v_1)}, \frac{1.00}{(u_3, v_2)}, \frac{1.00}{(u_3, v_3)}\right\},$$

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_2, f_1) = \left\{\frac{1.00}{(u_1, v_1)}, \frac{1.00}{(u_1, v_2)}, \frac{1.00}{(u_1, v_3)}, \frac{1.00}{(u_2, v_1)}, \frac{0.85}{(u_2, v_2)}, \frac{1.00}{(u_2, v_3)}, \frac{1.00}{(u_3, v_1)}, \frac{0.45}{(u_3, v_2)}, \frac{0.70}{(u_3, v_3)}\right\},$$

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_2, f_2) = \left\{\frac{1.00}{(u_1, v_1)}, \frac{1.00}{(u_1, v_2)}, \frac{1.00}{(u_1, v_3)}, \frac{0.80}{(u_2, v_1)}, \frac{1.00}{(u_2, v_2)}, \frac{1.00}{(u_2, v_3)}, \frac{0.40}{(u_3, v_1)}, \frac{0.60}{(u_3, v_2)}, \frac{1.00}{(u_3, v_3)}\right\},$$

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_2, f_3) = \left\{\frac{1.00}{(u_1, v_1)}, \frac{1.00}{(u_1, v_2)}, \frac{1.00}{(u_1, v_3)}, \frac{1.00}{(u_2, v_1)}, \frac{0.90}{(u_2, v_2)}, \frac{1.00}{(u_2, v_3)}, \frac{1.00}{(u_3, v_1)}, \frac{0.50}{(u_3, v_2)}, \frac{0.70}{(u_3, v_3)}\right\},$$

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_3, f_1) = \left\{ \frac{1.00}{(u_1, v_1)}, \frac{1.00}{(u_1, v_2)}, \frac{1.00}{(u_1, v_3)}, \frac{1.00}{(u_2, v_1)}, \frac{0.35}{(u_2, v_2)}, \frac{0.60}{(u_2, v_3)}, \frac{1.00}{(u_3, v_1)}, \frac{0.65}{(u_3, v_2)}, \frac{0.90}{(u_3, v_3)} \right\},$$

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_3, f_2) = \left\{ \frac{1.00}{(u_1, v_1)}, \frac{1.00}{(u_1, v_2)}, \frac{1.00}{(u_1, v_3)}, \frac{0.30}{(u_2, v_1)}, \frac{0.50}{(u_2, v_2)}, \frac{1.00}{(u_2, v_3)}, \frac{0.60}{(u_3, v_1)}, \frac{0.80}{(u_3, v_2)}, \frac{1.00}{(u_3, v_3)} \right\},$$

$$\widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)(e_3, f_3) = \left\{ \frac{1.00}{(u_1, v_1)}, \frac{1.00}{(u_1, v_2)}, \frac{1.00}{(u_1, v_3)}, \frac{1.00}{(u_2, v_1)}, \frac{0.40}{(u_2, v_2)}, \frac{0.60}{(u_2, v_3)}, \frac{1.00}{(u_3, v_1)}, \frac{0.70}{(u_3, v_2)}, \frac{0.90}{(u_3, v_3)} \right\}.$$

The cylindrical extension of $\widetilde{M}'_A$ *is constructed and* $\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B)$ *is as given below:*

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_1, f_1) = \left\{ \frac{0.95}{(u_1, v_1)}, \frac{0.20}{(u_1, v_2)}, \frac{0.45}{(u_1, v_3)}, \frac{0.23}{(u_2, v_1)}, \frac{0.23}{(u_2, v_2)}, \frac{0.23}{(u_2, v_3)}, \frac{0.27}{(u_3, v_1)}, \frac{0.22}{(u_3, v_2)}, \frac{0.27}{(u_3, v_3)} \right\},$$

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_1, f_2) = \left\{ \frac{0.15}{(u_1, v_1)}, \frac{0.35}{(u_1, v_2)}, \frac{0.93}{(u_1, v_3)}, \frac{0.23}{(u_2, v_1)}, \frac{0.23}{(u_2, v_2)}, \frac{0.23}{(u_2, v_3)}, \frac{0.17}{(u_3, v_1)}, \frac{0.27}{(u_3, v_2)}, \frac{0.27}{(u_3, v_3)} \right\},$$

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_1, f_3) = \left\{ \frac{0.90}{(u_1, v_1)}, \frac{0.25}{(u_1, v_2)}, \frac{0.45}{(u_1, v_3)}, \frac{0.23}{(u_2, v_1)}, \frac{0.23}{(u_2, v_2)}, \frac{0.23}{(u_2, v_3)}, \frac{0.27}{(u_3, v_1)}, \frac{0.27}{(u_3, v_2)}, \frac{0.27}{(u_3, v_3)} \right\},$$

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_2, f_1) = \left\{ \frac{0.17}{(u_1, v_1)}, \frac{0.17}{(u_1, v_2)}, \frac{0.17}{(u_1, v_3)}, \frac{0.42}{(u_2, v_1)}, \frac{0.27}{(u_2, v_2)}, \frac{0.42}{(u_2, v_3)}, \frac{0.83}{(u_3, v_1)}, \frac{0.28}{(u_3, v_2)}, \frac{0.53}{(u_3, v_3)} \right\},$$

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_2, f_2) = \left\{ \frac{0.17}{(u_1, v_1)}, \frac{0.17}{(u_1, v_2)}, \frac{0.17}{(u_1, v_3)}, \frac{0.22}{(u_2, v_1)}, \frac{0.42}{(u_2, v_2)}, \frac{0.42}{(u_2, v_3)}, \frac{0.23}{(u_3, v_1)}, \frac{0.43}{(u_3, v_2)}, \frac{0.83}{(u_3, v_3)} \right\},$$

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_2, f_3) = \left\{ \frac{0.17}{(u_1, v_1)}, \frac{0.17}{(u_1, v_2)}, \frac{0.17}{(u_1, v_3)}, \frac{0.42}{(u_2, v_1)}, \frac{0.32}{(u_2, v_2)}, \frac{0.42}{(u_2, v_3)}, \frac{0.83}{(u_3, v_1)}, \frac{0.33}{(u_3, v_2)}, \frac{0.53}{(u_3, v_3)} \right\},$$

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_3, f_1) = \left\{ \frac{0.05}{(u_1, v_1)}, \frac{0.05}{(u_1, v_2)}, \frac{0.05}{(u_1, v_3)}, \frac{0.93}{(u_2, v_1)}, \frac{0.28}{(u_2, v_2)}, \frac{0.53}{(u_2, v_3)}, \frac{0.56}{(u_3, v_1)}, \frac{0.21}{(u_3, v_2)}, \frac{0.46}{(u_3, v_3)} \right\},$$

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_3, f_2) = \left\{ \frac{0.05}{(u_1, v_1)}, \frac{0.05}{(u_1, v_2)}, \frac{0.05}{(u_1, v_3)}, \frac{0.23}{(u_2, v_1)}, \frac{0.43}{(u_2, v_2)}, \frac{0.93}{(u_2, v_3)}, \frac{0.16}{(u_3, v_1)}, \frac{0.36}{(u_3, v_2)}, \frac{0.56}{(u_3, v_3)} \right\},$$

$$(\widetilde{ce}(\widetilde{M}'_A) \cap \widetilde{I}(\widetilde{M}_A, \widetilde{N}_B))(e_3, f_3) = \left\{ \frac{0.05}{(u_1, v_1)}, \frac{0.05}{(u_1, v_2)}, \frac{0.05}{(u_1, v_3)}, \frac{0.93}{(u_2, v_1)}, \frac{0.33}{(u_2, v_2)}, \frac{0.53}{(u_2, v_3)}, \frac{0.56}{(u_3, v_1)}, \frac{0.26}{(u_3, v_2)}, \frac{0.46}{(u_3, v_3)} \right\}.$$

Following the above discussion we compute $\widetilde{N}'_B$ *as given below:*

$$\begin{aligned} \widetilde{N}'_B(f_1) &= \{0.90/v_1,\ 0.26/v_2,\ 0.50/v_3\}, \\ \widetilde{N}'_B(f_2) &= \{0.23/v_1,\ 0.40/v_2,\ 0.90/v_3\}, \\ \widetilde{N}'_B(f_3) &= \{0.89/v_1,\ 0.23/v_2,\ 0.5/v_3\}. \end{aligned}$$

As an illustration, a pictorial representation of the above is given in the adjoining figures. Three parameter fuzzy soft sets $\widetilde{M}_A$ and $\widetilde{M}'_A$ for each parameters e_1, e_2 and e_3 are plotted in the first three figures one over the other. Pictures show that they are quite similar. Following them, the three parameter fuzzy soft sets $\widetilde{N}_B$ and $\widetilde{N}'_B$ are plotted for each parameter f_1, f_2 and f_3 in the next three figures. Here, $\widetilde{M}_A$ and $\widetilde{N}_B$ are related by the condition that 'if it is $\widetilde{M}_A$ then it is $\widetilde{N}_B$'. Now, fact is that it is not $\widetilde{M}_A$ but $\widetilde{M}'_A$, a slightly modified $\widetilde{M}_A$. Our problem is to compute the consequence of this fact based on the given condition. From the next three figures we observe that the consequence and the rule-consequent are almost similar. This shows that the result is significant. The procedure is such that a small change in the antecedent of the conditional statement produces a small change in the consequent.

4 Fuzzy soft reasoning — modus ponens

Now, let us consider a rule-based system defined by means of a set of r imprecise rules. With the above idea let us now design a modus ponens type rule of inference scheme with fuzzy soft set. Here, we have

$$\begin{array}{cccc} \widetilde{M}_A^i & \Longrightarrow & \widetilde{N}_B^i; & i=1,2,\ldots,r \\ \widetilde{M}_A' & & & \\ \hline & & \widetilde{N}'. & \end{array}$$

Let $U = \{u_1, u_2, \ldots, u_m\}$ and $V = \{v_1, v_2, \ldots, v_n\}$ be two universal sets. Let $A = \{e_1, e_2, \ldots, e_p\}$ and $B = \{f_1, f_2, \ldots, f_q\}$ be two parameter sets associated with U and V respectively. Also, let $\widetilde{M}_A^i = \left\{\widetilde{M}_A^i(e_1), \widetilde{M}_A^i(e_2), \ldots, \widetilde{M}_A^i(e_p)\right\}$, where $\widetilde{M}_A^i(e_j)$ is a fuzzy subset of U for each $j = 1, 2, \ldots p$. Again, let $\widetilde{N}_B^i$ is defined by $\widetilde{N}_B^i = \left\{\widetilde{N}_B^i(f_1), \widetilde{N}_B^i(f_2), \ldots \widetilde{N}_B^i(f_q)\right\}$, where $\widetilde{N}_B^i(f_k)$ is a fuzzy subset of V for each $k = 1, 2, \ldots, q$.

ALGORITHM A1:

Step 1 (Translation) For each $i = 1, 2, \ldots, r$, let us use ALGORITHM MODUS PONENS to get

$$\widetilde{N}_B'^i = \left\{\widetilde{N}_B'^i(f_1), \widetilde{N}_B'^i(f_2), \ldots, \widetilde{N}_B'^i(f_q)\right\},$$

where $\widetilde{N}_B'^i(f_k)$ is a fuzzy subset of V for each $k = 1, 2, \ldots, q$.

Step 2 (Aggregation) Now we have r number of fuzzy set $\left[\widetilde{N}_B'^1(f_1), \widetilde{N}_B'^2(f_1), \ldots, \widetilde{N}_B'^r(f_1)\right]$ to form $\widetilde{N}_B'(f_1)$ and $\left[\widetilde{N}_B'^1(f_2), \widetilde{N}_B'^2(f_2), \ldots, \widetilde{N}_B'^r(f_2)\right]$ to form $\widetilde{N}_B'(f_2)$ and so on. Here, we take specificity measure for the fuzzy soft set $\widetilde{N}_B'^i$ for each $i = 1, 2, \ldots, r$ as

$$Sp_{fs}(\widetilde{N}_B'^i) = \left[Sp_f(\widetilde{N}_B'^i(f_1)), Sp_f(\widetilde{N}_B'^i(f_2)), \ldots, Sp_f(\widetilde{N}_B'^i(f_q))\right].$$

Now, we find the i for which $(Sp_f(\widetilde{N}_B'^i(f_1)) + Sp_f(\widetilde{N}_B'^i(f_2)) + \ldots + Sp_f(\widetilde{N}_B'^i(f_q))$ is the highest and take the corresponding fuzzy soft set as the final output $\widetilde{N}_B'$. For example, $\widetilde{N}_B' = \left\{\widetilde{N}_B'^l(f_1), \widetilde{N}_B'^l(f_2), \ldots \widetilde{N}_B'^l(f_q)\right\}$ where

$$(Sp_f(\widetilde{N}_B'^l(f_1)) + Sp_f(\widetilde{N}_B'^l(f_2)) + \ldots + Sp_f(\widetilde{N}_B'^l(f_q)),$$

is the highest.

In case there is a tie, we take the weighted average to get to the desired $\widetilde{N}_B' = \left\{\widetilde{N}_B'(f_1), \widetilde{N}_B'(f_2), \ldots, \widetilde{N}_B'(f_q)\right\}$ with

$$\widetilde{N}_B'(f_k)(u) = \frac{\sum_{t=1}^{s} Sp_f(\widetilde{N}_B'^{i_t}(f_k))\widetilde{N}_B'^{i_t}(f_k)(u)}{\sum_{t=1}^{s} Sp_f(\widetilde{N}_B'^{i_t}(f_k)},$$

where, $k = 1, 2, \ldots, q$ and $i_1, i_2, \ldots, i_s$ correspond to the highest specificity.

As we observed, for $f \in B$ the smaller the set $\{(u, v) : \widetilde{M}_e(u) - \widetilde{M}_e'(u) \leq \widetilde{N}_f(v)\}$ for $e \in A$, the more flat/less specific is it's contributing factor $(\widetilde{M}_e' * (\widetilde{M}_e \to \widetilde{N}_f))$. Hence, we use specificity measure to filter the/those rule(s), for which most/ two of most e's are responsible to produce the largest

$$\{(u, v) : \widetilde{M}_e(u) - \widetilde{M}_e'(u) \leq \widetilde{N}_f(v)\},$$

for a particular input. This reflects the robustness of the system.

Let us illustrate the same with a typical example as considered by the authors in [17]. Here is a real-life problem of managing risk of hypertension, a chronic medical condition. Systolic blood pressure (SBP), diastolic blood pressure (DBP), age and body mass index (BMI) are taken as input parameters defining the physical condition of the patient and hypertension risk is the output parameter. In this paper, we take the knowledge base in terms of fuzzy IF-THEN rules as given in MEDDIAG [19] and use the concept of fuzzy soft inference to determine the risk of hypertension of a patient whose preconditions are known.

Example 4.1. *First let us suppose the collection of parameters* $A=\{e_1, e_2, e_3, e_4\}$ *where, by* e_1, e_2, e_3, e_4 *we mean* SBP, DBP, AGE, BMI *respectively and also suppose* $B=\{f_1\}$ *where* f_1=*Hypertention Risk. Let us consider the real interval* $[20, 280]$ *and discretize it to get* $U = \{u_1, u_2, \ldots, u_i, \ldots, u_{63}\}$ *where,*

$u_1=20$, $u_2=21.25$, $u_3=22.5$, $u_4=23.75$, $u_5=25$, $u_6=26.25$, $u_7=27.5$, $u_8=28.75$, $u_9=30$, $u_{10}=31.25$, $u_{11}=32.5$, $u_{12}=33.75$, $u_{13}=35$, $u_{14}=36.25$, $u_{15}=37.5$, $u_{16}=38.75$, $u_{17}=40$, $u_{18}=41.25$, $u_{19}=42.5$, $u_{20}=43.75$, $u_{21}=45$, $u_{22}=50$, $u_{23}=52.5$, $u_{24}=55$, $u_{25}=57.25$, $u_{26}=60$, $u_{27}=62.5$, $u_{28}=65$, $u_{29}=67.5$, $u_{30}=70$, $u_{31}=72.5$, $u_{32}=75$, $u_{33}=77.5$, $u_{34}=80$, $u_{35}=82.5$, $u_{36}=85$,
$u_{37}=87.5$, $u_{38}=90$, $u_{39}=92.5$, $u_{40}=95$, $u_{41}=100$, $u_{42}=105$, $u_{43}=110$, $u_{44}=115$, $u_{45}=120$, $u_{46}=125$, $u_{47}=130$, $u_{48}=135$, $u_{49}=140$, $u_{50}=150$, $u_{51}=160$, $u_{52}=170$, $u_{53}=180$, $u_{54}=190$, $u_{55}=200$, $u_{56}=210$, $u_{57}=220$, $u_{58}=230$, $u_{59}=240$, $u_{60}=250$, $u_{61}=260$, $u_{62}=270$, $u_{63}=280$.
Similarly, considering the real interval $[150, 300]$ *and do the same to get* $V = \{v_1, v_2, v_3, \ldots, v_r, \ldots, v_{41}\}$ *where,*
$v_1 = 150, v_2 = 152.5, v_3 = 155, v_4 = 157.5, v_5 = 160, v_6 = 162.5, v_7 = 165, v_8 = 167.5, v_9 = 170, v_{10} = 172.5,$
$v_{11} = 175, v_{12} = 177.5, v_{13} = 180, v_{14} = 182.5, v_{15} = 185, v_{16} = 187.5, v_{17} = 190, v_{18} = 192.5, v_{19} = 195, v_{20} = 197.5,$
$v_{21} = 200, v_{22} = 205, v_{23} = 210, v_{24} = 215, v_{25} = 220, v_{26} = 225, v_{27} = 230, v_{28} = 235, v_{29} = 240, v_{30} = 245,$
$v_{31} = 250, v_{32} = 255, v_{33} = 260, v_{34} = 265, v_{35} = 270, v_{36} = 275, v_{37} = 280, v_{38} = 285, v_{39} = 290, v_{40} = 295, v_{41} = 300$
We choose 4 copies of U *to distribute those 4 parameters. The fuzzy values mild, moderate and severe are used for* SBP *and* DBP*, and young, middle age, old and very old for* AGE*, and low, normal, high and very high for* BMI*. The three fuzzy sets corresponding to the parameter* SBP *are*

$$Mild = \left\{\frac{0.125}{u_{34}}, \frac{0.25}{u_{35}}, \frac{0.375}{u_{36}}, \frac{0.5}{u_{37}}, \frac{0.75}{u_{38}}, \frac{1}{u_{39}}, \frac{0.75}{u_{40}}, \frac{0.5}{u_{41}}, \frac{0.25}{u_{42}}\right\},$$
$$Moderate = \left\{\frac{0.25}{u_{40}}, \frac{0.5}{u_{41}}, \frac{0.75}{u_{42}}, \frac{1}{u_{43}}, \frac{0.75}{u_{44}}, \frac{0.5}{u_{45}}, \frac{0.25}{u_{46}}\right\},$$
$$Severe = \left\{\frac{0.16}{u_{44}}, \frac{0.33}{u_{45}}, \frac{0.5}{u_{46}}, \frac{0.66}{u_{47}}, \frac{1}{u_{48}}, \frac{0.66}{u_{49}}, \frac{0.33}{u_{50}}\right\},$$

The free fuzzy sets corresponding to the parameter DBP *are*

$$Mild = \left\{\frac{0.25}{u_{26}}, \frac{0.50}{u_{27}}, \frac{0.75}{u_{28}}, \frac{1}{u_{29}}, \frac{0.75}{u_{30}} \frac{0.5}{u_{31}}, \frac{0.25}{u_{32}}\right\},$$
$$Moderate = \left\{\frac{0.125}{u_{30}}, \frac{0.375}{u_{31}}, \frac{0.5}{u_{32}}, \frac{1}{u_{33}}, \frac{0.75}{u_{34}}, \frac{0.5}{u_{35}}, \frac{0.25}{u_{36}}\right\},$$
$$Severe = \left\{\frac{0.125}{u_{33}}, \frac{0.25}{u_{34}}, \frac{0.375}{u_{35}}, \frac{0.5}{u_{36}}, \frac{0.625}{u_{37}} \frac{0.75}{u_{38}}, \frac{1}{u_{39}}, \frac{0.75}{u_{40}}, \frac{0.5}{u_{41}}, \frac{0.25}{u_{42}}\right\},$$

The fuzzy sets corresponding to the parameter BMI *are*

$$Low = \left\{\frac{0.25}{u_2}, \frac{0.5}{u_3}, \frac{0.75}{u_4}, \frac{1}{u_5}, \frac{0.75}{u_6}, \frac{0.5}{u_7}, \frac{0.25}{u_8}\right\},$$
$$Normal = \left\{\frac{0.25}{u_6}, \frac{0.5}{u_7}, \frac{0.75}{u_8}, \frac{1}{u_9}, \frac{0.75}{u_{10}}, \frac{0.5}{u_{11}}, \frac{0.25}{u_{12}}\right\},$$
$$High = \left\{\frac{0.25}{u_{10}}, \frac{0.5}{u_{11}}, \frac{0.75}{u_{12}}, \frac{1}{u_{13}}, \frac{0.75}{u_{14}}, \frac{0.5}{u_{15}}, \frac{0.25}{u_{16}}\right\},$$
$$Very\ high = \left\{\frac{0.25}{u_{14}}, \frac{0.5}{u_{15}}, \frac{0.75}{u_{16}}, \frac{1}{u_{17}}, \frac{0.75}{u_{18}}, \frac{0.5}{u_{19}}, \frac{0.25}{u_{20}}\right\},$$

The fuzzy sets corresponding to the parameter set AGE are

$$Young = \left\{\frac{0.125}{u_2}, \frac{0.25}{u_3}, \frac{0.375}{u_4}, \frac{0.5}{u_5}, \frac{0.625}{u_6}, \frac{0.75}{u_7}, \frac{0.875}{u_8}, \frac{1}{u_9}, \frac{0.875}{u_{10}}, \frac{0.75}{u_{11}}, \frac{0.625}{u_{12}}, \frac{0.5}{u_{13}}, \frac{0.375}{u_{14}}, \frac{0.25}{u_{15}}, \frac{0.125}{u_{16}}\right\},$$

$$Middle\ age = \left\{ \frac{0.0625}{u_{11}}, \frac{0.125}{u_{12}}, \frac{0.1875}{u_{13}}, \frac{0.25}{u_{14}}, \frac{0.3125}{u_{15}}, \frac{0.375}{u_{16}}, \frac{0.4375}{u_{17}}, \frac{0.5}{u_{18}}, \frac{0.5625}{u_{19}}, \frac{0.625}{u_{20}}, \frac{0.6875}{u_{21}}, \frac{0.75}{u_{22}}, \frac{1}{u_{23}}, \frac{0.75}{u_{24}}, \frac{0.5}{u_{25}}, \frac{0.25}{u_{26}}\right\},$$

$$Old = \left\{\frac{0.33}{u_{23}}, \frac{0.5}{u_{24}}, \frac{0.66}{u_{25}}, \frac{0.83}{u_{26}}, \frac{1}{u_{27}}, \frac{0.75}{u_{28}}, \frac{0.5}{u_{29}}, \frac{0.25}{u_{30}}\right\},$$

$$Very\ old = \left\{\frac{0.25}{u_{26}}, \frac{0.5}{u_{27}}, \frac{0.75}{u_{28}}, \frac{1}{u_{29}}, \frac{0.833}{u_{30}}, \frac{0.75}{u_{31}}, \frac{0.66}{u_{32}}, \frac{0.5}{u_{33}}, \frac{0.33}{u_{34}}, \frac{0.17}{u_{35}}\right\}.$$

Likewise to distribute $f_1(Hypertension\ Risk)$ on V we use fuzzy sets, mild, moderate and severe and define them as following

$$Mild = \left\{ \frac{0.167}{v_2}, \frac{0.333}{v_3}, \frac{0.5}{v_4}, \frac{0.667}{v_5}, \frac{0.833}{v_6}, \frac{1}{v_7}, \frac{0.833}{v_8}, \frac{0.667}{v_9}, \frac{0.5}{v_{10}}, \frac{0.33}{v_{11}}, \frac{0.167}{v_{12}}\right\},$$

$$Moderate = \left\{\frac{0.25}{v_{10}}, \frac{0.5}{v_{11}}, \frac{0.75}{v_{12}}, \frac{1}{v_{13}}, \frac{0.75}{v_{14}}, \frac{0.5}{v_{15}}, \frac{0.25}{v_{16}}\right\},$$

$$Severe = \left\{ \frac{0.125}{v_{14}}, \frac{0.25}{v_{15}}, \frac{0.375}{v_{16}}, \frac{0.5}{v_{16}}, \frac{0.625}{v_{17}}, \frac{0.75}{v_{18}}, \frac{0.875}{v_{19}}, \frac{1}{v_{20}}, \frac{0.83}{v_{21}}, \frac{0.66}{v_{22}}, \frac{0.5}{v_{23}}, \frac{0.33}{v_{24}}, \frac{0.17}{v_{25}}\right\}.$$

We have chosen the rule base represented by Table 1 as used in [17]. We use 16 rules(80% of the data) as the knowledge base and 4 rules(20% of the data) as input for comparison. We use 3-fold cross-validation technique to test the algorithm's accuracy. First fold contains Rule Nos. 1,2, $\cdots$, 16 to use as the knowledge base and Rule Nos. 17, 18, 19, 20 to test the algorithm. The second fold contains Rule Nos. 1, 2, $\cdots$, 8 and Rule Nos. 13, 14, $\cdots$, 20 to use as the knowledge base and Rule Nos. 9, 10, 11, 12 to test the algorithm and likewise Rule Nos. 5, 6, $\cdots$, 20 to use as the knowledge base and Rule Nos. 1,2,3,4 as a testing data in the third fold. Each input is chosen in a way so as to compare our output with the results given in the said work for a better understanding of the effectiveness of our proposed reasoning system. Each antecedent fuzzy soft set consists of four parameters — SBP, DBP, AGE and BMI. There are 3 folds of four sets of inputs, linguistic values of which are given in each row of the Tables 2, 3, 4.

INPUT	SBP	DBP	AGE	BMI
1	mild	severe	old	normal
2	mild	moderate	young	high
3	severe	mild	middle age	very high
4	moderate	moderate	very old	normal

Table 2: 1st fold fuzzy soft inputs

INPUT	SBP	DBP	AGE	BMI
1	severe	mild	middle age	low
2	moderate	moderate	young	normal
3	mild	severe	old	high
4	mild	severe	very old	very high

Table 3: 2nd fold fuzzy soft inputs

Rule	If				then
	SBP	DBP	AGE	BMI	Hyper-tension risk
1	mild	severe	young	low	mild
2	moderate	moderate	middle age	normal	mild
3	severe	mild	old	high	moderate
4	severe	mild	very old	very high	severe
5	mild	mild	very old	normal	mild
6	moderate	moderate	old	very high	moderate
7	mild	severe	middle age	high	severe
8	moderate	mild	young	low	severe
9	severe	mild	middle age	low	moderate
10	moderate	moderate	young	normal	moderate
11	mild	severe	old	high	moderate
12	mild	severe	very old	very high	mild
13	moderate	moderate	young	low	mild
14	severe	mild	middle age	normal	severe
15	severe	mild	old	normal	severe
16	moderate	moderate	very old	low	mild
17	mild	severe	old	normal	mild
18	mild	moderate	young	high	mild
19	severe	mild	middle age	very high	severe
20	moderate	moderate	very old	normal	mild

Table 1: Fuzzy soft rules

INPUT	SBP	DBP	AGE	BMI
1	mild	severe	young	low
2	moderate	moderate	middle age	normal
3	severe	mild	old	high
4	severe	mild	very old	very high

Table 4: 3rd fold fuzzy soft inputs

In the following figures, we have compared our proposed algorithm's output with the corresponding rule-consequent from the knowledge base as given in Table 1. We provide the *IF* part of the data given in Table 1 as the input for the algorithm and compare the output with the corresponding rule-consequent. Figure 1 to Figure 4 represent the comparison for the Rule Nos. 17, 18, 19, 20. Figure 5 to Figure 8 represent the comparison for the Rule Nos. 9, 10, 11, 12 and Figure 9 to Figure 12 represent the comparison for the Rule No. 1, 2, 3, 4.

The system's output, a one-parameter fuzzy soft set $\widetilde{N}'_B$ is depicted by the black-coloured graph while, the red-coloured graph represents the fuzzy set *mild* has given in Figure 1, Figure 2, Figure 4, Figure 8, Figure 9 and Figure 10. Whereas, the fuzzy set *moderate* in Figure 3, Figure 6, Figure 7, Figure 11. Similarly, fuzzy set *severe* is represented in Figure 3 and Figure 12 as they are the rule-consequent to the corresponding inputs.

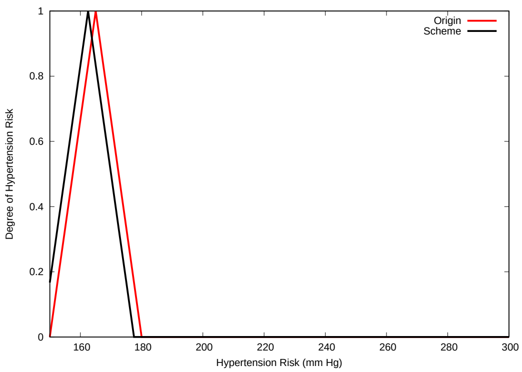

Figure 1: $\widetilde{M}_A(e_1)$ and $\widetilde{M}'_A(e_1)$

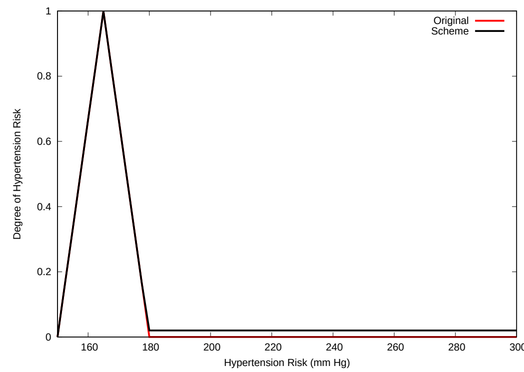

Figure 2: $\widetilde{M}_A(e_2)$ and $\widetilde{M}'_A(e_2)$

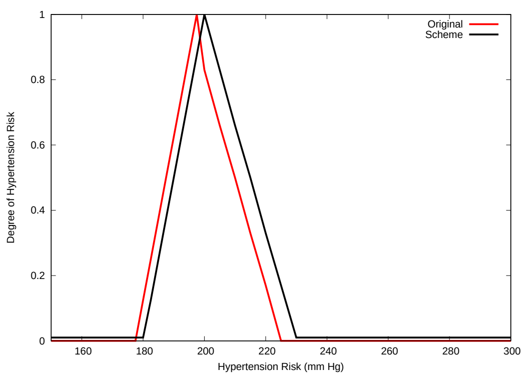

Figure 3: $\widetilde{M}_A(e_3)$ and $\widetilde{M}'_A(e_3)$

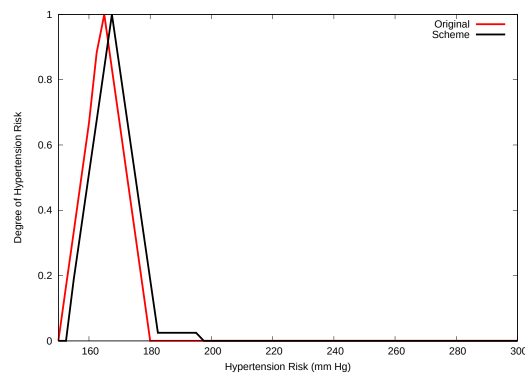

Figure 4: $\widetilde{N}_B(f_1)$ and $\widetilde{N}'_B(f_1)$

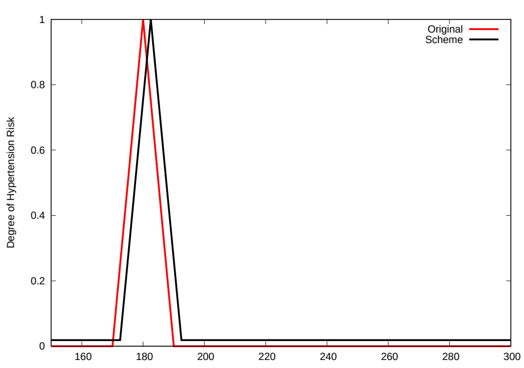

Figure 5: $\widetilde{N}_B(f_2)$ and $\widetilde{N}'_B(f_2)$

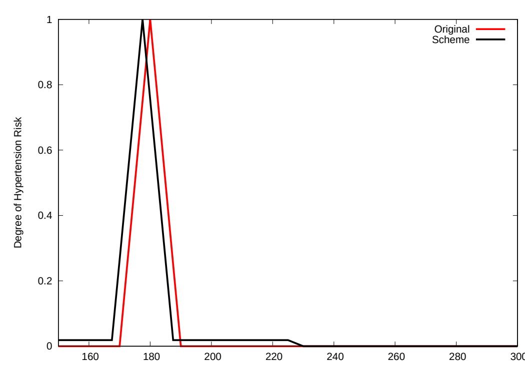

Figure 6: $\widetilde{N}_B(f_3)$ and $\widetilde{N}'_B(f_3)$

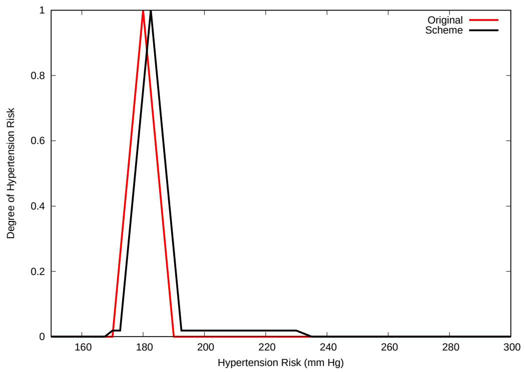

Figure 7: $\widetilde{N}_B^1$ and $\widetilde{N}_B'^1$

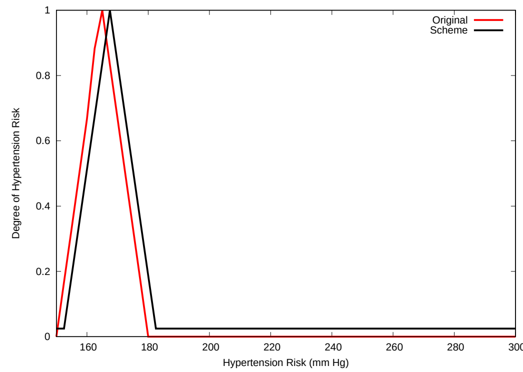

Figure 8: $\widetilde{N}_B^2$ and $\widetilde{N}_B'^2$

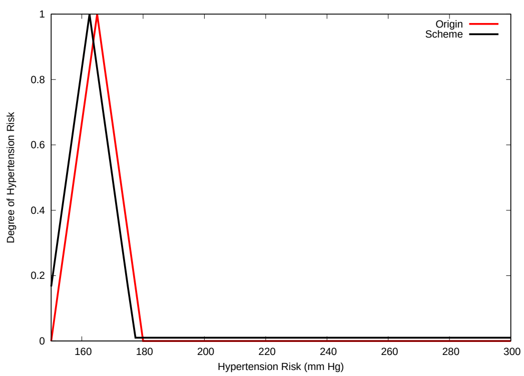

Figure 9: $\widetilde{N}_B^3$ and $\widetilde{N}_B'^3$

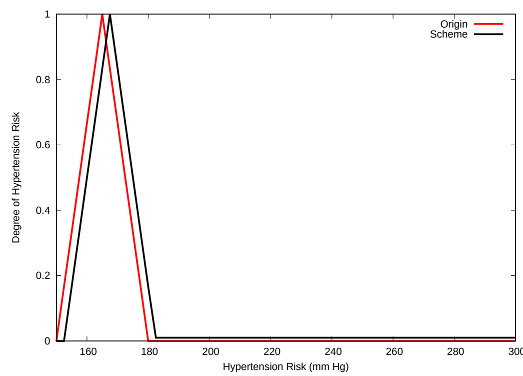

Figure 10: $\widetilde{N}_B^4$ and $\widetilde{N}_B'^4$

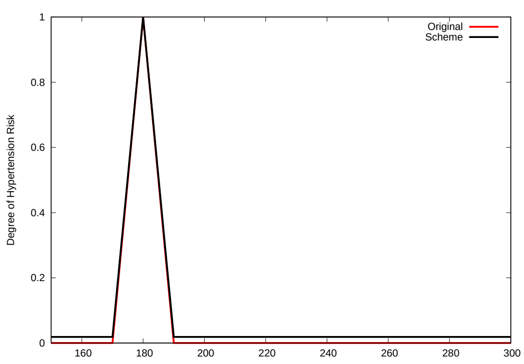

Figure 11: $\widetilde{N}_B^5$ and $\widetilde{N}_B'^5$

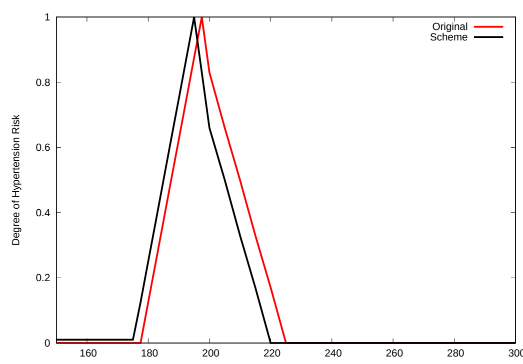

Figure 12: $\widetilde{N}_B^6$ and $\widetilde{N}_B'^6$

Table 5 contains the data from Table 2 [17], which the authors used as an input data for their fuzzy system (ALGORITHM SAR-HTN).

INPUT	SBP	DBP	AGE	BMI
1	mild	mild	young	high
2	moderate	mild	middle age	normal
3	mild	severe	young	high
4	moderate	moderate	middle age	normal
5	mild	severe	old	very high

Table 5: fuzzy soft inputs

Here, we present a pictorial comparison between a well established fuzzy system proposed in [17] and our proposed system. Their output data for the above mentioned input is presented in Table 5 [17] in red and labelled as SBF Scheme and that generated by our scheme is presented in black.

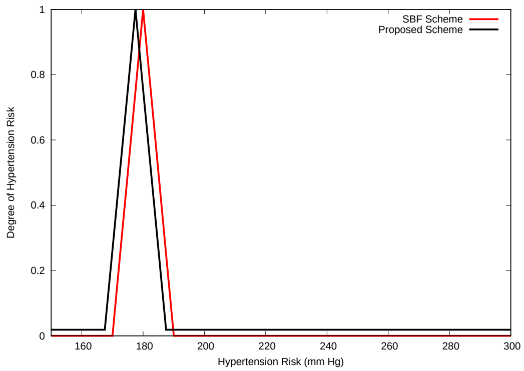

Figure 13: SAR-HTN vs Proposed Scheme

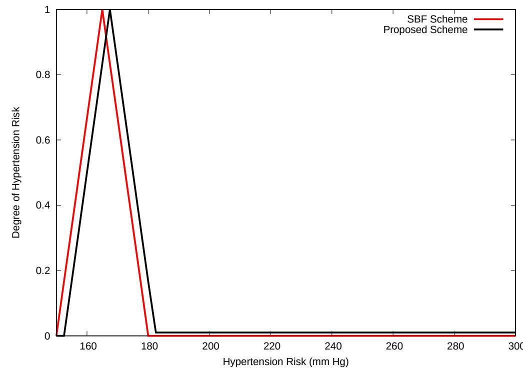

Figure 14: SAR-HTN vs Proposed Scheme

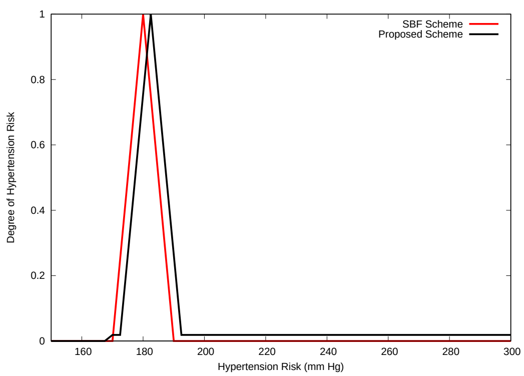

Figure 15: SAR-HTN vs Proposed Scheme

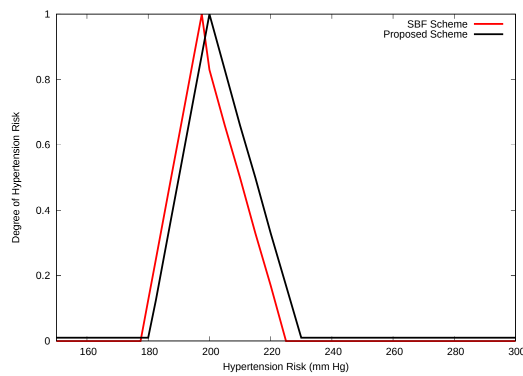

Figure 16: SAR-HTN vs Proposed Scheme

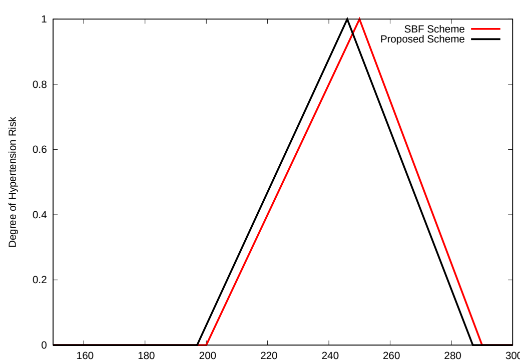

Figure 17: SAR-HTN vs Proposed Scheme

It can be seen from the comparison that our proposed scheme has performed well compared to a well established fuzzy system.

Here, we would like to mention the fact that our proposed algorithm is in an early stage of development. A detailed (analytical and practical) comparison with some fuzzy systems and also with type-2 fuzzy systems are needed. However, its potential is apparent. Where due to shortage of data, linear-regression fails to provide good results, the proposed algorithm can perform well. Linear regression is very sensitive to outliers (anomalies) while the proposed algorithm is not. Linear regression assumes that there is no relationship among independent variables which may cause multi-collinearity problem in the model. Main limitation of linear regression is the assumption of linearity between dependent and independent variables. The algorithm proposed here suffers from no such limitations as it is based on a one-to-one correspondence between the input data and the output data.

In the sequel, we took help of the logical connectives described in [2] and designed a well-known rule based system for inexact/uncertain reasoning. We use this system to simulate a real life medical diagnostic problem to prove it's competency. The above figures reveal that the results are potentially good. We cannot also deny the naturality and simplicity in the design of an uncertain system with fuzzy soft set.

5 Conclusions

The concept of a soft set initiated by Molodtsov was meant to model uncertainty (the boundary of the set depends on the parameters) in decision making using crisp, deterministic and precise mathematical constructs. Fuzzy soft set theory is a more general soft set model which makes descriptions of the objective world more general, realistic, practical and appropriate in most cases of decision making. We have studied the theory of fuzzy soft sets in a new perspective and initiated several results related to fuzzy soft sets. We have seen that the class of all fuzzy soft sets forms a bounded complete lattice which satisfies De Morgan's algebra. This prompted us to study approximate reasoning with fuzzy soft sets and examine the corresponding logic. Some properties of fuzzy implication for fuzzy soft sets are investigated. It has already been established that approximate reasoning is a significant topic of research because of its scope of applications in different forms of decision making. It is hoped that with modelling of modus ponens using fuzzy soft set theory, decision making under uncertainty can be made more versatile. It has been shown that our proposal could be moulded/tuned substantially in order to consider approximate reasoning with fuzzy soft set. This study is aimed at designing a medical diagnostic support system for the management of hypertension. It has the potential to assist medical experts in the tedious and complicated task of diagnosing hypertension and the designed system can provide schemes that will assist medical personnel in the process of offering primary health care to the patients.

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