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Original paper

A novel hesitant fuzzy linguistic term sets approach and its application on acceptance sampling plans

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Abstract

Hesitant fuzzy linguistic term set (HFLTS) is an approach giving ability to obtain more flexible decision-making (DM) process by integrating linguistic fuzzy modeling (LFM) with hesitative expert judgments. Although HFLTS is widely-studied in the literature and many enhancements are made on HFLTS procedure, none of these enhancements gives ability to continue with precise fuzzy modeling (PFM) in decision process. LFM has a big drawback about accuracy because of the dependency between the size of term set and the comprehensiveness of fuzzy sets (FSs). This issue creates a very critical difficulty in modeling of DM problems that need sensitive evaluations by using HFLTS. This paper aims to solve this problem by proposing a novel HFLTS methodology that is usable for DM problems that need sensitive calculations in the decision stage. The proposed methodology integrates 2-tuple LFM and linguistic fuzzy modifiers with HFLTS to overcome the accuracy problem and obtain more sensitive and flexible decision procedure. This paper also presents an envelopment transformation technique to aggregate expert assessments as a fuzzy membership function instead of membership grades. It becomes possible to keep interpretability in a certain level and achieve sensitive results at the same time with the help of these modifications. The proposed HFLTS approach is analyzed on a real case example from manufacturing industry for acceptance sampling plans (ASP) that is a DM problem requiring sensitive calculations. As another originality of the paper, the main formulations of ASP are derived based on hesitant fuzzy defectiveness information. The obtained results are also compared with some existing enhancements of the HFLTS and the success of the proposed methodology is proved in terms of sensitive calculation.

Keywords: Hesitant fuzzy sets, linguistic term sets, 2-tuple linguistic statements, linguistic fuzzy modifiers, acceptance sampling plans.

1 Introduction

Most of the engineering techniques need certain information to obtain reasonable and usable results. This necessity causes a critical problem while modeling real case problems that generally include uncertainties. It is an obligation to completely reflect these uncertainties for successfully modeling of the problem. One of the popular techniques named the fuzzy set theory (FST) was suggested in the literature to model the real case uncertainties. These uncertainties can be occurred by many factors such as human error, expert hesitancy, lack of expertness about the event, and lack of information about the event. Recently, FST was extended to model such complicated uncertainties better, depending on the nature of the uncertainty. For example, Torra [40] suggested hesitant fuzzy set (HFS) theory to formulate the scenario of lack of expertness by allowing multiple membership degrees for each set element. HFS gives flexibility to make hesitative assessments and ability to work with multiple experts. HFS has been often handled in the literature. For example, Xu [47] analyzed the hesitant fuzzy aggregation operators for distance, similarity, correlation and entropy measures, preference relations and multi-criteria decision making (MCDM) models. Most of the papers on HFS theory

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in the literature mainly focused on MCDM problems. Modeling with Fuzzy Sets (FSs) should start with understanding the root cause of the uncertainty. When it is understood, the best suitable FS extension is decided to reach reasonable results. If there are usable historical data, the parameters of FS can be defined easily. However, in some cases such as changing some parameters in production process makes the historical data useless. In such circumstances, the best practice is to get assessments from multiple experts. FST use numeric values to model uncertainty, but decision makers generally use the words to express their judgments. For this type of cases, linguistic fuzzy modeling (LFM) approach is a beneficial practice to think with words and convert them to corresponding FSs by employing linguistic approximation.

LFM was integrated with HFSs by Rodrguez et al. [36] with the name hesitant fuzzy linguistic term set (HFLTS) to obtain an opportunity to make hesitative assessments in MCDM problems. In HFLTS, expert opinions are aggregated by envelopment technique to obtain fuzzy intervals that are used to build preference relations, and choice degrees to find the best alternative. Rodrguez et al. [37] extended this approach to be able to work with multiple experts in group MCDM (GMCDM) problems. Various envelopment techniques [4, 24, 25, 37, 38] were offered in the literature to minimize the loss of information while aggregating the expert assessments in HFLTS process. Some of these studies have focused on the improvement of the capability of HFLTS to work with the information having unusual nature. Yu et al. [50] developed an optimization model to work with multi-granular HFLTSs. Liao et al. [17] offered a new score function and extended it for unbalanced hesitant fuzzy linguistic information (HFLI). Zhang et al. [52] proposed algorithms to transform unbalanced HFLTSs into balanced ones and proposed an approach to deal with multi granular unbalanced hesitant linguistic fuzzy information. A large part of the studies (more than half of the studies according to Liao et al. [20]) concentrated on aggregation of the assessments [4, 7, 19, 24, 31], and the ranking of the alternatives based on preference relations or distance measures [16, 15, 27, 28] in MDCM. In some studies, new frameworks with HFLI were offered. Wu & Xu [45] presented a kind of fuzzy restriction approach named possibility distribution-based methodology, developed aggregation operators, and comparison rules for group MCDM problem. Xu et al. [48] modified the HFLTS approach to obtain risk and return oriented approaches in MCDM process. Lin et al. [22] increased the sensitivity analysis capability of hesitant decision process by building a score value-based improvement suggestion mechanism for nonoptimal alternatives. Gou et al. [6] offered double hierarchy linguistic term sets (LTSs) consisting of two independent term sets used hierarchically and many studies were conducted based on double hierarchy LTSs in the context of MCDM [8, 14, 26, 51]. Xu et al. [49] combined the HFLTS with linear programming for multi-dimensional preference analysis in MCDM. HFLTS were also combined with MCDM techniques such as VIKOR [46], AHP [3], TOPSIS [8, 30, 46], ELECTRE [21, 51], ANP [30], TODIM [18, 23], ARAS [2] etc. and also integrated with the quality tools such as FMEA [42] and QFD [5, 9]. Pang et al. [33] integrated HFSs with probabilistic linguistic information to overcome the limitations of existing method causing loss of information in aggregation and ranking steps in GMCDM problems. Some studies [8, 18, 41, 42] integrated probabilistic LTS approach with various MCDM approaches such as VIKOR [8]. Some other studies focused on providing consensus between the experts in GMCDM problems [29, 34, 35, 53]. HFLTS has also integrated with 2-tuple LFM in line with the needs of GMCDM problem. Beg and Rashid [1] reformulated the HFLTS method for the expert assessments consist of a linguistic term and multiple numerical values (in other words, a set of 2-tuple statements having the same linguistic term.). Tan et al. [39] changed the schema of linguistic statements which is quite different from the cognitive process of humans, to collect hesitative assessments. The experts select multiple linguistic terms and assigns a numerical score to each term such as very heavy 0.2, heavy 0.7. Wei and Liao [44] aggregated ordinary HFLTS statements in 2-tuple form to minimize the loss of information. Romero et al. [38] proposed a method based on 2-tuple statements using similar aggregation approach with the studies [24] and [4]. The method aggregates the assessments as trapezoidal FSs even if the original assessments are not trapezoidal. Since the obtained trapezoidal FSs are too comprehensive, they were useful for comparison but not for sensitive calculation.

All the above-mentioned studies concentrated on the improvement of the capabilities of HFLTS in line with the needs of the MCDM problems. However, using LFM with the problems that need sensitive calculations is not applicable in terms of accuracy. One of the biggest problems about LFM is the interpretability-accuracy trade off depending on the size of LTS. Since the width of corresponding FSs are directly dependent on the size of term set, it is possible to encounter information loss. To obtain more sensitive results in LFM without any additional mechanism, the size of the LTS should be big but, the logical meanings of the verbal terms are dramatically lost while the size of term set is getting bigger. The quality of the results obtained is affected by term set building decisions and loss of information is faced for any size of LTS. Even if the possible biggest term set not causing loss of interpretability is determined, loss of information is still encounter because of the dependency between width of FS and size of term set. This is not an issue for MCDM problems because the best alternative is decided by ranking the alternatives in exploitation phase by just considering the membership grades. It is clear that some new mechanisms are needed to improve the accuracy of LFM while protecting the interpretability at a high level. Such mechanisms are not necessary for MCDM problems because considering just the relative greatness of the membership values are enough to reach a decision. Some characteristics of

FSs such as shape and width are not important for these problems. However, the accuracy improvement is inevitable if the modeling continues with precise fuzzy modeling (PFM). None of the above-mentioned studies brings a solution for this accuracy issue. Such weaknesses of HFLTS had been also noticed by some researchers. Liao et al. [20] and Wang et al. [43] reviewed a great number of studies related to HFLTS and discussed the developments, issues, and the challenges about the HFLTS theory. Both agreed on the following arguments: the studies on preference relations are insufficient, aggregation methods have limited novelty because of being just the extensions of the classical ones, and linguistic fuzzy modifiers (LFMs) should be integrated with HFLTS. Moreover, Liao et al. [20] criticized the complexity and low applicable potential of HFLTS extensions and encouraged the studies related with clustering algorithms, and MCDM models. Wang et al. [43] analyzed the lack of paid attention to uncertainty caused by hesitancy in the decision process and encouraged the studies focusing on the multi-granularity, unbalanced linguistic term sets (LTSs), comparison strategies, approximation procedures, and modeling of multiple uncertainties.

In this study, a novel methodology that integrates 2-tuple LFM and fuzzy modifiers with HFLTS has been suggested to obtain more accurate results. Differently from the above reviewed papers, the HFLTS has been enhanced to make it suitable for other type of DM problems that need to continue with PFM by providing an envelopment conversion to intuitionistic fuzzy set (IFS). The 2-tuple approach [32] has been employed as a shifting modifier because of its popularity and ease of usage. The suggested HFLTS approach is a generic solution for various DM problems requiring PFM in later steps of decision process. It gives opportunity to keep interpretability in a certain level, minimize loss of information, and achieve sensitive results at the same time. The proposed methodology has been applied for a special type of DM problem named acceptance sampling plans (ASPs). HFLTS has been integrated the with ASPs to obtain more flexible calculations and make better sensitivity analysis. As another originality of this study, ASPs based on hesitant fuzzy linguistic defectiveness information has been formulated. In the literature, there is no study analyzing ASPs under hesitancy yet and this paper contains a significant level of innovation and originality in this sense.

The rest of this paper has been organized as follows: A brief information about preliminaries of FST, HFS, and HFLTS is briefly summarized in Section 2. The suggested improvements that construct a novel HFLTS approach are detailed in Section 3. One of the specific DM problems named ASP is analyzed based on suggested HFLTS approach, an illustrative example from manufacturing industry is presented, and the results of the proposed methodology are compared with some other studies in Sections 4 and 5, respectively. Finally, the obtained results and future research directions are discussed in Section 6.

2 Preliminaries

In some cases, the uncertainty can be occurred by lack of expertise about the event and the experts can have some hesitancies about their evaluations. To avoid these uncertainties, working with multiple experts or taking multiple assessments from an expert can used for an alternative evaluation method. The HFS that is a type of FS and is defined in terms of a function which returns a subset of values in [0,1] cite1 can be successfully used for these cases. Since HFS contain multiple membership degrees for each set member $x \in X$, it is not practical to use the FS directly in formulations of some real-case engineering problems that can includes many types of uncertainties. For this reason, some transformations are required. For most of the cases, assessments are merged by using data envelopment technique (ET) to obtain an Intuitionistic FS (IFS). The most popular ET is presented in Definition 2.1.

Definition 2.1. *For a given HFS $h(x)$, the envelope of $h(x)$ is an IFS represented as $\tilde{A}_{env(h)} = \{x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) | x \in X\}$ satisfying Eq.* (1) [40]:

$$\mu_{\tilde{A}}(x) = h^-(x),\ \nu_{\tilde{A}(x)} = 1 - h^+(x), \tag{1}$$

where $h^-(x) = min(h(x))$ is the lower bound and $h^+(x) = max(h(x))$ is the upper bound.

The quality of fuzzy models is assessed with two features: Interpretability and accuracy. These two are contradicting features in practice. The accuracy refers to the closeness of real system and the interpretability refers to the capability for express the system behavior in an understandable way. The LFM is concentrating on the enhancement of interpretability by enabling assessments with words [10]. LFM has a drawback about the accuracy because of deciding the size of term set at the beginning as a constant. It is a challenging work to improve the accuracy of LFM while protecting the interpretability at a high level. To increase the accuracy in LFM, improvements should be made in modeling process or model structure [10]. Model structure can be extended in four ways: (i) using linguistic modifiers such as powering, expensing, and shifting modifiers, (ii) using double consequent rules such as between statements, (iii) using weighted rules and (iv) using hierarchical knowledge-base to make possible using learning methods in knowledge-base [10]. Since considered DM problems are not suitable for extending modeling process, the dependency between the width of FSs and the size of term set has been diluted by using fuzzy modifiers in this study.

Fuzzy modifiers make several transformations on FSs such as shifting, powering depending on the type of the modifier. Some example modifiers are normalization, complement, piecewise linear, powering, dilatation modifiers [13]. In the scope of this study, intensifying Kuzmin modifier with linear transformation presented in Definition 2.2 is preferred to narrow the FS. In this way, it becomes possible to get sensitive results as if working with a bigger term set despite using a relatively smaller one. It helps to break the interpretability-accuracy trade-off originating from pre-determined LTS size.

Definition 2.2. *Let $M = \{m_i\} = \{m_0, , m_h\}$ be a LTS for intensifying Kuz'min modifier with linear transformation* [12]*, $I2T_M = (m_i, \alpha)$ be a 2-tuple statement for modifier, Δ^{-1} be a function converting a 2-tuple statement $x_i = (m_i, \alpha_i)$ into numerical equivalent of it and, $\tilde{\tilde{A}}_{env(H_{S(x)})}$ be an IFS. The modified IFS $\tilde{\tilde{A}}'$ is obtained as a narrowed FS by shifting the support points towards the kernel with the ratio $r_i = 1 - \frac{\Delta^{-1}(m_i,\alpha)}{h} = 1 - \frac{(i+\alpha)}{h}$. More specifically, for a z-shaped FS $\tilde{A}$ having membership function $\mu_{\tilde{A}} = (kernel, support) = (a, b)$, membership function of the modified FS $\tilde{A}'$ is revised as $\mu_{\tilde{A}'} = (kernel, support) = (a, (a + r_i(b - a)))$.*

2-tuple approach is an extended version of LTS giving ability to make more flexible assessments. In 2-tuple approach, fuzzy information is represented with a linguistic term ($s_i \in S = \{s_0, , s_g\}$) and a companion numerical value ($\alpha \in [-0.5, 0.5)$) named difference of information (DOI) as (s_i, α). DOI acts as a kind of shifting modifier and designates the measure of the movement of the FS between the processor and successor linguistic terms. In this way, dependency of the kernel and support points of FS to the verbal linguistic terms is minimized. Fig. 1 shows the corresponding FS of an example 2-tuple statement with the help of a term set [32]. The concept of HFLTS has been introduced by

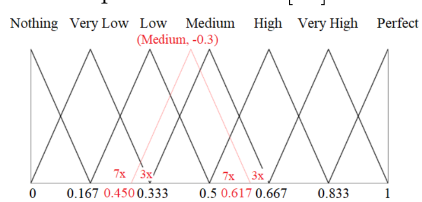

Figure 1: Corresponding FS for 2-tuple linguistic statement (Medium, -0.3) with its semantics.

Rodrguez et al. [36] to be able to work with hesitative expert assessments in DM problems. To give flexibility about hesitancy to the expert, the assessments are supported by a second term: context-free grammar. In this way, a single expert can make multiple assessments and more elaborated linguistic expressions are obtained. Since the concept uses a simple and problem dependent context-free grammar, relation sets that are used while building statements should be decided problem specific [37]. HFLTS approach consists of mainly five phases [36]:

(i) *Definition*: (a) Semantics and syntax of LTS and, (b) context-free grammar are defined by the help of Definition 2.3.
(ii) *Gathering*: Assessments are gathered from experts in context-free grammar format.
(iii) *Transformation*: Context-free grammar statements are converted to HFLTS statements by using the formulation given in 2.4.
(iv) *Aggregation*: Linguistic intervals are obtained by envelopment of linguistic expressions presented in Definition 2.5.
(v) *Exploitation*: Preference relation between alternatives is built by using linguistic intervals and the best alternative is selected.

For an LTS ($S = \{s_i\} = \{s_0, , s_g\}$), an HFLTS statement ($H_S(x)$) is represented with an ordered finite subset of the consecutive linguistic terms [36]. The subset is procured by using a context-free grammar. The experts make assessment as a composite term by using the primary term set and the context-free grammar (i.e. between Medium and Perfect, lower than Medium etc.). Definition 2.3 presents the structure of composite terms. Transformation from a composite term into subset of linguistic terms is operated by a transformation rule as shown in Definition 2.4. Fig. 2 shows the equivalent HFLTS values of some linguistic context-free grammar statements. Since multiple linguistic statements for each set member cannot be used in the later steps of HFLTS directly, the statements are aggregated by using the envelopment technique presented in Definition 3.1. For the example HFLTS statement $H_S(x) = \{\text{medium, high, perfect}\}$, the envelope is reached as $env(H_S(x)) = [H_{S^-}(x), H_{S^+}(x)] = [\text{medium, perfect}]$ [36].

Definition 2.3. *Let $S = \{s_i\} = \{s_0, , s_g\}$ be a linguistic primary term set, $U = \{u_j\}$ be a unary relation set, $B = \{b_k\}$ be a binary relation set, $C = \{c_l\}$ be a conjunction set, and $D = \{d_m\}$ be a disjunction set. A composite term t is* *an ordered pair constructed as in Eq.* (2) *and a context-free grammar statement* II *is defined in the form as shown in Eq.*(3) [36]*:*

$$t \overset{\text{def}}{=} (\langle u_j\rangle, \langle s_i\rangle) \text{ or } (\langle s_i\rangle, \langle d_m\rangle, \langle s_j\rangle) \text{ or } (\langle b_k\rangle, \langle s_i\rangle, \langle c_l\rangle, \langle s_j\rangle) \ , u_j \in U, s_i, s_j \in S, b_k \in B, c_l \in C, d_m \in D, \tag{2}$$

$$II \overset{\text{def}}{=} \langle s_i\rangle \text{ or } \langle t\rangle, \ \ s_i \in S. \tag{3}$$

Definition 2.4. *Let* E_{G_H} *be a rule set transforming linguistic context-free grammar statement II into HFLTS statement* H_S *where* S *is LTS such that* $E_{G_H} : II \to H_s$*. The transformation is done by one of the following rules* [36]*:*

- $E_{G_H}(s_i) = \{s_i | s_i \in S\}$,
- $E_{G_H}(\langle u_j\rangle = lessthan, \langle s_i\rangle) = \{s_k | s_k \in S \text{ and } s_k \le s_i\}$,
- $E_{G_H}(\langle u_j\rangle = greaterthan, \langle s_i\rangle) = \{s_k | s_k \in S \text{ and } s_k \ge s_i\}$,
- $E_{G_H}(\langle b_k\rangle = between, \langle s_i\rangle, \langle c_l\rangle = and, \langle s_j\rangle) = \{s_m | s_m \in S \text{ and } s_i \le s_m \le s_j\}$.

Definition 2.5. *The envelope of an HFLTS* ($env(H_S(x))$) *is a linguistic interval as shown in Eq.* (4) [36]*:*

$$env(H_S(x)) = [H_S^-(x), H_S^+(x)], \tag{4}$$

where $H_S^-(x) = min(s_i(x)) = s_j(x)$ *is lower bound and* $H_S^+(x) = max(s_i(x)) = s_j(x)$ *is upper bound satisfying* $s_i(x) \in H_S(x)$ *and* $s_i(x) \ge s_j(x)$ *for all* i*.*

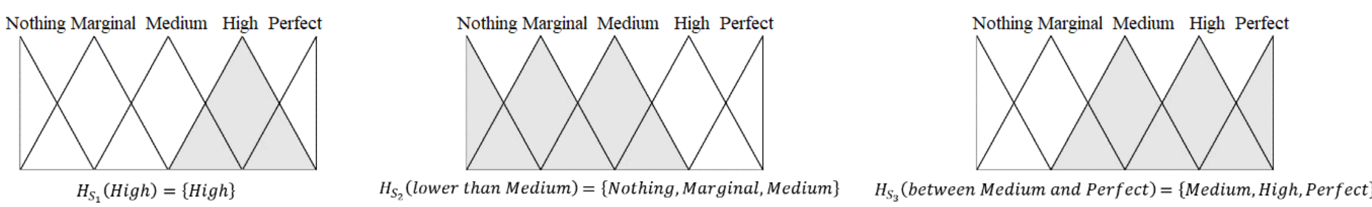

Figure 2: HFLTS equivalents of example linguistic context-free grammar statement.

3 A novel approach based on hesitant fuzzy linguistic term sets

In some cases, some of the input variables may not be determined numerically because of data insufficiency in DM process. It can be caused from two factors: (i) the historical data may become useless because of some factors such as changing some parameters in process or (ii) parameter tuning by a learning mechanism cannot be applied because of some reasons such as data insufficiency, data unsuitableness, and cost of the tuning operation in terms of time and money. In such circumstances, getting assessments from experts by using HFLTS can be a good solution because it gives flexibility to get linguistic hesitative assessments from multiple experts when experts do not reach a decision on a single linguistic term. Since the HFLTS approach has been built in line with the needs of MCDM problem, it focuses on ranking alternatives and does not take the FS shape into account. Most of the studies work with membership degrees or linguistic terms instead of membership functions. Thus, it is not possible to use HFLTS methodology directly for some type of DM problems that need sensitive calculation in decision phase. As presented in Section 1, there are various studies in the literature enhancing the HFLTS methodology but none of them gives ability to continue with PFM. Most of these studies focus on improving the exploitation phase or transformation phases.

This study focuses to analyze DM problems that the historical data is not available to decide/tune FS parameters. It aims to propose a novel approach based on HFLTS approach to provide a generalized methodology for modeling of the DM problems which require accurate and sensitive results. As an example of this kind of problems, results should be calculated sensitively with the help of PFM by turning the linguistic fuzzy assessments into numeric values to reach a decision in ASP problems. This can be done by presenting an approach to convert the hesitant linguistic assessments into numeric fuzzy membership functions. For this reason, Definitions 1 and 5 have been combined and the envelopment method presented in Definition 6 has been developed. By this way, the IFSs have been handled as membership functions in formulations.

Definition 3.1. *The linguistic interval $[H_S^-(x), H_S^+(x)]$ obtained by the envelope of an HFLTS ($env(H_S(x))$) builts a linguistic IFS represented as $\tilde{\tilde{A}}_{env(H_S(x))} = \{x, \mu_{\tilde{\tilde{A}}}(x), \nu_{\tilde{\tilde{A}}}(x) | x \in X\}$ with $\mu_{\tilde{\tilde{A}}}(x)$ and $\nu_{\tilde{\tilde{A}}}(x)$ defined as in Eq.* (5)*:*

$$\mu_{\tilde{\tilde{A}}}(x) = H_S^-(x),\ \ \nu_{\tilde{\tilde{A}}}(x) = 1 - H_S^+(x). \tag{5}$$

Despite this enhancement, combining LFM with the DM problems which require accurate and sensitive results is a challenging issue. Because the dependency between the size of term set and width of corresponding FSs causes loss of information. To obtain accurate and sensitive results in LFM without any additional mechanism, the size of the LTS should be big. On the other hand, defectiveness of items is a qualitative concept so the logical meanings of the verbal terms are dramatically lost, and subjectivity of the assessments is dramatically increased while the size of term set is getting bigger. Even if the possible biggest term set not causing loss of interpretability is determined, loss of information is still faced. Because the width of FS is directly dependent on term set size in LFM. In other words, the quality of the obtained results is directly dependent on the term set building decisions and loss of information is faced in all situations: working with a small, medium, or large LTSs. This is not an issue for MCDM problems because the best alternative is decided by ranking the alternatives in exploitation phase by just considering the membership grades. For example, the term set seen in Fig. 1 offered by Martnez & Herrera [32] having FSs with width 0.334 is a convenient LTS for MCDM problems but the width of FS is extremely big for item defectiveness concept to reach reasonable results. A linguistic statement such as $(Medium, -0.3) = (0.283, 0.450, 0.617)$ is not suitable to obtain accurate results for acceptance probability, rejection probability, average output quality etc.

In this study, model structure improvement has been preferred to build a more common linguistic approach. The suggested procedure combines the advantages and eliminates the drawbacks of 2-tuple and HFLTS approaches. A step has been added to the definition phase to improve the accuracy by reducing the loss of information caused by the size of LTS and width of FS dependency with the help of an intensifying linguistic modifier. This has provided us an opportunity to keep interpretability in a certain level and achieve sensitive results at the same time by adopting a relatively small LTS. Indeed, reasonable results have been reached on a numerical ASP example from manufacturing industry despite using a small LTS in Section 5. In the proposed modified HFLTS approach, the assessments are made in 2-tuple form, so the context-free grammar has been redefined in 2-tuple form as shown in refdef7. The offered HFLTS approach gives ability to make assessments with or without DOI of 2-tuple, and with or without context-free grammar. The procedure is also usable with the fully linguistic 2-tuple approach offered by Ik [10] in which DOI is also a linguistic variable. This flexibility has also been demonstrated on the numerical example.

Definition 3.2. *Let $S = \{s_i\} = \{s_0, , s_g\}$ be a linguistic primary term set, $U = \{u_j\}$ be a unary relation set, $B = \{b_k\}$ be a binary relation set, $C = \{c_l\}$ be a conjunction set, and $s_{i_{2T}}$ be a 2-tuple statement. A composite 2-tuple term t_{2T} is an ordered pair presented in Eq.* (6) *and a 2-tuple context-free grammar statement II_{2T} is defined in the form given in Eq.* (7)*:*

$$t_{2T} \stackrel{\text{def}}{=} (\langle u_j \rangle, \langle s_{i_{2T}} \rangle) \text{ or } (\langle b_k \rangle, \langle s_i \rangle, \langle c_l \rangle, \langle s_j \rangle)\ , u_j \in U, s_i, s_j \in S, b_k \in B, c_l \in C, \tag{6}$$

$$II \stackrel{\text{def}}{=} \langle s_i \rangle \text{ or } \langle t_{2T} \rangle,\ s_i \in S, \tag{7}$$

Intensifying modifiers were discussed around some specific keywords in the literature. Very and extremely keywords were analyzed in the scope of intensifying modifiers, and roughly and more or less keywords in the scope of negative intensifying weakening modifiers [88]. These keywords are generally used in the same statement with the primary linguistic terms. However, using the modifier terms together with context-free grammar in the same linguistic statements may be too confusing. Another issue is occurred in the aggregation of the statements in case of working with multiple experts. For these reasons, most modification is more suitable for this study. The main objective is to increase sensitivity of the HFLTS, so all the mentioned keywords are used in the interest of intensifying modification with the help of a guiding label such as availability of semi-defective items over fully defectives. There is no need for weakening modifiers because the initial FS are the limiting value and associating extremely keyword with this limiting value is a plausible approach. In the light of these directions, the term set has been decided as follows: roughly, more or less, moderate, very, extremely. The expert assessments about modifiers have been combined by an aggregation operator and the FS has been transformed in a post modification step. To provide simplicity, all the expert assessments handled with the same importance level and arithmetic mean has been used as the aggregation operator as presented in Definition 3.3.

Definition 3.3. *Let $S = \{s_i\} = \{s_0, , s_g\}$ be a linguistic primary term set, $x_j = \{x_1, , x_n\}$ be linguistic statements, $*Delta^{-1}$ be a function converting a 2-tuple statement $(r_j, \alpha_j) = (s_i, \alpha_j)$ into its numerical equivalent $\beta_j = j + \alpha_j$ and, Δ be the inverse of $*Delta^{-1}$ producing a 2-tuple statement from a number. The arithmetic mean ($\bar{x}$) of these n* *linguistic statements is defined as shown in Eq. 8, if statements are ordinary linguistic statements and is acquired as shown in Eq. 9* [32] *and they are 2-tuple statements.*

$$\bar{x} = \begin{cases} x_{\frac{n+1}{2}}, & if\, n\ is\ odd \\ \frac{1}{2}\left(x_{\frac{n}{2}} + x_{\frac{n}{2}+1}\right), & if\, n\ is\ even \end{cases}, \tag{8}$$

$$\bar{x} = \Delta\left(\sum_{j=1}^{n}\left(\frac{1}{n}\Delta^{-1}(r_j, \alpha_j)\right)\right) = \Delta\left(\frac{1}{n}\sum_{j=1}^{n}\beta_j\right). \tag{9}$$

The enhanced HFLTS approach has additional steps in definition, gathering and aggregation phases but the transformation phase has been protected as is. Once the linguistic intervals are obtained at the end of aggregation phase, the current exploitation phase has been followed for MCDM. However, the obtained linguistic interval has been converted into IFS for the problems requiring fuzzy modeling such as ASP as illustrated in Fig. 3. Renewed HFLTS phases have been formed as follows:

1. *Definition*: (a) Semantics and syntax of primary LTS, (b) semantics and syntax of LTS for intensifying modifier, (c) context-free grammar are defined and, (d) definition space and FS shape are determined problem specific.
2. *Gathering*: Assessments are gathered from experts (a) for decision variable in 2-tuple context-free grammar format and, (b) for intensifying multiplier in 2-tuple format.
3. *Transformation*: Context-free grammar statements are converted to HFLTS statements.
4. *Aggregation*: (a) Linguistic intervals are obtained by envelopment of linguistic expressions, (b) corresponding IFS is procured from linguistic interval. For MCDM problems, (c) two aggregation operators are decided, (d) pessimistic and optimistic collective preference relations are obtained by the first aggregation operator, (e) pessimistic and optimistic collective preferences for each alternative are computed by the second aggregation operator. For other type of problems, (c) the linguistic statements of modifier are aggregated and, (d) the IFS is narrowed with the ratio presented in Definition 2.2.
5. *Exploitation*: (a) Vector of intervals of collective preferences are built for the alternatives (b) preference relation between alternatives is built, (c) a non-dominance choice degree is applied, (d) set of alternatives is ranked and the best one is selected.

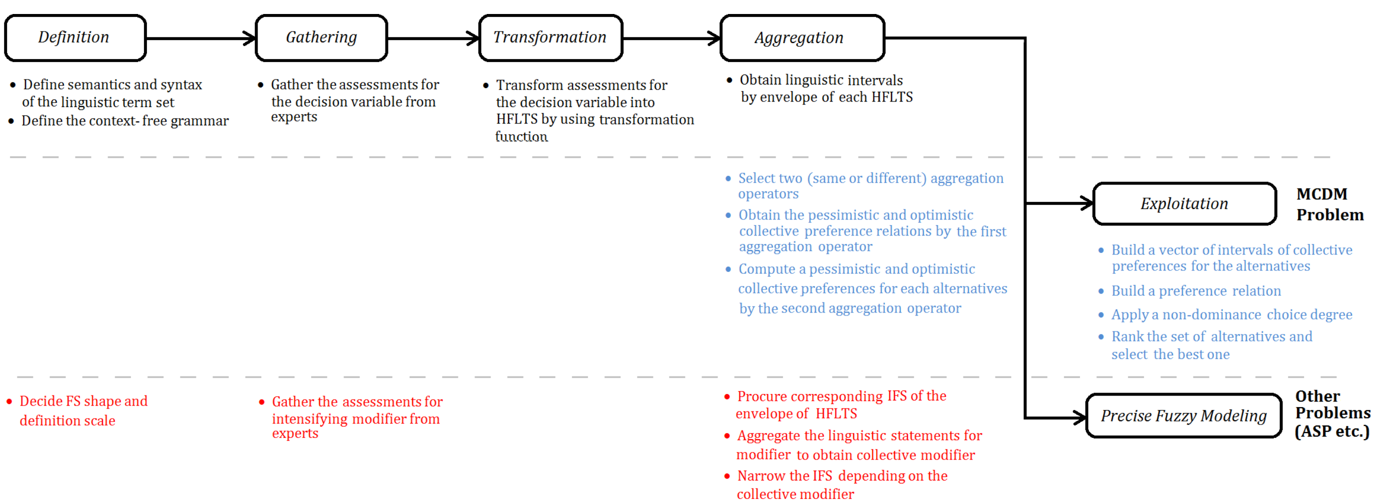

Figure 3: The stages of the proposed methodology based on enhanced HFLTS.

The symbolic representation of the enhanced HFLTS approach has been introduced in Definition 3.4 and 4-tuple representation of modified HFLTS statements has been presented in Definition 3.5.

Definition 3.4. *Let $S = \{s_i\} = \{s_0, , s_g\}$ be a primary LTS, E_{G_H} be a rule set transforming linguistic 2-tuple context-free grammar statement $II2T_S$ into HFLTS statement H_S, $M = \{m_j\} = \{m_0, , m_h\}$ be a LTS for intensifying modifier, $I2T_M$ be a 2-tuple statement for the linguistic intensifying modifier, ϕ be an aggregation operator, P_D be preference relation matrix between alternatives in DM problems and X^{ND} be non-dominated alternatives depending on the non-dominance degrees of alternatives (NDD). The enhanced $HFLTS$ approach $HFLTS^{'}$ is a set of successive steps as formulated in Eq. 10.*

$$HFLTS^{'} : II2T_S \overset{E_{G_H}}{\rightarrow} H_S \overset{env}{\rightarrow} \tilde{\tilde{A}}_{env(H_S)} \overset{I2T_M}{\rightarrow} \tilde{\tilde{A}}' \overset{\phi}{\rightarrow} P_D \overset{NDD}{\rightarrow} X^{ND}. \tag{10}$$

Definition 3.5. *Let $HFLTS'$ be the enhanced HFLTS approach, $H_{S,M}$ be a 4-tuple statement entered in definition and gathering phases of $HFLTS'$, S be the primary LTS, M be the term set for linguistic intensifying modifier, $X = [a, b]$ be the definition space and, l be the FS shape in words. $H_{S,M}$ is represented in 4-tuple form such that $H_{S,M} = \langle X,\ l,\ II2T_S,\ I2T_M \rangle$.*

HFLTS is generally discussed with the triangular and trapezoidal shaped FSs in the literature, but the FS shape has no effect on the results of MCDM problems because of using membership grades in formulations. However, the proposed HFTLS approach considers the FS shape in formulations so the results are affected by the FS shape. Both the symmetric and non-symmetric FSs can be employed by the proposed HFLTS approach. Example 3.6 shows an example for a z-shaped FS that is a non-symmetric FS.

Example 3.6. *Let S = non-defective (N), insensibly defective (I), slightly defective (S), partially defective (P), almost totally defective (A), totally defective (T) be the primary LTS and M = roughly (R), more or less (L), moderate (M), very (V), extremely (E) be the LTS for intensifying modifier labeled as Approximate proportion of semi defective items over the fully defective items. The unary relations less than and greater than are not suitable with ASP problem so only the binary relation between with the conjunction and is usable. Corresponding IFS for the 4-tuple $HFLTS'$ statement $H_{S,M} = \langle [0, 1], (z\ shaped), (between\ (N, +0.2)\ and\ (S, -0.5)), (M, 0) \rangle$ is found as seen in Fig. 4.*

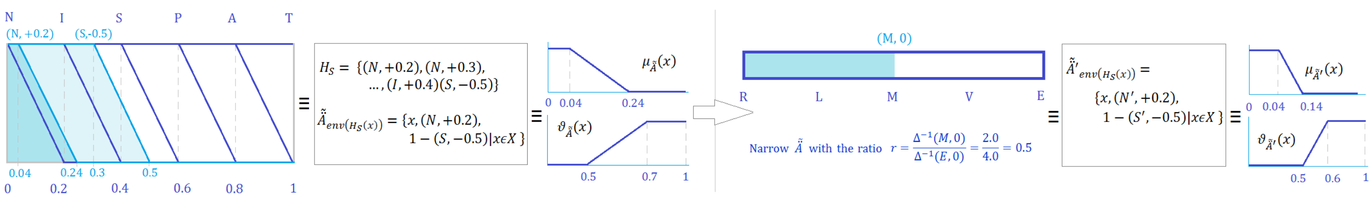

Figure 4: IFS for the 4-tuple HFLTS statement [0,1],(z shaped),(Between (N,+0.2) and (S,-0.5)),(M,0).

4 Acceptance sampling plans based on hesitant fuzzy linguistic term sets

The effectiveness in terms of time and cost has a high significance in supply chains especially in inspection stages. To obtain high quality with a lower cost, the quality of the produced items is measured statistically when a pre-determined risk is acceptable. ASP is a certain set of rules offers inspecting a small set of items instead of all the produced items by guaranteeing a specific quality level. Sample size (n) and maximum allowed defective item count (c) are ASP parameters which are adjusted to satisfy the mentioned risk margins. Double ASP (DASP) is a two-phased ASP used for reaching lower risks with relatively small sample sizes. ASP is a special type of DM problem including qualitative information to define defectiveness level of items. Unlike the classical MCDM problems, ASP problems do not need comparison between the alternatives, but they need PFM to obtain more sensitive results and calculations.

The defectiveness ratio of the parties is defined as a certain value for a traditional ASP. Depending on this, the inspection process is also thought as a certain event. However, in some real cases, for example when some changes are made in production process, the historical data becomes useless, and a certain value cannot be decided for defectiveness ratio of the items. For these cases, the best way is to use expert opinion. Since the expert opinions about the defectiveness ratio can include some hesitancy, the inspection procedure should be designed with respect to this type of uncertainty. According to Definition 6, the envelopment of hesitant fuzzy statements generates IFSs. This means, the intuitionistic fuzzy ASPs formulated by Ik & Kaya [11] become usable with the hesitant fuzzy linguistic expert opinions. In intuitionistic fuzzy binomial ASPs, the defectiveness of the items is an IFS ($\tilde{\tilde{A}} = \{x, \mu_{\tilde{\tilde{A}}}(x) = \tilde{p}, \nu_{\tilde{\tilde{A}}}(x)\tilde{q}|x \in X\}$) satisfying $\tilde{p} + \tilde{q} \leq 1$ with the hesitancy/non-determinacy degree $\tilde{\pi} = 1 - \tilde{p} - \tilde{q}$ where $\tilde{p}$ and $\tilde{q}$ represent the fuzzy defective and non-defective item proportions in a lot, respectively [11]. Plan parameters n and c are also FSs in the mentioned study. In this scenario, the fuzziness is caused by the expert judgements about the item defectiveness thus, the formulations have been revised, only the defectiveness of the items has been handled as FS and the plan parameters n and c have been considered as certain values in this study.

ASPs with hesitant defectiveness of items have three outcomes: acceptance, rejection, and non-determinacy. The outcomes are realized based on the conditions shown in Fig. 5 while n (n_1 and n_2 in DASPs), c (c_1 and c_2 in DASPs), τ (τ_1 and τ_2 in DASPs), d (d_1 and d_2 in DASPs) and i (i_1 and i_2 in DASPs) are representing the sample size, maximum allowed defective and non-determinate item counts, and observed defective and non-determinate item counts, respectively. The formulations of hesitant ASPs have been presented in Definitions 4.1-4.5. The symbols $\oplus$, $\ominus$, and $\otimes$ have been used for algebraic addition, subtraction, and multiplication operations between FSs in the rest of the paper. These are also algebraic operations, but they are defined with the help of the ordinary ones. The mathematical symbols of them have been differentiated from the ordinary ones to not bring any confusion.

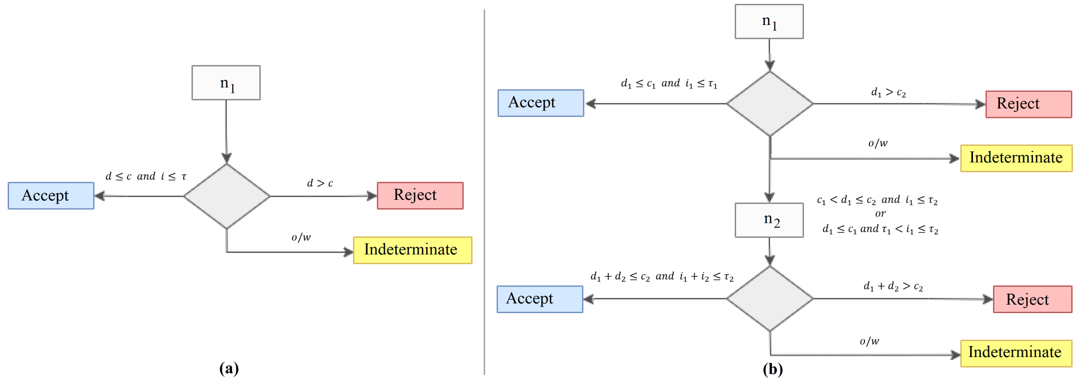

Figure 5: Intuitionistic fuzzy (a) single and (b) double ASPs for binomial distributions.

Definition 4.1. *Let $\tilde{\lambda}_p$, $\tilde{\lambda}_{p_1}$, $\tilde{\lambda}_{p_1}$ be the defectiveness frequencies and $\tilde{\lambda}_i$,$\tilde{\lambda}_{i_1}$, $\tilde{\lambda}_{i_2}$ be the non-determinacy frequencies of the items in a lot for SAPSs and DASPs. Lot acceptance probabilities for the single and double intuitionistic fuzzy ASPs ($\tilde{P}_a$) are calculated as shown in Eqs. 11-12.*

$$\begin{aligned}\tilde{P}_{a_{SASP}} &= \tilde{P}\{d \leqslant c,\ i \leqslant \tau \mid \tilde{p}, \tilde{q}\} \\ &= \tilde{P}_{a_{Binomial}} = \sum_{d=0}^{c}\left(\binom{n}{d} \otimes \tilde{p}^{d} \otimes \left[\sum_{i=0}^{\tau}\left(\binom{n-d}{i} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i} \otimes \tilde{q}^{(n-i-d)}\right)\right]\right) \\ &\approxeq \tilde{P}_{a_{Poisson}} = \sum_{d=0}^{c}\left(\frac{\tilde{\lambda}_p^{d}}{d!} \otimes \left[\sum_{i=0}^{\tau}\left(\frac{(\tilde{\lambda}_i)^{i}}{i!} \otimes e^{-(\tilde{\lambda}_p \oplus \tilde{\lambda}_i)}\right)\right]\right).\end{aligned} \tag{11}$$

$$\begin{aligned}\tilde{P}_{a_{DASP}} = \tilde{P}\{d_1 \leqslant c_1, i_1 \leqslant \tau_1 \mid \tilde{p}, \tilde{q}\} &\oplus \tilde{P}\{d_1 + d_2 \leqslant c_2, i_1 + i_2 \leqslant \tau_2 \mid c_1 < d_1 \leqslant c_2, i_1 \leqslant \tau_2 \text{ and } \tilde{p}, \tilde{q}\} \\ &\oplus \tilde{P}\{d_1 + d_2 \leqslant c_2, i_1 + i_2 \leqslant \tau_2 \mid d_1 \leqslant c_1, \tau_1 < i_1 \leqslant \tau_2 \text{ and } \tilde{p}, \tilde{q}\}.\end{aligned}$$

$$\begin{aligned}= \tilde{P}_{a_{Binomial}} = &\sum_{d_1=0}^{c_1}\left(\binom{n_1}{d_1} \otimes \tilde{p}^{d_1} \otimes \left[\sum_{i_1=0}^{\tau_1}\binom{n_1-d_1}{i_1} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_1} \otimes \tilde{q}^{(n_1-i_1-d_1)}\right]\right) \\ &\oplus \sum_{d_1=c_1+1}^{c_2}\left(\binom{n_1}{d_1} \otimes \tilde{p}^{d_1} \otimes \left[\sum_{i_1=0}^{\tau_2}\left(\binom{n_1-d_1}{i_1} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_1} \otimes \tilde{q}^{(n_1-i_1-d_1)} \otimes\right.\right.\right. \\ &\left.\left.\left.\left[\sum_{d_2=0}^{c_2-d_1}\left(\binom{n_2}{d_2} \otimes \tilde{p}^{d_2} \otimes \left[\sum_{i_2=0}^{\tau_2-i_1}\binom{n_2-d_2}{i_2} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_2} \otimes \tilde{q}^{(n_2-i_2-d_2)}\right]\right)\right]\right)\right]\right) \\ &\oplus \sum_{d_1=0}^{c_1}\left(\binom{n_1}{d_1} \otimes \tilde{p}^{d_1} \otimes \left[\sum_{i_1=\tau_1+1}^{\tau_2}\left(\binom{n_1-d_1}{i_1} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_1} \otimes \tilde{q}^{(n_1-i_1-d_1)} \otimes\right.\right.\right. \\ &\left.\left.\left.\left[\sum_{d_2=0}^{c_2-d_1}\left(\binom{n_2}{d_2} \otimes \tilde{p}^{d_2} \otimes \left[\sum_{i_2=0}^{\tau_2-i_1}\binom{n_2-d_2}{i_2} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_2} \otimes \tilde{q}^{(n_2-i_2-d_2)}\right]\right)\right]\right)\right]\right)\end{aligned}$$

$$\begin{aligned}\approxeq \tilde{P}_{a_{Poisson}} = &\sum_{d_1=0}^{c_1}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!} \otimes \left[\sum_{i_1=0}^{\tau_1}\frac{\tilde{\lambda}_{i_1}^{i_1}}{i_1!} \otimes e^{-(\tilde{\lambda}_{i_1} \oplus \tilde{\lambda}_{p_1})}\right]\right) \\ \oplus &\sum_{d_1=c_1+1}^{c_2}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!} \otimes \left[\sum_{i_1=0}^{\tau_2}\left(\frac{\tilde{\lambda}_{i_1}^{i_1}}{i_1!} \otimes e^{-(\tilde{\lambda}_{i_1} \oplus \tilde{\lambda}_{p_1})} \otimes \left[\sum_{d_2=0}^{c_2-d_1}\left(\frac{\tilde{\lambda}_{p_2}^{d_2}}{d_2!} \otimes \left[\sum_{i_2=0}^{\tau_2-i_1}\frac{\tilde{\lambda}_{i_2}^{i_2}}{i_2!} \otimes e^{-(\tilde{\lambda}_{i_2} \oplus \tilde{\lambda}_{p_2})}\right]\right)\right]\right)\right]\right) \\ \oplus &\sum_{d_1=0}^{c_1}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!} \otimes \left[\sum_{i_1=\tau_1+1}^{\tau_2}\left(\frac{\tilde{\lambda}_{i_1}^{i_1}}{i_1!} \otimes e^{-(\tilde{\lambda}_{i_1} \oplus \tilde{\lambda}_{p_1})} \otimes \left[\sum_{d_2=0}^{c_2-d_1}\left(\frac{\tilde{\lambda}_{p_2}^{d_2}}{d_2!} \otimes \left[\sum_{i_2=0}^{\tau_2-i_1}\frac{\tilde{\lambda}_{i_2}^{i_2}}{i_2!} \otimes e^{-(\tilde{\lambda}_{i_2} \oplus \tilde{\lambda}_{p_2})}\right]\right)\right]\right)\right]\right).\end{aligned} \tag{12}$$

Definition 4.2. *Let $\tilde{\lambda}_p$, $\tilde{\lambda}_{p_1}$ and, $\tilde{\lambda}_{p_2}$ be defectiveness frequencies and $\tilde{\lambda}_i$, $\tilde{\lambda}_{i_1}$ and, $\tilde{\lambda}_{i_2}$ be non-determinacy frequencies of the items in a lot for single ASPs (SAPSs) and DASPs. Lot rejection probability for the single and double intuitionistic fuzzy ASPs ($\tilde{P}_r$) are calculated as in Eqs. 13-14.*

$$\begin{aligned}\tilde{P}_{r_{SASP}} &= \tilde{P}\{d > c \mid \tilde{p}, \tilde{q}\}\\ &= \tilde{P}_{r_{Binomial}} = \sum_{d=c+1}^{n}\left(\binom{n}{d} \otimes \tilde{p}^d \otimes \left[\sum_{i=0}^{n-d}\left(\binom{n-d}{i} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^i \otimes \tilde{q}^{(n-i-d)}\right)\right]\right)\\ &\approxeq \tilde{P}_{r_{Poisson}} = \sum_{d=c+1}^{n}\left(\frac{\tilde{\lambda}_p^d}{d!} \otimes \left[\sum_{i=0}^{n-d}\left(\frac{(\tilde{\lambda}_i)^i}{i!} \otimes e^{-(\tilde{\lambda}_p \oplus \tilde{\lambda}_i)}\right)\right]\right).\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{P}_{r_{DASP}} = \tilde{P}\{d_1 > c_2 \mid \tilde{p}, \tilde{q}\} &\oplus \tilde{P}\{d_1 + d_2 > c_2 \mid c_1 < d_1 \leqslant c_2, i_1 \leqslant \tau_2 \; and \; \tilde{p}, \tilde{q}\}\\ &\oplus \tilde{P}\{d_1 + d_2 > c_2 \mid d_1 \leqslant c_1, \tau_1 < i_1 \leqslant \tau_2 \; and \; \tilde{p}, \tilde{q}\}.\end{aligned}$$

$$\begin{aligned}\tilde{P}_{r_{Binomial}} = &\sum_{d_1=c_2+1}^{n_1}\left(\binom{n_1}{d_1} \otimes \tilde{p}^{d_1} \otimes \left[\sum_{i_1=0}^{n_1-d_1}\binom{n_1-d_1}{i_1} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_1} \otimes \tilde{q}^{(n_1-i_1-d_1)}\right]\right)\\ &\oplus \sum_{d_1=c_1+1}^{c_2}\left(\binom{n_1}{d_1} \otimes \tilde{p}^{d_1} \otimes \left[\sum_{i_1=0}^{\tau_2}\left(\binom{n_1-d_1}{i_1} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_1} \otimes \tilde{q}^{(n_1-i_1-d_1)} \otimes\right.\right.\right.\\ &\left.\left.\left.\left[\sum_{d_2=c_2-d_1+1}^{n_2}\left(\binom{n_2}{d_2} \otimes \tilde{p}^{d_2} \otimes \left[\sum_{i_2=0}^{n_2-d_2}\binom{n_2-d_2}{i_2} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_2} \otimes \tilde{q}^{(n_2-i_2-d_2)}\right]\right)\right]\right)\right]\right)\\ &\oplus \sum_{d_1=0}^{c_1}\left(\binom{n_1}{d_1} \otimes \tilde{p}^{d_1} \otimes \left[\sum_{i_1=\tau_1+1}^{\tau_2}\left(\binom{n_1-d_1}{i_1} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_1} \otimes \tilde{q}^{(n_1-i_1-d_1)} \otimes\right.\right.\right.\\ &\left.\left.\left.\left[\sum_{d_2=c_2-d_1+1}^{n_2}\left(\binom{n_2}{d_2} \otimes \tilde{p}^{d_2} \otimes \left[\sum_{i_2=0}^{n_2-d_2}\binom{n_2-d_2}{i_2} \otimes (1 \ominus \tilde{p} \ominus \tilde{q})^{i_2} \otimes \tilde{q}^{(n_2-i_2-d_2)}\right]\right)\right]\right)\right]\right)\\ \approxeq \tilde{P}_{r_{Poisson}} = &\sum_{d_1=c_2+1}^{n_1}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!} \otimes \left[\sum_{i_1=0}^{n_1-d_1}\frac{\tilde{\lambda}_{i_1}^{i_1}}{i_1!} \otimes e^{-(\tilde{\lambda}_{i_1} \oplus \tilde{\lambda}_{p_1})}\right]\right)\\ &\oplus \sum_{d_1=c_1+1}^{c_2}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!} \otimes \left[\sum_{i_1=0}^{\tau_2}\left(\frac{\tilde{\lambda}_{i_1}^{i_1}}{i_1!} \otimes e^{-(\tilde{\lambda}_{i_1} \oplus \tilde{\lambda}_{p_1})} \otimes \left[\sum_{d_2=c_2-d_1+1}^{n_2}\left(\frac{\tilde{\lambda}_{p_2}^{d_2}}{d_2!} \otimes \left[\sum_{i_2=0}^{n_2-d_2}\frac{\tilde{\lambda}_{i_2}^{i_2}}{i_2!} \otimes e^{-(\tilde{\lambda}_{i_2} \oplus \tilde{\lambda}_{p_2})}\right]\right)\right]\right)\right]\right)\\ &\oplus \sum_{d_1=0}^{c_1}\left(\frac{\tilde{\lambda}_{p_1}^{d_1}}{d_1!} \otimes \left[\sum_{i_1=\tau_1+1}^{\tau_2}\left(\frac{\tilde{\lambda}_{i_1}^{i_1}}{i_1!} \otimes e^{-(\tilde{\lambda}_{i_1} \oplus \tilde{\lambda}_{p_1})} \otimes \left[\sum_{d_2=c_2-d_1+1}^{n_2}\left(\frac{\tilde{\lambda}_{p_2}^{d_2}}{d_2!} \otimes \left[\sum_{i_2=0}^{n_2-d_2}\frac{\tilde{\lambda}_{i_2}^{i_2}}{i_2!} \otimes e^{-(\tilde{\lambda}_{i_2} \oplus \tilde{\lambda}_{p_2})}\right]\right)\right]\right)\right]\right).\end{aligned} \tag{14}$$

Definition 4.3. *Let $\tilde{\lambda}_p$, $\tilde{\lambda}_{p_1}$ and, $\tilde{\lambda}_{p_2}$ be defectiveness frequencies of the items in a lot for SAPSs and DASPs. Average outgoing quality ($\widetilde{AOQ}$) is calculated as in Eq. 15.*

$$\widetilde{AOQ} = \tilde{P}_a \otimes \tilde{p} = \tilde{P}_a \otimes \tilde{\lambda}_p \oslash n = \tilde{P}_a \otimes \tilde{\lambda}_{p_1} \oslash n_1 = \tilde{P}_a \otimes \tilde{\lambda}_{p_2} \oslash n_2. \tag{15}$$

If τ is exceeded, the lot is regarded as non-determinate and sampling is repeated. In other words, sampling is repeated with the probability of $(1 - \tilde{P}_a - \tilde{P}_r)$. However, repetition may not be allowed for some products. Average total inspection (ATI) is calculated depending on the allowed repetition number.

Definition 4.4. *Let m be the allowed repetition number. $\tilde{P}_a^I$ be acceptance probability in the first step and, $\tilde{P}_a^{II}$ be acceptance probability in the second step of DASP. $\widetilde{ATI}$ for non-repetitive ASPs ($\widetilde{ATI}_{(0)}$), and $\widetilde{ATI}$ for repetitive ASPs ($\widetilde{ATI}_{(m)}$) are derived as in Eqs. 16-19.*

$$\widetilde{ATI}_{(0)_{SASP}} = n \oplus (1 \ominus \tilde{P}_a) \otimes (N - n). \tag{16}$$

$$\widetilde{ATI}_{(m)_{SASP}} = n \oplus \tilde{P}_r \otimes (N-n) \oplus (1 \ominus \tilde{P}_a \ominus \tilde{P}_r) \otimes \widetilde{ATI}_{(m-1)}. \tag{17}$$

$$\begin{aligned}\widetilde{ATI}_{(0)_{DASP}} =&(n_1 \otimes \tilde{P}_a^I) \oplus \Big((n_1+n_2) \otimes \tilde{P}_a^{II})\Big) \oplus \Big(N \otimes (1 \ominus \tilde{P}_a)\Big)\\ =&n_1 \otimes \tilde{P}\{d_1 \leqslant c_1, i_1 < \tau_1 \mid \tilde{p}, \tilde{q}\}\\ &\oplus (n_1+n_2) \otimes \tilde{P}\{d_1+d_2 \leqslant c_2, i_2 \leqslant \tau_2 \mid c_1 < d_1 \leqslant c_2, i_1 \leqslant \tau_1 \; and \; \tilde{p}, \tilde{q}\}\\ &\oplus \big(N \otimes (1-\tilde{P}_a)\big).\end{aligned} \tag{18}$$

$$\begin{aligned}\widetilde{ATI}_{(m)_{DASP}} =&(n_1 \otimes \tilde{P}_a^I) \oplus \Big((n_1+n_2) \otimes \tilde{P}_a^{II})\Big) \oplus \Big(N \otimes \tilde{P}_r\Big) \oplus \Big((1 \ominus \tilde{P}_a \ominus \tilde{P}_r) \otimes \widetilde{ATI}_{(m-1)}\Big)\\ =&n_1 \otimes \tilde{P}\{d_1 \leqslant c_1, i_1 < \tau_1 \mid \tilde{p}, \tilde{q}\}\\ &\oplus (n_1+n_2) \otimes \tilde{P}\{d_1+d_2 \leqslant c_2, i_2 \leqslant \tau_2 \mid c_1 < d_1 \leqslant c_2, i_1 \leqslant \tau_1 \; and \; \tilde{p}, \tilde{q}\}\\ &\oplus (N \otimes \tilde{P}_r) \oplus \big((1 \ominus \tilde{P}_a \ominus \tilde{P}_r) \otimes (\widetilde{ATI}_{(m-1)})\big).\end{aligned} \tag{19}$$

Definition 4.5. *Let $\tilde{P}^I$ be the probability of termination of the DASP in the first step as approval, rejection, or non-determination. $\widetilde{ASN}$ is calculated as in Eq. 20.*

$$\begin{aligned}\widetilde{ASN} &= n_1 \oplus \Big(n_2 \otimes (1 \ominus \tilde{P}^I)\Big)\\ &= n_1 \oplus n_2 \otimes \Big(1 \ominus \Big(\tilde{P}\{d_1 \leqslant c_1 \mid \tilde{p}, \tilde{q}\} \oplus \tilde{P}\{d_1 > c_2 \mid \tilde{p}, \tilde{q}\} \oplus \tilde{P}\{i_1 > \tau_2, d_1 \leqslant c_1 \mid \tilde{p}, \tilde{q}\}\Big)\Big).\end{aligned} \tag{20}$$

5 An illustrative example from manufacturing industry

In this section, the proposed HFLTS approach has been demonstrated and analyzed on a numerical example from an engine manufacturing company. The results have been compared with the results of the studies of Rodrguez et al. [37], Chen & Hong [4], Liu & Rodrguez [24] and Romero et al. [38]. These have been preferred as benchmark methods because all gives ability to work with multiple experts and offer a different envelopment technique. In fact, none of these studies give ability to continue with PFM originally but we have adapted them by adhering to their aggregation and envelopment procedures. To obtain comparable results, defectiveness of the items has been modeled as a triangular FS. The term set presented in Fig. 6 and the context-free grammar presented in Example 3.6 have been considered appropriate for this numerical example. The context-free grammar has been integrated with the 2-tuple LFM as presented in Definition 3.2. The set $\{roughly, more\ or\ less, moderate, very, extremely\}$ has been used for the linguistic intensifying modifier.

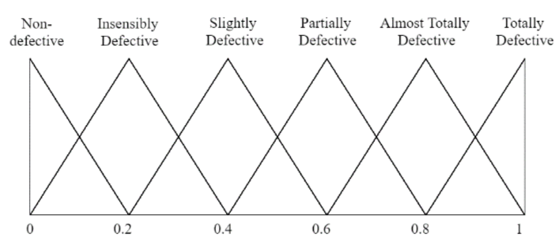

Figure 6: The term set used in numerical example.

The company purchases the valves of the engines from a supplier, and the acceptance sampling procedure is performed for these valves. The valves are sometimes purchased from alternative suppliers due to the fluctuations in demand. In such cases, the defectiveness proportion of the valves are not surely known because of insufficiency of the historical data. The company has three experts (E_1, E_2, E_3) to ensure assessments about defectiveness of items as hesitant linguistic statements. The expert evaluations for approximate defective item proportion inside fully defectives are given below respectively:

- $E_1 = \{II2T_{S_1}, I2T_{M_1}\}$: Between insensible defective +0.2 and slightly defective -0.4, more or less.
- $E_2 = \{II2T_{S_2}, I2T_{M_2}\}$: Insensible defective -0.3 or insensible defective -0.4, moderate +0.2.
- $E_3 = \{II2T_{S_3}, I2T_{M_3}\}$: Insensible defective, moderate

5.1 Aggregation of the expert evaluations

The context-free grammar statements are turned into hesitant linguistic statements by the transforming rule set given in Definition 3.2 as $E_i = \{II2T_{S_i}, I2T_{M_i}\} \overset{E_{G_H}}{\to} (H_{S_i}, I2T_{M_i})$. Note that, this rule set does not contain a rule for disjunction expression or. The rule can be defined basically as $E_{G_H}(\langle s_i\rangle, \langle d_m\rangle = or, \langle s_j\rangle) = \{s_i, s_j\}$. The following hesitant linguistic sets are reached from context-free grammar assessments:

$$E_1 = (\{(I, +0.2), (I, +0.3), , (S, -0, 5), (S, -0.4)\}, (L, 0)),$$
$$E_2 = (\{(I, -0.4), (I, -0.3)\}, (M, +0.2)),$$
$$E_3 = (\{(I, 0)\}, (M, 0)).$$

The hesitant linguistic statements are combined with envelopment technique explained in Definition 3.1 to turn them into IFSs. Linguistic statements for intensifying modifier are aggregated as in Definition 3.3. In this way, the following IFS and modifier are obtained:

$$\tilde{\tilde{A}}_{env(H_S)} = \{x, H_{S^-}, 1 - H_{S^+}|x \in X\} = \{x, (I, -0.4), 1 - (S, -0.4)|x \in X\},$$
$$\overline{I2T}_{M_i} = \Delta\Big(\sum_{i=1}^{n} \frac{1}{n}\Delta(r_i, \alpha_i)\Big) = \Delta\big(\frac{1}{3}(1.0 + 2.2 + 2.0)\big) = (M, -0.3).$$

The obtained set is rewritten as a numeric triangular IFS as $\tilde{\tilde{A}}_{env(H_S)} = \{x, (0, 0.12, 0.32), (0.48, 0.68, 0.88)|x \in X\}$. It should be narrowed depending on Definition 2 with the ratio $r = 1 - \frac{\Delta^{-1}(M,-0.3)}{4} = \frac{1.7}{4} = 0.425$. So, the final IFS is found as $\ddot{\tilde{A}}_{env(H_S)} = x, (0.035, 0.12, 0.205), (0.595, 0.68, 0.765)|x \in X$ and therefore non-determinacy degree is found as $\pi_{\tilde{\tilde{A}}} = 0.2$. The obtained final assessment is reasonably sensitive even though a small LTS has been employed.
Table 1 shows the numerical equivalent of the assessments for the item defectiveness by using the methods presented in benchmark studies and the method proposed in this study. The benchmark studies [24],[37], and [4] do not use 2-tuple LFM in data collection phase, so the above assessments are not directly usable in calculations. For this reason, the nearest primary terms have been regarded as the expert assessments in calculations. In study of Rodrguez et al.**4**, linguistic intervals are calculated for the alternatives. The derived interval P^R=$[\Delta^{-1}(Insensibly\ Defective, 0)$, $\Delta^{-1}(Insensibly\ Defective, 0.333)]$ can be used as the item defectiveness by turning into its numerical equivalent $[0.2, 0.2666]$ with the help of Fig. 6. Chen & Hong [4] builds a trapezoidal FS $T = (a^i_L, a^i_M, a^j_M, a^j_R)$ for the hesitant statement between a_i and a_j where $a_i = (a^i_L, a^i_M, a^i_R)$ and $a_i = (a^j_L, a^j_M, a^j_R)$ are two triangular FSs. In our example, the trapezoidal fuzzy item defectiveness is found as $(0, 0.2, 0.4, 0.6)$. Liu & Rodrguez [24] uses similar approach to find the trapezoidal FS and reaches the same results with [4] if $i + 1 = j$ is satisfied for the statement between a_i and a_j. According to proposal of Romero et al. [38], the aggregated assessments is also obtained as a trapezoidal FS with the parameters $(0.02, 0.187, 0.22, 0.42)$.

There are two studies in the literature combining 2-tuple approach with HFLTS, but these are not suitable to use as benchmark studies because of their linguistic statement schema. Tan et al [39] adds a numerical second term to the linguistic statement to use it as weight. This approach is quite different from the ordinary 2-tuple LFM whose second term is in [-0.5, 0.5) and not compatible with the expert assessments given above. Beg & Rashid [1] groups the 2-tuple statements having the same linguistic term with a new representation such as $\{(s_2, -0.2), (s_2, 0, 3), (s_2, 0.1)\} = (s_2, (-0.2, 0.3, 0.1))$. This approach does not allow to handle the assessment of the first expert in our numerical example. Even if the assessment of the first expert split into two parts as if two separate assessments, the aggregation of the expert assessments obtained as $(Slightly\ Defective, (-0.4, 0.2)) = \{(Slightly\ Defective, -0.4), (Slightly\ Defective, 0.2)\}$. This means, there are two separate assessments to use in calculations but the ASP formulations we have used does not support multiple inputs. As a workaround solution, the calculation can be made multiple times and multiple results are obtained but this procedure makes the sensitivity analysis complicated.

The numerical form of the obtained final assessment presented in Table 1 shows that the benchmark studies have two big problems about representing the real case properly:

- *They do not protect the fuzzy set shape of the assessments:* Even though the assessments are triangular FSs, one of them turns it into interval valued number and the three of them turn it into trapezoidal FSs after the aggregation operation.
- *They ignore the non-determinacy arising from the difference of expert opinions:* According to the HFS theory, the hesitative expert evaluations hinder the complete information about the event and cause non-determinacy. However, the benchmark studies ignore the non-determinacy and consider the assessments as if a complete information case is available.

Table 1: Aggregated assessments in numerical form for the proposed and benchmark methods

Method - Abbreviation	Numerical Aggregated Assessments
Rodrguez et al. [37] RD	Interval valued type-1 FS: $\mu = [0.2, 0.2666]$
Chen & Hong [4] - CH	Trapezoidal type-1 FS: $\mu = (0, 0.2, 0.4, 0.6)$
Lui & Rodrguez [24] LR	Trapezoidal type-1 FS: $\mu = (0, 0.2, 0.4, 0.6)$
Romero et al. [38] - ROM	Trapezoidal type-1 FS: $\mu = (0.02, 0.187, 0.22, 0.42)$
Proposed Method	Triangular intuitionistic FS: $\mu = (0.035, 0.12, 0.205)$, $\nu = (0.595, 0.68, 0.765)$

5.2 Acceptance sampling results

Suppose the company has decided to operate ASPs having similar plan parameters with MIL-STD-105E normal inspection plan for the lots having 500 items. Acceptance quality limit (AQL) has been decided as 2.5% for defective items, and as 6.5% for non-determinate items. Depending on this, ASP parameters are found as $n = 50, c = 3, \tau = 4$ for SASP and as $n_1 = 32, n_2 = 32, c_1 = 1, c_2 = 4, \tau_1 = 2, \tau_2 = 4$ for DASP. Results have been obtained as in Table 2 for the given plan parameters and expert assessments by using Definitions 4.1-4.5.

Table 2: Single and double ASP results for the proposed and benchmark methods

	Single ASP			
	Proposed Method	**RD**	**LR/CH**	**ROM**
$\widetilde{P}_a$	(0, 0.0013, 0.0158)	[0.0002, 0.0057]	(0, 0, 0.0057, 1)	(0, 0.0022, 0.0101, 0.9822)
$\widetilde{P}_r$	(0.0973, 0.8655, 0.9955)	[0.9943, 0.9998]	(0, 0.9943, 1, 1)	(0.0178, 0.9899, 0.9978, 1)
$\{\widetilde{AOQ}_{min}, \widetilde{AOQ}_{max}\}$	{0, 0.0006}	{0.0001, 0.0011}	{0, 0.0388}	{0, 0.0388}
$\widetilde{ATI}_{r=0}$	(492.877, 499.434, 499.993)	[497.455, 499.907]	(50, 497.455, 500, 500)	(57.9911, 495.459, 499, 500)
$\widetilde{ATI}_{r=1}$	(93.792, 500.215, 505.148)	[497.455, 499.907]	(50, 497.455, 500, 500)	(57.9911, 495.459, 499, 500)
	Double ASP			
	Proposed Method	RD	LR/CH	ROM
$\widetilde{P}_a$	(0, 0.0015, 0.0204)	[0.0006, 0.0084]	(0, 0, 0.0084, 1)	(0, 0.0039, 0.0138, 0.9923)
$\widetilde{P}_r$	(0.021, 0.4369, 0.8364)	[0.9916, 0.9994]	(0, 0.9916, 1, 1)	(0.0077, 0.9862, 0.9961, 1)
$\{\widetilde{AOQ}_{min}, \widetilde{AOQ}_{max}\}$	{0, 0.0007}	{0, 0.0421}	{0.0002, 0.0017}	{0, 0.0421}
$\widetilde{ATI}_{r=0}$	(490.474, 499.284, 499.978)	[496.094, 499.701]	(32, 496.094, 499.999, 500)	(39.651, 493.646, 498.168, 500)
$\widetilde{ATI}_{r=1}$	(11.2011, 493.934, 499.861)	[496.094, 499.701]	(32, 496.094, 499.999, 500)	(39.651, 493.646, 498.168, 500)
$\{\widetilde{ASN}_{min}, \widetilde{ASN}_{max}\}$	{32.7087, 37.9123}	{33.3424, 38.3121}	{32, 52.4846}	{32.0104, 52.4846}

5.3 Discussion

When the expert assessments are analyzed, it is seen that each of experts has different personal characteristic. The first expert is more pessimistic than the others. Moreover, his/her evaluation is more hesitative than the others. On the other hand, the third expert makes a less sensitive assessment. These differentiated characteristics of the experts cause variation between the assessments and affects the certainty of the results. The proposed approach provides more reliable results than the benchmark studies for this scenario as detailed in Table 3.

Comparative analysis presented in Table 3 shows that, changing the shape of FS and ignoring the non-determinacy caused by the variation of the expert assessments are big drawbacks of the benchmark methods in terms of accuracy for DM problems requiring sensitive calculation in decision phase. Since the benchmark methods are developed in line with the requirements of MCDM problems, they do not provide high sensitivity analysis capabilities out of the exploitation phase. The proposed HFLTS approach brings a novel contribution for hesitant modeling of the problems that need sensitive calculation in decision cycle. Apart from ASP problems, some other type of DM problems for instance engineering economy problems such as internal rate of return analysis while comparing investment alternatives can be modeled with the proposed HFLTS methods successfully.

6 Conclusions

HFLTS is an efficient technique to solve MCDM problems by using qualitative and hesitative experts evaluations. The context-free grammar gives a great flexibility to model express opinions in nearly natural language. Since HFLTS is designed for MCDM problems, it does not take the shape of FS into account in decision phase. It just uses membership degrees instead of membership functions to determine the best alternative by ranking the alternatives in decision process.

However, some types of DM problems such as ASP, investment analysis in engineering economy etc. do not require exploitation phase but require accurate and sensitive numerical results to reach the best decision. This is just possible to give an ability to continue with PFM after aggregating the linguistic assessments. In the literature, several enhancements were done on the HFLTS methodology, but most of them have improved the exploitation and transformation phases. Additionally, there is no study which gives ability to continue with PFM and none of these extensions can be used to model the DM problems requiring sensitive calculation like ASP.

In the scope of this study, the HFLTS approach has been modified to make it usable for the DM problems requiring accurate and sensitive evaluations and numerical fuzzy calculations to reach the final decision. Combining LFM with the DM problems requiring sensitive results is a challenging issue because of the dependency between the size of term set and width of corresponding FSs. The quality of the obtained results is directly dependent to the term set building decisions and loss of information is faced in all situations: working with a small, medium, or large LTSs. This is not the case for all MCDM problems because the best alternative is decided by ranking the alternatives in exploitation phase by just considering the membership grades.

In this study, 2-tuple approach and linguistic fuzzy modifiers have been integrated with the HFLTS to obtain more sensitive calculations and reduce the loss of information caused by the dependency between LTS size and FS width. In addition, an envelopment method has been offered to present an approach to convert the hesitant linguistic assessments into numeric fuzzy membership functions to continue with PFM. As another contribution, one of DM problem ASP has been analyzed and re-formulated by using hesitative defectiveness information. These modifications have ensured to keep interpretability in a certain level and achieve sensitive results at the same time by adopting a relatively small LTS.

The proposed HFLTS methodology has been analyzed on a numerical ASP example from manufacturing industry and the results obtained have been compared with some other enhancements of HFLTS. The comparative analysis shows that, changing the shape of FS just because of the employed envelopment technique and ignoring the non-determinacy caused by the variation of the expert assessments are important drawbacks of the benchmark methods in terms of accuracy.

These methods do not present a sensitivity analysis capability out of the exploitation phase, because they are developed in line with the requirements of MCDM problems. The proposed HFLTS methodology provides a novel contribution to the literature for hesitant modeling of the problems that need sensitive calculation. The numerical results show that the proposed methodology avails to reach reasonable, reliable, sensitive, and accurate results by using hesitative expert assessments. Moreover, it is more qualified than the benchmark methods in terms of representing the real case with the help of protecting the FS shape and regarding the non-determinacy caused by hesitation.

As a future direction, the proposed HFLTS approach can be combined with different type of fuzzy modifiers, envelopment procedures or it can be experienced on another real case DM problem. Moreover, the other FS extensions can be analyzed for ASPs and the obtained results can be compared.

Table 3: Comparison of the proposed and benchmark methodologies

	Proposed Methodology	**Benchmark Methodologies**
Reliability of Results	• Considers the variations caused by hesitancy as non-determinacy in modeling. In this way, the uncertainty caused by the hesitancies of experts is represented with a separate term in the results. • Does not ignore any part of the assessments, so it has no reliability issue. The obtained results are reasonable and suitable for real case use. For example; acceptance probability has been obtained as a triangular FS having parameters (0, 0.0013, 0.0158) and this result shows that the incoming lot have quality issues in terms of the quality standards.	• Prefer to model the problem as if the complete information case is available by using type-1 FS or interval valued type-1 FS. • Since they handle the variation by increasing comprehensiveness of the FSs, results are obtained as too wide in FSs. For example, the acceptance probability has defined between 0 and 1 in Table 2 for the methods of Chen & Hong [4] and Liu & Rodrguez [24]. Such an information is not reliable and not usable in real-case applications. • Rodrguez et al.s [37] method uses the kernel points of the assessments as interval limits and ignores the support points that are remained out of this interval. By this way, the results have been obtained as too narrowed intervals. Such approach can bring problems about the reliability of the results because of ignoring some part of initial FSs in calculations.
Capability of Sensitivity Analysis	• Obtained results such as $\tilde{P}_a = (0, 0.0015, 0.0204)$ are suitable to analyze the result with sensitivity analysis.	• Results of the methods of Chen & Hong [4], Liu & Rodrguez [24] and Romero et al. [38] are produced in a too wide range to take action after the sensitivity analysis such as $\tilde{P}_a = (0, 0, 0.0084, 1)$. They do not produce any meaningful output in sensitivity analysis. • On the other hand, the results of Rodrguez et al.s [37] method seems suitable for sensitivity analysis, but these results have reliability problem because of ignoring the values between support and kernel points of the FSs.
Consideration of FS Shape and Accuracy	• As an advantage, the results having the similar FS shape with the fuzzy item defectiveness information are obtained. • Since the defectiveness has been modeled as a triangular shaped FS, the results have been reached as triangular FSs.	• HFLTS methods offered by Chen & Hong [4], Liu & Rodrguez [24] and Romero et al. [38] have provided trapezoidal results because of the employed envelopment techniques. • Changing shape of FS just because of the employed envelopment technique while there is no necessity caused by the nature of the problem reveals issues about the accuracy of the results. • Results were found as intervals by the formulation offered by Rodrguez et al. [37]. This also brings some accuracy issues. • Another issue about the benchmark studies is just considering the membership. The non-determinacy caused by the hesitation of the experts are ignored and the non-determinate part is considered as non-defective. Even though the results show that the acceptance probability is high, the lot rejection ratio will be realized more than the calculated rejection probability in the real case problem. In other words, there are both accuracy and reliability issues about the results of the benchmark methods.

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