Iranian Journal of Fuzzy Systems

Volume 19, Number 5, (2022), pp. 79-94

Original paper

University of Sistan and Baluchestan

An extended MABAC method based on prospect theory for multiple attribute group decision making under probabilistic uncertain linguistic environment

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Abstract

Cyber security is a hot topic in recent years and one of the main performances in the decision-making is the choice of cyber security service providers (CSSPs). For enterprises and government units, the selection of appropriate NSSPs needs to be measured and evaluated from multiple perspectives, which is obviously a multiple attribute decision making (MADM) or multiple attribute group decision making (MAGDM). In this paper, the traditional MABAC method is improved by using the prospect theory, and the evaluation information is collected and sorted by using PULTS. The improved PUL-PT-MABAC method can not only deal with the uncertainty well, but also fully consider the influence of the psychological state of decision maker (DM) on the decision result. More importantly, we apply the improved PUL-PT-MABAC model to the selection of NSSPs and prove the reliability of this proposed method by taking advantage of comparative analysis with three existed methods. These results show that the model can deal with practical problems, and has good practicability and science.

Keywords: Multi-attribute group decision making (MAGDM), MABAC method, probabilistic uncertain linguistic term set, prospect theory, cyber security service providers (CSSPs).

1 Introduction

The multiple attribute decision making (MADM) and multiple attribute group decision making (MAGDM) are the research hotspots in recent years, because we can easily meet the situation that we need to choose the optimal solution after comprehensive consideration from multiple aspects in our life and work [26, 31, 35, 46, 49]. The DMs usually have different psychological perception to the expected profit and the expected loss in the decision-making process. Moreover, this kind of psychological perception is reflected differently in different types of DMs. Therefore, a more reasonable and scientific MADM method should fully consider the DM's psychological perception. Unfortunately, most MADM methods do not take this influence factor into account. Prospect theory (PT) [15] is a method of making decisions by treating gains and losses differently. At the same time, MABAC [32] is a method to evaluate the advantages and disadvantages of alternatives by measuring the distance between each alternative and the border approximation area (BAA) which involves geometric averages. MABAC method has the advantages of simplicity, easy to understand and strong operability, compared with other methods. In recent years, it has received attention from all sides. Liu and Zhang [25] defined some operational rules of normal wiggly hesitant fuzzy set (NWHFS) and further established the MABAC model and PT-based MABAC model under NWHFS environment. Gu, Zheng, Tian and Xu [7] relied on PT to construct a relatively simple decision model in the probabilistic linguistic environment. Lu, Zhao, Khan and

Corresponding Author: C. Wei
Received: April 2021; Revised: July 2022; Accepted: August 2022.
https://doi.org/10.22111/IJFS.2022.7158

Uthansakul [27] selected hesitant fuzzy number as the tool of assessing information and created PT-based MADM model to answer the sustainable development of maritime transport. However, there is still a gap in the improvement of MABAC using PT and PULTS. Therefore, in this paper we decide to build a PT-based MABAC method under probabilistic uncertain linguistic environment as the research goal of this paper. This improved PUL-PT-MABAC method is based on the traditional MABAC method and incorporates the concept of PT. In other words, the new method fully considers the influence of DMs' cognition on decision-making, and introduces parameters to reflect DMs' different psychological perception of benefits and losses, so as to ensure that the MAGDM method is more in line with the actual decision-making situation. More importantly, all assessment information in this method will be collected using uncertain linguistic term, and will be collated using relevant knowledge of PULTS. The advantages of this approach are mainly that linguistic terms are easier to be understood and expressed by DMs, and PULTS can better cope with the complexity and uncertainty of information. The innovativeness of the paper can be summarized as follows: (1) the PUL-PT-MABAC method is proposed to solve the probabilistic uncertain linguistic MAGDM problems; (2) a case study for cyber security service providers (CSSPs) selection is used to show the developed approach; and (3) some comparative studies are provided with the existing methods. In order to do so, the remainder of this paper is arranged in the following way. Firstly, we tend to introduce the basic knowledge of PULTS in Section 2. In Section 3, we briefly reviewed prospect theory to lay the foundation for the PUL-PT-MABAC model presented in this paper. The improved PUL-PT-MABAC method is established in Section 4. Furthermore, in Section 5, we attempt to apply the improved PUL-PT-MABAC model to the selection of NSSPs and have the satisfactory result. For further proving the reliability of this proposed method, in Section 6, three existed methods are selected for comparative analysis with the method presented in this paper. Finally, we summarize the content of this paper and look forward to the future research in Section 7.

2 Literature review

The increasing complexity of the environment makes MADM method or MAGDM method in the real number environment unable to deal with the uncertainty in the real problem. From the above literatures, we can also see that it has become a normal practice to use fuzzy set theory (FST) to expand the traditional MADM or MAGDM method. The fuzzy set theory originated from fuzzy set (FS) proposed by Professor Zadeh [45]. Afterward, on this basis, it evolved intuitionistic fuzzy set (IFS [1], picture fuzzy set (PFS) [3], hesitant fuzzy set (HFS) and so on. In the decision-making problem, decision makers (DMs) are more accustomed to using language to describe the qualitative indicators, so the linguistic term set (LTS) has been widely concerned as soon as it was proposed. There are also extensive researches on the combination of LTS and FST, such as probabilistic linguistic term set (PLTS) [33], probabilistic uncertain linguistic term set (PULTS) [22]. Pang, Wang and Xu [33] first proposed PLTS which described linguistic term (LT) by utilizing different probabilities and proved its operational laws and basic operators. Luo, Zeng and Chen [28] further completed the content of PLTS in correlation coefficient. Su, Zhao, Wei and Chen [36] defined the PT-TODIM method for probabilistic linguistic MAGDM. Wu and Zhang [43] focused on the security of water and attempt to complete the probabilistic linguistic TODIM. Wei, Wei and Guo [42] built the EDAS method for probabilistic linguistic MAGDM. Mo [29] brought D number theory into PLTS and finally proposed a decision-making model for emergency decision-making. Du and Liu [5] made use of PLTS to expand dual Muirhead mean (DMM) operators and constructed VIKOR decision-making method under probabilistic linguistic environment. Lin, Xu, Zhai and Yao [22] created PULTS which was similar to PLTS, but used uncertain linguistic term (ULT) instead of LT. Xie, Ren, Xu and Wang [44] proposed probabilistic uncertain linguistic preference relation and researched its application in the decision-making of virtual reality. Lei, Wei, Wu, Wei and Guo [18] established probabilistic uncertain linguistic QUALIFLEX method for the choice of green supplier. Bashir, Ali and Rashid [2] defined new operations, operators and some new measure to further popularize PULTS. In addition, [41] defined new similarity measures, such as, PULWDSM and PULWGDSM. He, Wei and Chen [9] built the Bidirectional projection method for probabilistic uncertain linguistic MAGDM. He, Wei and Chen [10] developed the Taxonomy method for probabilistic uncertain linguistic MAGDM. In [37] defined the Probabilistic uncertain linguistic EDAS method based on prospect theory. Wang, Wei, Lu, Wu, Wei and Chen [38] built the GRP and CRITIC method for probabilistic uncertain linguistic MAGDM. Zhao, Gao, Wei, Wei and Guo [50] built the probabilistic uncertain linguistic TODIM method based on prospect theory.

At present, there are many methods for solving MADM or MAGDM, including the Interactive Multi-Criteria Decision Making (TODIM) method [21, 47, 52, 53], the multiple attributive border approximation area comparison (MABAC) method [4, 19] and EDAS method [11, 12, 16, 17] and so on. These methods have their own characteristics. For instance, MABAC is a method to evaluate the advantages and disadvantages of alternatives by measuring the distance between each alternative and the border approximation area (BAA) which involves geometric averages. MABAC method has the advantages of simplicity, easy to understand and strong operability, compared with other methods. Zhao, Wei, Chen and Wei [51] built the intuitionistic fuzzy MABAC method based on cumulative prospect theory for MAGDM. Gong, Li, Yin and Liu [6] improved MABAC for q-rung orthopair fuzzy MAGDM about teaching evaluation in higher education. Liu and Cheng [23] rebuilt the MABAC method in line with regret theory (RT) and probability multi-valued neutrosophic set. Jiang, Wei and Guo [13] built the Picture fuzzy MABAC method based on prospect theory for MAGDM. Liu, Hou, Liu and Lin [24] believed that the integration of bipolar fuzzy set and MABAC was an effective assessment tool to resolve occupational health and safety. Z-number and RT were used to supplement the MBAC method in the model. Zhang, Wei and Chen [48] built the CPT-MABAC method for spherical fuzzy MAGDM. Shen, Wang, Qiao and Wang [34] built the new MABAC model with q-ROFNs. Liao, Gao, Wei and Chen [20] defined the CPT-MABAC-based MAGDM method with probabilistic hesitant fuzzy information.

3 Preliminary knowledge

It has been studied by many scholars to define rules that tie language and mathematics closely together. The key knowledge about PULTS is shown in the following.

Definition 3.1. [30] *There is a linguistic term set (LTS)* $\mathrm{X} = \{\chi_{\lambda} | \lambda = -\hat{\Upsilon}, \cdots, -2, -1, 0, 1, 2, \cdots, \hat{\Upsilon}\}$, *where* χ_{λ} *is a linguistic term (LT) corresponding to a description. For example,* $\mathrm{X} = \{\chi_{-3} = \text{extremely poor}, \chi_{-2} = \text{very poor}, \chi_{-1} = \text{poor}, \chi_0 = \text{medium}, \chi_1 = \text{great}, \chi_2 = \text{very great}, \chi_3 = \text{extremely great}\}$ *is a LTS including seven elements. For realizing the transformation from a LT* χ_{λ} *to a crisp* Ω, *the transfer function* $\ddot{T}$ *is created as:*

$$\begin{aligned} &\ddot{T} : \left[\chi_{-\hat{\Upsilon}}, \chi_{\hat{\Upsilon}}\right] \to [0,1], \\ &\ddot{T}\left(\chi_{\lambda}\right) = \tfrac{\lambda + \hat{\Upsilon}}{2\hat{\Upsilon}} = \Omega. \end{aligned} \tag{1}$$

At the same time, it is essential to create another transfer function $\ddot{T}^{-1}$ *(like Eq. (2)) which does the opposite of* $\ddot{T}$.

$$\begin{aligned} &\ddot{T}^{-1} : [0,1] \to \left[\chi_{-\hat{\Upsilon}}, \chi_{\hat{\Upsilon}}\right], \\ &\ddot{T}^{-1}\left(\Omega\right) = \chi_{(2\Omega-1)\hat{\Upsilon}} = \chi_{\lambda}. \end{aligned} \tag{2}$$

Based on the conception of LTS, the probabilistic uncertain linguistic term set (PULTS) is defined by Lin, Xu, Zhai and Yao [22].

Definition 3.2. [22] *If there is a LTS* $\mathrm{X} = \left\{\chi_{\lambda} \middle| \lambda = -\hat{\Upsilon}, \cdots, -2, -1, 0, 1, 2, \cdots, \hat{\Upsilon}\right\}$, *the PULTS can be described as:*

$$Pul\left(\widehat{p}\right) = \left\{ \left[\chi_L^{(\ell)}, \chi_U^{(\ell)}\right] \left(\widehat{p}^{(\ell)}\right) \middle| \widehat{p}^{(\ell)} \geq 0; \ell = 1, 2, \cdots, \#Pul\left(\widehat{p}\right); \sum_{\ell=1}^{\#Pul\left(\widehat{p}\right)} \widehat{p}^{(\ell)} \leq 1 \right\}. \tag{3}$$

Where is a probabilistic uncertain linguistic term (PULT) $\left[\chi_L^{(\ell)}, \chi_U^{(\ell)}\right]\left(\widehat{p}^{(\ell)}\right)$ *including the lower limit* $\chi_L^{(\ell)}$ *and the upper limit* $\chi_U^{(\ell)}$ *(*$\chi_L^{(\ell)}, \chi_U^{(\ell)} \in \mathrm{X}$ *and* $\chi_L^{(\ell)} \leq \chi_U^{(\ell)}$*), and* $\widehat{p}^{(\ell)}$ *is also an item of PULT to depict the probability of uncertain linguistic term (ULT)*$\left[\chi_L^{(\ell)}, \chi_U^{(\ell)}\right]$. *Final,* $\#Pul\left(\widehat{p}\right)$ *is the total number of different ULT.*

When different ULT are not independent units, it is necessary to split the non-basic ULT into base units and merge the same base units. For example, the element $[\chi_{-3}, \chi_{-1}]\,(0.7)$ of PULTS $Pul\left(\widehat{p}\right) = \left\{\left[\chi_L^{(\ell)}, \chi_U^{(\ell)}\right]\left(\widehat{p}^{(\ell)}\right) \middle| \widehat{p}^{(\ell)} \geq 0; \ell = 1, 2, \cdots, \#Pul\left(\widehat{p}\right)\right\}$ can be divided into two base units, $[\chi_{-3}, \chi_{-2}]\,(0.35)$ as well as $[\chi_{-2}, \chi_{-1}]\,(0.35)$. And if there is another PULT $[\chi_{-3}, \chi_{-2}]\,(0.2)$ in the PULTS $Pul\left(\widehat{p}\right) = \left\{\left[\chi_L^{(\ell)}, \chi_U^{(\ell)}\right]\left(\widehat{p}^{(\ell)}\right) \middle| \widehat{p}^{(\ell)} \geq 0; \ell = 1, 2, \cdots, \#Pul\left(\widehat{p}\right)\right\}$, the probability of ULT $[\chi_{-3}, \chi_{-2}]$ will change to 0.55. In other words, the adjusted PULTS contains elements $[\chi_{-2}, \chi_{-1}]\,(0.55)$ and $[\chi_{-2}, \chi_{-1}]\,(0.35)$.

Definition 3.3. [22] $\bar{p}^{(\ell)} = \widehat{p}^{(\ell)} \Big/ \sum_{\ell=1}^{\#Pul\left(\widehat{p}\right)} \widehat{p}^{(\ell)}$ *is used to standardize the adjusted PULTS*

$$Pul\left(\widehat{p}\right) = \left\{ \left[\chi_L^{(\ell)}, \chi_U^{(\ell)}\right] \left(\widehat{p}^{(\ell)}\right) \middle| \widehat{p}^{(\ell)} \geq 0; \ell = 1, 2, \cdots, \#Pul(\bar{p}); \sum_{\ell=1}^{\#Pul(\bar{p})} \bar{p}^{(\ell)} \leq 1 \right\},$$

to the standardized PULTS

$$Pul(\bar{p})=\left\{\left[\chi_L^{(\ell)},\chi_U^{(\ell)}\right]\left(\bar{\bar{p}}^{(\ell)}\right)\middle|\bar{\bar{p}}^{(\ell)}\geq 0;\ell=1,2,\cdots,\#Pul(\bar{p});\sum_{\ell=1}^{\#Pul(\bar{p})}\bar{p}^{(\ell)}=1\right\}.$$

Definition 3.4. [22] *If there are two PULTSs* $Pul_1\left(\widehat{p}_1\right)=\left\{\left[\chi_{L_1}^{(\ell)},\chi_{U_1}^{(\ell)}\right]\left(\widehat{p}_1^{(\ell)}\right)\middle|\widehat{p}_1^{(\ell)}\geq 0;\ell=1,2,\cdots,\#Pul\left(\widehat{p}_1\right)\right\}$ *as well as* $Pul_2\left(\widehat{p}_2\right)=\left\{\left[\chi_{L_2}^{(\ell)},\chi_{U_2}^{(\ell)}\right]\left(\widehat{p}_2^{(\ell)}\right)\middle|\widehat{p}_2^{(\ell)}\geq 0;\ell=1,2,\cdots,\#Pul\left(\widehat{p}_2\right)\right\}$*, and* $\#Pul_1\left(\widehat{p}_1\right)>\#Pul_2\left(\widehat{p}_2\right)$*, the approach to ensure calculation of PULTS is to add* $\#Pul_1\left(\widehat{p}_1\right)-\#Pul_2\left(\widehat{p}_2\right)$ *minimal ULTs of* $Pul_2\left(\widehat{p}_2\right)$ *as its own elements and probabilities of these ULTs are zero.*

Definition 3.5. [22] *If there is a PULTS* $Pul\left(\widehat{p}\right)=\left\{\left[\chi_L^{(\ell)},\chi_U^{(\ell)}\right]\left(\widehat{p}^{(\ell)}\right)\middle|\widehat{p}^{(\ell)}\geq 0;\ell=1,2,\cdots,\#Pul\left(\widehat{p}\right)\right\}$*, Eq. (4) and Eq. (5) represent the excepted degree (ED)* $\ddot{E}\left(Pul\left(\widehat{p}\right)\right)$ *and the standard deviation degree (DD)* $\ddot{D}\left(Pul\left(\widehat{p}\right)\right)$*, respectively.*

$$\ddot{E}\left(Pul\left(\widehat{p}\right)\right)=\frac{\sum_{\ell=1}^{\#Pul\left(\widehat{p}\right)}\left(\frac{\ddot{T}\left(\chi_L^{(\ell)}\right)\cdot\widehat{p}^{(\ell)}+\ddot{T}\left(\chi_U^{(\ell)}\right)\cdot\widehat{p}^{(\ell)}}{2}\right)}{\sum_{\ell=1}^{\#Pul\left(\widehat{p}\right)}\widehat{p}^{(\ell)}}. \tag{4}$$

$$\ddot{D}\left(Pul\left(\widehat{p}\right)\right)=\frac{\sqrt{\sum_{\ell=1}^{\#Pul\left(\widehat{p}\right)}\left(\frac{\ddot{T}\left(\chi_L^{(\ell)}\right)\cdot\widehat{p}^{(\ell)}+\ddot{T}\left(\chi_U^{(\ell)}\right)\cdot\widehat{p}^{(\ell)}}{2}-\ddot{E}\left(Pul\left(\widehat{p}\right)\right)\right)^2}}{\sum_{\ell=1}^{\#Pul\left(\widehat{p}\right)}\widehat{p}^{(\ell)}}. \tag{5}$$

At the same time, the ED and the DD are the way to determine the relationship of size between the PULTS $Pul_1\left(\widehat{p}_1\right)=\left\{\left[\chi_{L_1}^{(\ell)},\chi_{U_1}^{(\ell)}\right]\left(\widehat{p}_1^{(\ell)}\right)\middle|\widehat{p}_1^{(\ell)}\geq 0;\ell=1,2,\cdots,\#Pul\left(\widehat{p}_1\right)\right\}$ *and the PULTS*

$$Pul_2\left(\widehat{p}_2\right)=\left\{\left[\chi_{L_2}^{(\ell)},\chi_{U_2}^{(\ell)}\right]\left(\widehat{p}_2^{(\ell)}\right)\middle|\widehat{p}_2^{(\ell)}\geq 0;\ell=1,2,\cdots,\#Pul\left(\widehat{p}_2\right)\right\}.$$

(1) *If* $\ddot{E}\left(Pul_1\left(\widehat{p}_1\right)\right)>\ddot{E}\left(Pul\left(\widehat{p}_2\right)\right)$*, then* $Pul\left(\widehat{p}_1\right)>Pul_2\left(\widehat{p}_2\right)$*;*
(2) *If* $\ddot{E}\left(Pul_1\left(\widehat{p}_1\right)\right)=\ddot{E}\left(Pul_2\left(\widehat{p}_2\right)\right)$*, then compare the value of DD:*
  i. If $\ddot{D}\left(Pul_1\left(\widehat{p}_1\right)\right)>\ddot{D}\left(Pul_2\left(\widehat{p}_2\right)\right)$*, then* $Pul_1\left(\widehat{p}_1\right)>Pul_2\left(\widehat{p}_2\right)$*;*
  ii. If $\ddot{D}\left(Pul_1\left(\widehat{p}_1\right)\right)=\ddot{D}\left(Pul_2\left(\widehat{p}_2\right)\right)$*, then* $Pul_1\left(\widehat{p}_1\right)=Pul_2\left(\widehat{p}_2\right)$*;*

Definition 3.6. [40] *The Hamming distance between* $Pul_1\left(\widehat{p}_1\right)=\left\{\left[\chi_{L_1}^{(\ell)},\chi_{U_1}^{(\ell)}\right]\left(\widehat{p}_1^{(\ell)}\right)\middle|\widehat{p}_1^{(\ell)}\geq 0;\ell=1,2,\cdots,\#Pul\right\}$ *and* $Pul_2\left(\widehat{p}_2\right)=\left\{\left[\chi_{L_2}^{(\ell)},\chi_{U_2}^{(\ell)}\right]\left(\widehat{p}_2^{(\ell)}\right)\middle|\widehat{p}_2^{(\ell)}\geq 0;\ell=1,2,\cdots,\#Pul\right\}$ *is defined as:*

$$d\left(Pul_1\left(\widehat{p}_1\right),Pul_2\left(\widehat{p}_2\right)\right)=\frac{1}{2\#Pul}\sum_{\ell=1}^{\#Pul}\left(\begin{array}{c}\left|\ddot{T}\left(\chi_{L_1}^{(\ell)}\right)\cdot\widehat{p}_1^{(\ell)}-\ddot{T}\left(\chi_{L_2}^{(\ell)}\right)\cdot\widehat{p}_2^{(\ell)}\right|\\+\left|\ddot{T}\left(\chi_{U_1}^{(\ell)}\right)\cdot\widehat{p}_1^{(\ell)}-\ddot{T}\left(\chi_{U_2}^{(\ell)}\right)\cdot\widehat{p}_2^{(\ell)}\right|\end{array}\right). \tag{6}$$

4 Prospect theory

The prospect theory (PT) Tversky and Amos [15] devised consists of two main functions: the value function $VF\left(\dot{I}_a\right)$ and the weighting function$\tilde{G}\left(h_s\right)$.

$$VF\left(\dot{I}_a\right)=\left\{\begin{array}{c}\left(\dot{I}_a-\dot{I}_0\right)^{\wp},\dot{I}_a\geq\dot{I}_0\\-\Re\cdot\left(\dot{I}_0-\dot{I}_a\right)^{\Im},\dot{I}_a<\dot{I}_0\end{array}\right., \tag{7}$$

where $\dot{I}_0$ is the standard income, $\dot{I}_a$ is the anticipated income, and $\wp$, $\Im$ as well as $\Re$ are three parameters. The values of these parameters are related to the type of decision maker. For risk-averter, $\gamma \leq \xi$ and $\kappa < 1$, because decision makers with risk-averse characteristics always prefer gains to losses.

$$WF\left(\dot{P}_a\right) = \begin{cases} \frac{\dot{P}_a^{\hat{\vartheta}}}{\left(\dot{P}_a^{\hat{\vartheta}}+\left(1-\dot{P}_a\right)^{\hat{\vartheta}}\right)^{\frac{1}{\hat{\vartheta}}}}, \dot{I}_a \geq \dot{I}_0 \\ \frac{\dot{P}_a^{\hat{\omega}}}{\left(\dot{P}_a^{\hat{\omega}}+\left(1-\dot{P}_a\right)^{\hat{\omega}}\right)^{\frac{1}{\hat{\omega}}}}, \dot{I}_a < \dot{I}_0 \end{cases} \tag{8}$$

where $\dot{P}_a$ is the probability of the anticipated income $\dot{I}_a$, and $\hat{\vartheta}$ as well as $\hat{\omega}$ are two parameters expressing the curvature of the weighting function $WF\left(\dot{P}_a\right)$.

Traditional expected value theory (TEVT) held that the anticipated income and its probability were two important elements in decision making, but Tversky and Amos [15] pointed TEVT was unmatched in lots of circumstances and concluded the prospect function PF, which consists of $VF\left(\dot{I}_a\right)$ and $WF\left(\dot{P}_a\right)$, was more appropriate for decision making.

$$PF = \sum_{a=1}^{\nu} VF\left(\dot{I}_a\right) \cdot WF\left(\dot{P}_a\right). \tag{9}$$

5 MABAC method for MAGDM based on PT under PULTS

The conception of prospect theory is regarded as a profound depiction of decision-maker's psychological state. In this section, the effective combination of PT and traditional MABAC method is elaborated, and ULT is used for all assessment information. Assume there are three collections$Al = \{Al_1, Al_2, \cdots, Al_\varphi\}$, $At = \{At_1, At_2, \cdots, At_\nu\}$and $Dm = \{Dm_1, Dm_2, \cdots, Dm_\tau\}$, which respectively express φ alternatives, ν attributes and τ decision makers.

Step 1. Gather assessment information from decision maker Dm_s to form ULT evaluation matrix $\Pi^{(s)} = \left(\left[\chi_{L_{fh}}^{(s)}, \chi_{U_{fh}}^{(s)}\right]\right)_{\varphi\times\nu}$ $(\chi_{L_{fh}}^{(s)}, \chi_{U_{fh}}^{(s)} \in \mathrm{X}; \chi_{L_{fh}}^{(s)} \leq \chi_{U_{fh}}^{(s)}; f = 1, 2, \cdots, \varphi; h = 1, 2, \cdots, \nu; s = 1, 2, \cdots, \tau)$.

Step 2. Transforming the negative attribute into positive attribute means that $\left[\chi_{\breve{a}}, \chi_{\breve{b}}\right]$ is changed into $\left[\chi_{-\breve{b}}, \chi_{-\breve{a}}\right]$ when the corresponding attribute is negative.

Step 3. Summarize information from τ ULT evaluation matrices $\Pi^{(s)} = \left(\left[\chi_{L_{fh}}^{(s)}, \chi_{U_{fh}}^{(s)}\right]\right)_{\varphi\times\nu} (s = 1, 2, \cdots, \tau)$ to construct PULTS evaluation matrix $\tilde{\Delta} = \left(Pul_{fh}\left(\widehat{p}_{fh}\right)\right)_{\varphi\times\nu} = \left(\left\{\left[\chi_{L_{fh}}^{(\ell)}, \chi_{U_{fh}}^{(\ell)}\right]\left(\widehat{p}_{fh}^{(\ell)}\right)\middle| \ell = 1, 2, \cdots, \#Pul\right\}\right)_{\varphi\times\nu}$, where $\left[\chi_{L_{fh}}^{(\ell)}, \chi_{U_{fh}}^{(\ell)}\right]$ is an indivisible basic ULT.

Step 4. Calculate the border approximation area (BAA) $PulBAA_h$ $(h = 1, 2, \ldots, \nu)$ in line with Eq. (10).

$$PulBAA_h = \left\{\left[\hat{\chi}_{L_h}^{(\ell)}, \hat{\chi}_{U_h}^{(\ell)}\right]\left(\hat{p}_h^{(\ell)}\right)\middle| \ell = 1, 2, \cdots, \#Pul\right\} h = 1, 2, \ldots, \nu, \tag{10}$$

where $\left[\hat{\chi}_{L_h}^{(\ell)}, \hat{\chi}_{U_h}^{(\ell)}\right]\left(\hat{p}_h^{(\ell)}\right) = \left[\ddot{T}^{-1}\left(\sqrt[\varphi]{\prod_{f=1}^{\varphi} \ddot{T}\left(\chi_{L_{fh}}^{(\ell)}\right)}\right), \ddot{T}^{-1}\left(\sqrt[\varphi]{\prod_{f=1}^{\varphi} \ddot{T}\left(\chi_{U_{fh}}^{(\ell)}\right)}\right)\right]\left(\sqrt[\varphi]{\prod_{f=1}^{\varphi} \widehat{p}_{fh}^{(\ell)}}\right)$.

Step 5. The CRITIC method [40] is a special tool for obtaining the original attribute weight $\tilde{\psi}_h$ $(h = 1, 2, \cdots, \nu$, $\tilde{\psi}_h \geq 0$ and $\sum_{h=1}^{\nu} \tilde{\psi}_h = 1)$. And the detailed process is shown as Eq. (11)-(14).

$$Cc_{h\kappa} = \frac{\sum_{f=1}^{\varphi}\left(\begin{array}{c}\left(\sum_{\ell=1}^{\#Pul}\frac{\left(\ddot{T}\left(\chi_{L_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)}-\ddot{T}\left(\bar{\chi}_{L_h}^{(\ell)}\right)\cdot\bar{p}_h^{(\ell)}\right)+\left(\ddot{T}\left(\chi_{U_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)}-\ddot{T}\left(\bar{\chi}_{U_h}^{(\ell)}\right)\cdot\bar{p}_h^{(\ell)}\right)}{2}\right)\cdot\\ \left(\sum_{\ell=1}^{\#Pul}\frac{\left(\ddot{T}\left(\chi_{L_{f\kappa}}^{(\ell)}\right)\cdot\widehat{p}_{f\kappa}^{(\ell)}-\ddot{T}\left(\bar{\chi}_{L_\kappa}^{(\ell)}\right)\cdot\bar{p}_\kappa^{(\ell)}\right)+\left(\ddot{T}\left(\chi_{U_{f\kappa}}^{(\ell)}\right)\cdot\widehat{p}_{f\kappa}^{(\ell)}-\ddot{T}\left(\bar{\chi}_{U_\kappa}^{(\ell)}\right)\cdot\bar{p}_\kappa^{(\ell)}\right)}{2}\right)\end{array}\right)}{\left(\begin{array}{c}\sqrt{\sum_{f=1}^{\varphi}\left(\sum_{\ell=1}^{\#Pul}\frac{\left(\ddot{T}\left(\chi_{L_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)}-\ddot{T}\left(\bar{\chi}_{L_h}^{(\ell)}\right)\cdot\bar{p}_h^{(\ell)}\right)+\left(\ddot{T}\left(\chi_{U_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)}-\ddot{T}\left(\bar{\chi}_{U_h}^{(\ell)}\right)\cdot\bar{p}_h^{(\ell)}\right)}{2}\right)^2}\cdot\\ \sqrt{\sum_{f=1}^{\varphi}\left(\sum_{\ell=1}^{\#Pul}\frac{\left(\ddot{T}\left(\chi_{L_{f\kappa}}^{(\ell)}\right)\cdot\widehat{p}_{f\kappa}^{(\ell)}-\ddot{T}\left(\bar{\chi}_{L_\kappa}^{(\ell)}\right)\cdot\bar{p}_\kappa^{(\ell)}\right)+\left(\ddot{T}\left(\chi_{U_{f\kappa}}^{(\ell)}\right)\cdot\widehat{p}_{f\kappa}^{(\ell)}-\ddot{T}\left(\bar{\chi}_{U_\kappa}^{(\ell)}\right)\cdot\bar{p}_\kappa^{(\ell)}\right)}{2}\right)^2}\end{array}\right)}, \tag{11}$$

where $h, \kappa = 1, 2, \cdots, \nu$ and

$$PulAV_h\left(\bar{p}_h\right) = \left\{ \left[\bar{\chi}_{L_h}^{(\ell)}, \bar{\chi}_{U_h}^{(\ell)}\right]\left(\bar{p}_h^{(\ell)}\right) = \left[\begin{array}{l} \ddot{T}^{-1}\left(\frac{1}{\varphi}\sum\limits_{f=1}^{\varphi}\ddot{T}\left(\chi_{L_{fh}}^{(\ell)}\right)\right), \\ \ddot{T}^{-1}\left(\frac{1}{\varphi}\sum\limits_{f=1}^{\varphi}\ddot{T}\left(\chi_{U_{fh}}^{(\ell)}\right)\right) \end{array}\right]\left(\frac{1}{\varphi}\sum_{f=1}^{\varphi}\widehat{p}_{fh}^{(\ell)}\right) \middle| \ell = 1, 2, \cdots, \#Pul \right\},$$

$$PulAV_\kappa\left(\bar{p}_\kappa\right) = \left\{ \left[\bar{\chi}_{L_\kappa}^{(\ell)}, \bar{\chi}_{U_\kappa}^{(\ell)}\right]\left(\bar{p}_\kappa^{(\ell)}\right) = \left[\begin{array}{l} \ddot{T}^{-1}\left(\frac{1}{\varphi}\sum\limits_{f=1}^{\varphi}\ddot{T}\left(\chi_{L_{f\kappa}}^{(\ell)}\right)\right), \\ \ddot{T}^{-1}\left(\frac{1}{\varphi}\sum\limits_{f=1}^{\varphi}\ddot{T}\left(\chi_{U_{f\kappa}}^{(\ell)}\right)\right) \end{array}\right]\left(\frac{1}{\varphi}\sum_{f=1}^{\varphi}\widehat{p}_{f\kappa}^{(\ell)}\right) \middle| \ell = 1, 2, \cdots, \#Pul \right\}.$$

$$Sd_h = \sqrt{\frac{\sum\limits_{f=1}^{\varphi}\left(\sum\limits_{\ell=1}^{\#Pul}\frac{1}{2}\left(\begin{array}{c} \left(\ddot{T}\left(\chi_{L_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)} - \ddot{T}\left(\bar{\chi}_{L_h}^{(\ell)}\right)\cdot\bar{p}_h^{(\ell)}\right) \\ + \left(\ddot{T}\left(\chi_{U_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)} - \ddot{T}\left(\bar{\chi}_{U_h}^{(\ell)}\right)\cdot\bar{p}_h^{(\ell)}\right) \end{array}\right)\right)^2}{\varphi - 1}},\ h = 1, 2, \cdots, \upsilon, \tag{12}$$

$$Ic_h = Sd_h \cdot \sum_{\kappa=1}^{\nu}(1 - Cc_{h\kappa}), h = 1, 2, \cdots, \nu. \tag{13}$$

$$\tilde{\psi}_h = \frac{Ic_h}{\sum\limits_{h=1}^{\nu} Ic_h}, h = 1, 2, \cdots, \nu. \tag{14}$$

Step 6. The weighting function is brought in this step for adjusting the original attribute weight to obtain the relative attribute weights $\tilde{\beta}_f\left(\tilde{\psi}_h\right)$ $(f = 1, 2, \ldots, \varphi; h = 1, 2, \ldots, \nu)$.

$$\tilde{\beta}_f\left(\tilde{\psi}_h\right) = \begin{cases} \left(\tilde{\psi}_h\right)^{\hat{\vartheta}} \Big/ \left(\left(\tilde{\psi}_h\right)^{\hat{\vartheta}} + \left(1 - \tilde{\psi}_h\right)^{\hat{\vartheta}}\right)^{\frac{1}{\hat{\vartheta}}}, Pul_{fh}\left(\widehat{p}_{fh}\right) \geq PulBAA_h, \\ \left(\tilde{\psi}_h\right)^{\hat{\omega}} \Big/ \left(\left(\tilde{\psi}_h\right)^{\hat{\omega}} + \left(1 - \tilde{\psi}_h\right)^{\hat{\omega}}\right)^{\frac{1}{\hat{\omega}}}, Pul_{fh}\left(\widehat{p}_{fh}\right) < PulBAA_h. \end{cases} \tag{15}$$

Step 7. Utilize the value function to deal with the Hamming distance under different relationship between $Pul_{fh}\left(\widehat{p}_{fh}\right)$ and $PulBAA_h$. And finally acquire the weighted distance WD_{fh} based on Equations (16) and (17).

$$d\left(Pul_{fh}\left(\widehat{p}_{fh}\right), PulBAA_h\right) = \frac{1}{2\cdot\#Pul}\sum_{\ell=1}^{\#Pul}\left(\begin{array}{c} \left|\ddot{T}\left(\chi_{L_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)} - \ddot{T}\left(\hat{\chi}_{L_h}^{(\ell)}\right)\cdot\hat{p}_h^{(\ell)}\right| \\ + \left|\ddot{T}\left(\chi_{U_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)} - \ddot{T}\left(\hat{\chi}_{U_h}^{(\ell)}\right)\cdot\hat{p}_h^{(\ell)}\right| \end{array}\right). \tag{16}$$

$$\ddot{\mathrm{M}}\left(Pul_{fh}\left(\widehat{p}_{fh}\right), PulBAA_h\right) = \frac{1}{2\cdot\#Pul}\sum_{\ell=1}^{\#Pul}\left(\begin{array}{c} \left(\ddot{T}\left(\chi_{L_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)} - \ddot{T}\left(\hat{\chi}_{L_h}^{(\ell)}\right)\cdot\hat{p}_h^{(\ell)}\right) \\ + \left(\ddot{T}\left(\chi_{U_{fh}}^{(\ell)}\right)\cdot\widehat{p}_{fh}^{(\ell)} - \ddot{T}\left(\hat{\chi}_{U_h}^{(\ell)}\right)\cdot\hat{p}_h^{(\ell)}\right) \end{array}\right). \tag{17}$$

$$WD_{fh} = \begin{cases} \tilde{\beta}_f\left(\tilde{\psi}_h\right)\cdot\left(d_h\left(Pul_{fh}\left(\widehat{p}_{fh}\right), PulBAA_h\right)\right)^{\wp} & , \ddot{\mathrm{M}}\left(Pul_{fh}\left(\widehat{p}_{fh}\right), PulBAA_h\right) > 0 \\ 0 & , \ddot{\mathrm{M}}\left(Pul_{fh}\left(\widehat{p}_{fh}\right), PulBAA_h\right) = 0 \\ -\Re\cdot\tilde{\beta}_f\left(\tilde{\psi}_h\right)\cdot\left(d_h\left(Pul_{fh}\left(\widehat{p}_{fh}\right), PulBAA_h\right)\right)^{\Im} & , \ddot{\mathrm{M}}\left(Pul_{fh}\left(\widehat{p}_{fh}\right), PulBAA_h\right) < 0. \end{cases} \tag{18}$$

where $\wp, \Im, \Re$ are parameters and $f = 1, 2, \cdots, \varphi,\ h = 1, 2, \cdots, \nu$.

Step 8. The equation (19) can help us to work out the overall weighted distance $Owd_f$of alternative Al_f $(f = 1, 2, \ldots, \varphi)$ from BBA.

$$Owd_f = \sum_{h=1}^{\nu} WD_{fh}. \tag{19}$$

Step 9. Rank the alternatives in descending order, according to the value of overall weighted distance Owd_f. And the best alternative is the first one with the biggest value of Owd_f $(f = 1, 2, \ldots, \varphi)$.

6 Numerical example

Cyber security (CS) is to ensure that the hardware, software and data in the network system will not be damaged, changed or leaked by accidental or malicious forces under any circumstances. Because cyber security involves a wide range of content and large groups, such as individuals, enterprises, and even countries, so CS is a very serious and particularly important issue. The consequences of information leakage caused by malicious attacks on information networks are very serious, which will not only damage the privacy of citizens, but also threaten the security of the country. In recent years, under the impetus of informatization, preventing malicious attack and ensuring cyber security has become one of the problems that many countries and enterprises attach great importance to. Many countries have put in place special policies to urge companies and government agencies to improve every part of the network system, to ensure that citizens information and important national information are not affected by hackers. Because of this, the cyber security industry has developed rapidly in recent years. Many cyber security companies recruit professional technical personnel and provide such services as assessment service. Therefore, some enterprises and institutions need to make decisions according to the information collected selecting the appropriate cyber security service providers (CSSPs) to ensure that their network system meets the requirements and norms of the country. There is a company who needs to choose the most suitable supplier from five CSSPs Al_f $(f = 1, 2, 3, 4, 5)$ in order to cooperate with him. Five decision makers $Dm_s(s = 1, 2, 3, 4, 5)$ evaluate the five suppliers using uncertain linguistic term in the following four aspects: (1) At_1 is the equipment performance; (2) At_2 is the maintenance difficulty; (3) At_3 is the evaluating capability; (4) At_4 is the emergency processing capability.

In addition, the linguistic term set is as follows:

$$\mathrm{X} = \left\{ \begin{array}{l} \chi_{-3} = \text{extremely poor}(\dot{\mathrm{E}}\dot{\mathrm{P}}), \chi_{-2} = \text{very poor}(\dot{\mathrm{V}}\dot{\mathrm{P}}), \\ \chi_{-1} = \text{poor}(\dot{\mathrm{P}}), \chi_0 = \text{medium}(\dot{\mathrm{M}}), \chi_1 = \text{good}(\dot{\mathrm{G}}), \\ \chi_2 = \text{very good}(\dot{\mathrm{V}}\dot{\mathrm{G}}), \chi_3 = \text{extremely good}(\dot{\mathrm{E}}\dot{\mathrm{G}}) \end{array} \right\}. \tag{20}$$

Step 1. The assessment information collected is shown in the following Table 1 to Table 5.

Table 1: The ULT matrix $\Pi^{(1)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{V}\dot{P}, \dot{M}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{V}\dot{G}]$
Al_2	$[\dot{P}, \dot{G}]$	$[\dot{P}, \dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{M}]$
Al_3	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{M}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$
Al_4	$[\dot{P}, \dot{M}]$	$[\dot{V}\dot{P}, \dot{P}]$	$[\dot{M}, \dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$
Al_5	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$

Table 2: The ULT matrix $\Pi^{(2)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$	$[\dot{P}, \dot{M}]$	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$
Al_2	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{V}\dot{P}, \dot{P}]$	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{P}, \dot{M}]$
Al_3	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{M}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{V}\dot{P}, \dot{P}]$
Al_4	$[\dot{V}\dot{P}, \dot{M}]$	$[\dot{P}, \dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$
Al_5	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{M}]$	$[\dot{G}, \dot{E}\dot{G}]$	$[\dot{M}, \dot{V}\dot{G}]$

Step 2. The negative attribute $[\chi_{\breve{a}}, \chi_{\breve{b}}]$ is changed into $[\chi_{-\breve{b}}, \chi_{-\breve{a}}]$ and the outcomes are shown in Table 6 to Table 10.

Step 3. Summarize information from τ ULT evaluation matrices $\Pi^{(s)} = \left(\left[\chi_{L_{fh}}^{(s)}, \chi_{U_{fh}}^{(s)}\right]\right)_{\varphi\times\nu} (s = 1, 2, 3, 4, 5)$ to construct PULTS evaluation matrix $\tilde{\Delta} = (Pul_{fh}(\widehat{p}_{fh}))_{5\times4} = \left(\left\{ \left[\chi_{L_{fh}}^{(\ell)}, \chi_{U_{fh}}^{(\ell)}\right] \left(\widehat{p}_{fh}^{(\ell)}\right) \middle| \ell = 1, 2, \cdots, \#Pul \right\}\right)_{5\times4}$, just as

Table 3: The ULT matrix $\Pi^{(3)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{V}\dot{P}, \dot{P}]$	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$
Al_2	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{V}\dot{P}, \dot{M}]$	$[\dot{P}, \dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$
Al_3	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{V}\dot{P}, \dot{M}]$	$[\dot{P}, \dot{M}]$	$[\dot{P}, \dot{M}]$
Al_4	$[\dot{M}, \dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{G}]$	$[\dot{M}, \dot{G}]$
Al_5	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$

Table 4: The ULT matrix $\Pi^{(4)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{P}, \dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$
Al_2	$[\dot{M}, \dot{G}]$	$[\dot{V}\dot{P}, \dot{M}]$	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$
Al_3	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{V}\dot{P}, \dot{P}]$	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{M}]$
Al_4	$[\dot{P}, \dot{M}]$	$[\dot{P}, \dot{M}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$
Al_5	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$	$[\dot{P}, \dot{M}]$	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$	$[\dot{M}, \dot{G}]$

Table 5: The ULT matrix $\Pi^{(5)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{G}, \dot{E}\dot{G}]$
Al_2	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{M}]$	$[\dot{M}, \dot{G}]$	$[\dot{M}, \dot{V}\dot{G}]$
Al_3	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$	$[\dot{P}, \dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{P}, \dot{M}]$
Al_4	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{V}\dot{G}, \dot{E}\dot{G}]$
Al_5	$[\dot{M}, \dot{G}]$	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$

Table 6: The standardized ULT matrix from $\Pi^{(1)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{V}\dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{V}\dot{G}]$
Al_2	$[\dot{P}, \dot{G}]$	$[\dot{P}, \dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{P}, \dot{M}]$
Al_3	$[\dot{M}, \dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$
Al_4	$[\dot{P}, \dot{M}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{M}, \dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$
Al_5	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{P}, \dot{M}]$	$[\dot{G}, \dot{V}\dot{G}]$	$[\dot{G}, \dot{V}\dot{G}]$

Table 7: The standardized ULT matrix from $\Pi^{(2)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$
Al_2	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$
Al_3	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{V}}\dot{\mathrm{P}}, \dot{\mathrm{P}}]$
Al_4	$[\dot{\mathrm{V}}\dot{\mathrm{P}}, \dot{\mathrm{M}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$
Al_5	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$

Table 8: The standardized ULT matrix from $\Pi^{(3)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$
Al_2	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$
Al_3	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$
Al_4	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$
Al_5	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$

Table 9: The standardized ULT matrix from $\Pi^{(4)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$
Al_2	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$
Al_3	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$
Al_4	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$
Al_5	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$

Table 10: The standardized ULT matrix from $\Pi^{(5)}$

	At_1	At_2	At_3	At_4
Al_1	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$
Al_2	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$
Al_3	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{M}}]$
Al_4	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{P}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{V}}\dot{\mathrm{G}}, \dot{\mathrm{E}}\dot{\mathrm{G}}]$
Al_5	$[\dot{\mathrm{M}}, \dot{\mathrm{G}}]$	$[\dot{\mathrm{V}}\dot{\mathrm{P}}, \dot{\mathrm{M}}]$	$[\dot{\mathrm{M}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$	$[\dot{\mathrm{G}}, \dot{\mathrm{V}}\dot{\mathrm{G}}]$

Table 11.

Table 11: The PULTS matrix $\tilde{\Delta}$

	At_1	At_2	At_3	At_4
Al_1	$\{[\chi_0,\chi_1](0.3),[\chi_1,\chi_2](0.3),[\chi_2,\chi_3](0.4)\}$	$\{[\chi_{-1},\chi_0](0.3),[\chi_0,\chi_1](0.4),[\chi_1,\chi_2](0.3)\}$	$\{[\chi_0,\chi_1](0.3),[\chi_1,\chi_2](0.5),[\chi_2,\chi_3](0.2)\}$	$\{[\chi_0,\chi_1](0.5),[\chi_1,\chi_2](0.4),[\chi_2,\chi_3](0.1)\}$
Al_2	$\{[\chi_{-1},\chi_0](0.1),[\chi_0,\chi_1](0.5),[\chi_1,\chi_2](0.4)\}$	$\{[\chi_{-1},\chi_0](0.1),[\chi_0,\chi_1](0.5),[\chi_1,\chi_2](0.4)\}$	$\{[\chi_{-1},\chi_0](0.1),[\chi_0,\chi_1](0.7),[\chi_1,\chi_2](0.2)\}$	$\{[\chi_{-1},\chi_0](0.4),[\chi_0,\chi_1](0.1),[\chi_1,\chi_2](0.5)\}$
Al_3	$\{[\chi_0,\chi_1](0.5),[\chi_1,\chi_2](0.3),[\chi_2,\chi_3](0.2)\}$	$\{[\chi_{-1},\chi_0](0.1),[\chi_0,\chi_1](0.6),[\chi_1,\chi_2](0.3)\}$	$\{[\chi_{-1},\chi_0](0.2),[\chi_0,\chi_1](0.2),[\chi_1,\chi_2](0.6)\}$	$\{[\chi_{-2},\chi_{-1}](0.2),[\chi_{-1},\chi_0](0.6),[\chi_0,\chi_1](0.2)\}$
Al_4	$\{[\chi_{-2},\chi_{-1}](0.1),[\chi_{-1},\chi_0](0.5),[\chi_0,\chi_1](0.4)\}$	$\{[\chi_{-1},\chi_0](0.4),[\chi_0,\chi_1](0.4),[\chi_1,\chi_2](0.2)\}$	$\{[\chi_{-1},\chi_0](0.1),[\chi_0,\chi_1](0.5),[\chi_1,\chi_2](0.4)\}$	$\{[\chi_0,\chi_1](0.6),[\chi_1,\chi_2](0.2),[\chi_2,\chi_3](0.2)\}$
Al_5	$\{[\chi_0,\chi_1](0.4),[\chi_1,\chi_2](0.4),[\chi_2,\chi_3](0.2)\}$	$\{[\chi_{-2},\chi_{-1}](0.1),[\chi_{-1},\chi_0](0.5),[\chi_0,\chi_1](0.4)\}$	$\{[\chi_0,\chi_1](0.1),[\chi_1,\chi_2](0.4),[\chi_2,\chi_3](0.5)\}$	$\{[\chi_0,\chi_1](0.3),[\chi_1,\chi_2](0.5),[\chi_2,\chi_3](0.2)\}$

Step 4. Calculate the border approximation area (BAA) $PulBAA_h$ $(h = 1, 2, 3, 4)$ in line with Eq. (10). And the results are included in Table 12.

Table 12: The BAA for every attributes

$PulBAA$	
At_1	$\{[\chi_{-0.7794},\chi_{0.2875}](0.2268),[\chi_{0.2875},\chi_{1.3174}](0.3898),[\chi_{1.3174},\chi_{2.3345}](0.3031)\}$
At_2	$\{[\chi_{-1.2589},\chi_{-0.2337}](0.1644),[\chi_{-0.2337},\chi_{0.7764}](0.4743),[\chi_{0.7764},\chi_{1.7818}](0.3104)\}$
At_3	$\{[\chi_{-0.6478},\chi_{0.3659}](0.1431),[\chi_{0.3659},\chi_{1.3734}](0.4258),[\chi_{1.3734},\chi_{2.3783}](0.3438)\}$
At_4	$\{[\chi_{-0.7794},\chi_{0.2875}](0.3728),[\chi_{0.2875},\chi_{1.3174}](0.2993),[\chi_{1.3174},\chi_{2.3345}](0.2091)\}$

Step 5. The CRITIC method is used to derive the attribute weight $\tilde{\psi}_h$ $(h = 1, 2, 3, 4)$ by Eq. (11)-(14).

$$\tilde{\psi}_1 = 0.2391\ \tilde{\psi}_2 = 0.2405, \tilde{\psi}_3 = 0.1911, \tilde{\psi}_4 = 0.3293.$$

Step 6. The weighting function is brought in this step for adjusting the original attribute weight to obtain the modified attribute weights $\tilde{\beta}_f\left(\tilde{\psi}_h\right)$ $(f = 1, 2, 3, 4, 5; h = 1, 2, 3, 4)$ shown in Table 13.(Notes: the values of parameters $\hat{\vartheta} = 0.61$ and $\hat{\omega} = 0.69$ in Eq. (15) are derived from Kahneman [51]s the experimental proof.)

Table 13: The modified attribute weights

	At_1	At_2	At_3	At_4
Al_1	0.2844	0.2852	0.2551	0.3338
Al_2	0.2844	0.2852	0.2502	0.3338
Al_3	0.2844	0.2852	0.2551	0.3468
Al_4	0.2858	0.2852	0.2551	0.3338
Al_5	0.2844	0.2868	0.2551	0.3338

Step 7. Utilize the value function to deal with the Hamming distance shown in Table 14 under different relationship between $Pul_{fh}\left(\widehat{p}_{fh}\right)$ and $PulBAA_h$. And finally acquire the weighted distance WD_{fh} , which is shown in Table 15,

based on Equations (16) and (17). (Notes: the values of parameters $\wp = 0.88$, $\Im = 0.88$ and $\Re = 2.25$ in Eq. (18) are derived from Kahneman [51]'s the experimental proof.)

Table 14: The Hamming distance between each alternative and BAA

	At_1	At_2	At_3	At_4
Al_1	0.0719	0.0307	0.1011	0.1025
Al_2	0.0544	0.0439	0.0965	0.1142
Al_3	0.09	0.0384	0.1146	0.0777
Al_4	0.0428	0.0672	0.0214	0.0779
Al_5	0.0809	0.033	0.0714	0.0695

Table 15: The weighted distance

	At_1	At_2	At_3	At_4
Al_1	0.0281	0.0133	0.0339	0.045
Al_2	0.0219	0.0182	-0.0719	0.0495
Al_3	0.0342	0.0162	0.0379	-0.0823
Al_4	-0.0401	0.0265	0.0087	0.0353
Al_5	0.0311	-0.0321	0.0250	0.0320

Step 8. The equation (19) can help us to work out the overall weighted distance $Owd_f$of alternative Al_f $(f = 1, 2, 3, 4, 5)$ from BBA.

$$Owd_1 = 0.1203 \quad Owd_2 = 0.0177 \quad Owd_3 = 0.0059,$$

$$Owd_4 = 0.0303 \quad Owd_5 = 0.0560.$$

Step 9. Rank the alternatives in descending order, according to the value of overall weighted distance Owd_f. And the best alternative is the first one with the biggest value of Owd_f, $(f = 1, 2, 3, 4, 5)$.

$$Al_1 > Al_5 > Al_4 > Al_2 > Al_3.$$

To sum up, the optimal alternative is Al_1.

It is obvious that the value of parameters just as $\hat{\vartheta}$,$\hat{\omega}$,$\wp$,$\Im$,$\Re$ can make a change in the above calculative outcome. And there is no doubt that we need to select the perfect parameters according to the problem we study. The responsibility of this paper isnt to analyze the parameters but to establish a brilliant PULT MAGDM model.

7 Comparative analysis

In this section, we select three already existed methods, such as PULWA[15] operator, probabilistic uncertain linguistic MABAC (PUL-MABAC) method [39] and probabilistic uncertain linguistic EDAS (PUL-EDAS) method [8], to compare with the PUL-PT-MABAC method presented in this paper.

7.1 Compared with PULWA operator

The PULWA operator was proposed by Lin, Xu, Zhai and Yao [22]. Based on the information of PULTS matrix $\tilde{\Delta}$ and attribute weights $\tilde{\psi}_1 = 0.2391$,$\tilde{\psi}_2 = 0.2405$,$\tilde{\psi}_3 = 0.1911$,$\tilde{\psi}_4 = 0.3293$, we get the overall PULTS of each alternative by using PULWA, just as following.

$$\tilde{Q}_1 = \{[\chi_{-0.0721}, \chi_{0.2937}], [\chi_{0.2990}, \chi_{0.6942}], [\chi_{0.4057}, \chi_{0.6447}]\};$$
$$\tilde{Q}_2 = \{[\chi_{-0.1988}, \chi_{0}], [\chi_{0}, \chi_{0.4065}], [\chi_{0.3947}, \chi_{0.7894}]\};$$
$$\tilde{Q}_3 = \{[\chi_{-0.1940}, \chi_{0.0537}], [\chi_{-0.1258}, \chi_{0.3260}], [\chi_{0.2825}, \chi_{0.5830}]\};$$
$$\tilde{Q}_4 = \{[\chi_{-0.1631}, \chi_{0.1737}], [\chi_{-0.0537}, \chi_{0.3235}], [\chi_{0.2563}, \chi_{0.5423}]\};$$
$$\tilde{Q}_5 = \{[\chi_{-0.0481}, \chi_{0.1895}], [\chi_{0.2165}, \chi_{0.6735}], [\chi_{0.4185}, \chi_{0.7239}]\}.$$

According to the excepted degree of above overall PULTS of each alternative$\ddot{E}\left(\tilde{Q}_1\right) = 1.1326$, $\ddot{E}\left(\tilde{Q}_2\right) = 0.6959$, $\ddot{E}\left(\tilde{Q}_3\right) = 0.4626$, $\ddot{E}\left(\tilde{Q}_4\right) = 0.5395$, $\ddot{E}\left(\tilde{Q}_5\right) = 1.0869$, the descending order of these alternatives is $Al_1 > Al_5 > Al_2 > Al_4 > Al_3$ which means alternative Al_1 is the best one.

7.2 Compared with PUL-MABAC method

We can compare the new proposed method with the PUL-MABAC method [39] which is the common model without improving by PT. Based on exactly the same initial data and weight information, the common PUL-MABAC method can figure out the following results $PULSV_1 = 0.0776$, $PULSV_2 = 0.0427$, $PULSV_3 = 0.0271$, $PULSV_4 = 0.0357$,$PULSV_5 = 0.0480$, so the order and the optimal alternative are $Al_1 > Al_5 > Al_2 > Al_4 > Al_3$and Al_1 respectively.

7.3 Compared with PUL-EDAS method

At the same time, PUL-EDAS method [8] is also a wonderful method to prove the effectiveness of this method. PUL-EDAS mainly involves positive distance $PULPDA$, negative distance $PULNDA$ and appraisal score $PULAS$ (shown in Table 16). Obviously, the descending order of all alternatives is $Al_1 > Al_5 > Al_2 > Al_4 > Al_3$ in line with $PULAS$. Therefore, the alternative Al_1 with biggest appraisal score is the optimal decision.

$$PULPDA = \begin{array}{c} \\ Al_1 \\ Al_2 \\ Al_3 \\ Al_4 \\ Al_5 \end{array} \begin{array}{c} \begin{array}{cccc} At_1 & At_2 & At_3 & At_4 \end{array} \\ \begin{pmatrix} 0.0558 & 0.0078 & 0.0227 & 0.0321 \\ 0 & 0.0370 & 0 & 0 \\ 0.0220 & 0.0273 & 0 & 0 \\ 0 & 0 & 0 & 0.0321 \\ 0.0305 & 0 & 0.0631 & 0.0588 \end{pmatrix} \end{array},$$

$$PULNDA = \begin{array}{c} \\ Al_1 \\ Al_2 \\ Al_3 \\ Al_4 \\ Al_5 \end{array} \begin{array}{c} \begin{array}{cccc} At_1 & At_2 & At_3 & At_4 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0118 & 0 & 0.0421 & 0.0125 \\ 0 & 0 & 0.0178 & 0.1105 \\ 0.0964 & 0.0117 & 0.0259 & 0 \\ 0 & 0.0604 & 0 & 0 \end{pmatrix} \end{array}.$$

Table 16: Others results of PUL-EDAS method

	Al_1	Al_2	Al_3	Al_4	Al_5
PULNSP	0.778	0.23	0.3054	0.2729	1
PULNSN	1	0.6235	0	0.2255	0.6348
PULAS	0.889	0.4268	0.1527	0.2492	0.8174

There are some differences in the ranking of alternatives, but we can draw the conclusion that alternative Al_1 is the best choice from any of the above comparison methods, which fully proves that the new PUL-PT-MABAC method is reasonable and scientific. Although these three methods have their own characteristics, the new PUL-PT-MABAC method has obvious advantages in the description of DM's mental state. PUL-PT-MABAC method not only solves the problem of information fuzziness and uncertainty by using PULTS, but also inherits the advantages of PT. In attribute weights, PUL-PT-MABAC method firstly extracts the initial weights information from the evaluation information by CRITIC method, and then utilizes the weighting function to correct the initial weight to solve the possible information distortion in the subjective evaluation. When calculating the distance from BAA, PUL-PT-MABAC method treats the distance higher than BAA and the distance lower than BAA as profit and loss respectively by using the value function, which fully reflects the DM's different psychological perception of profit and loss. In general, PUL-PT-MABAC method has distinct superiority in completeness and rationality compared with existed MADM or MAGDM methods. Therefore, the establishment of improved PUL-PT-MABAC method is very valuable for solving the problem of MADM or MAGDM.

8 Conclusions

In this paper, the traditional MABAC method is improved by using the prospect theory, and the evaluation information is collected and sorted by using PULTS. The improved PUL-PT-MABAC method can not only deal with the uncertainty well, but also fully consider the influence of the DM's psychological state on the decision result, which is an important point that the PUL-PT-MABAC method is different from other methods. To sum up, the consideration of DM's psychological state in this new method is mainly reflected in two aspects: on the one hand, the adjustment of attribute weights avoids the information distortion caused by DM's subjective consciousness; On the other hand, considering that DMs often have different psychological perceptions of benefits and losses in practical problems, parameters are introduced to ensure that the model can have good applicability for different types of DMs. More importantly, the results of example and comparative analysis show that the model can deal with practical problems, and has good practicability and science. And the application of this model in cyber security also provides a new way to solve similar problems in cyber security.

In the future, the following works are worthy to investigated: (1) the application of this built model are worth to discuss in other fields; (2) we will also continue to study all kinds of advanced knowledge and concepts, and strive to put forward a more perfect and scientific MAGDM model in different fuzzy settings; (3) we will combine this with other relatively new decision-making techniques (e.g., WASPAS, MARCOS, etc.) which will be utilized in a wider range of fields in real life; (4) the consensus reaching process is another hot and important issue attracting a large number of researchers, thus, we will focus on solving large-scale group consensus and multi-attribute decision-making problems by using Granular Computing or some machine learning technologies.

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