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Sliding Mode Control Design for a Class of Nonlinear Fractional Systems with Application to Glucose-Insulin Systems

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ABSTRACT

Nonlinear fractional system in the presence of uncertainties and external disturbances are important issues these days. In this paper, we introduce a method for study and evaluating stability by using a suitable sliding surface and then due to the unknown upper bound of uncertainties and external disturbances we design robust adaptive fractional sliding mode controller. Finally, we illustrate the accuracy of our proposed method on two examples and ensure its effectiveness.

I. Introduction

Many mathematical theories have been evolved and extended over time. One instance of such theories is integer order calculus, which has been extended using the notion of fractional derivative. The major advantage of fractional calculus is that it is nonlocal. In fact, when we calculate ordinary derivative at a specific point, the resulting value depends only on that point and its around. This property is known as locality. But fractional derivatives are obtained by integrating a wide range of values and depends on the state of the system in the past.

It might be difficult to use classical differential equations for the modelling and analysis of systems with memory effects. Nevertheless, that fractional derivatives are non-local allows them to cover memory effects. As a tool, fractional calculus has been evolved to improve the efficiency of control loops in two primary ways, namely, to improve the quality of modelling and to enhance the efficiency of controllers. This tool has been beneficial to the finding of more accurate models for processes by providing a wider context for dynamic models.

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It is obvious that finding a more accurate model of a process allows us to design more suitable controllers for the handling of that process [1]. Besides, since traditional controllers are special cases of fractional controllers, the use of fractional controllers can improve the efficiency of control systems designed. [2, 3]

The first application of fractional calculus to the modelling of physical phenomena dates back to the 1930s [4]. Since then, this tool has been used in numerous fields of sciences. For example, in physics [5,6], in medical and biological sciences [7, 8], in economics [9] and so on [10,12]. As a special instance of nonlinear fractional order models, the fractional order model of glucose-insulin systems has been investigated by some researchers in recent years. In [13], the existence and uniqueness of fractional order derivatives for glucose-insulin regulation were proved using the fixed-point theorem and an iterative scheme. Also, a fuzzy fractional order predictive control system was proposed in [14] to stabilize the glucose level regulation. In [15] the effect of the incommensurate fractional order derivatives on a glucose-insulin regulatory model is studied. In [16] for blood glucose level control, a Reinforcement Learning method and its combination with sliding mode controllers is used to determine the injection dosage. Stability analysis of fractional systems is more complicated than that of integer order systems. Considering the structure of the response of linear time invariant fractional systems, the stability of the equilibrium point in these systems has been discussed [17]. However, in contrast to linear systems, the stability analysis of nonlinear fractional systems (NFS) runs slowly. Around two decades ago, the problem of structural stability of these systems was studied for the first time, considering the Riemann Liouville derivative and the use of Taylor's polynomial expansion. Furthermore, the concept of Lyapunov stability was introduced in [18], using Gronwall's lemma and Schwarz's inequality. On the other hand, in some references, the method of linearization has been used. However, a unique and specific basis theory has not been obtained yet.

In real world applications, model uncertainties and external disturbances can affect the performance of system. Dealing with these issues is one of the design problems of controllers. Sliding mode control (SMC) is an effective nonlinear robust control method. In recent decades, it has been widely used and has proved successful due to its good features such as high accuracy, ease of implementation and resistance to external disturbances and parametric uncertainties. [19]

This approach received attention after Utkin's work and was extended to control issues [20]. There are two main steps in the design of a sliding mode controller, namely, creating a suitable sliding surface that represents the desired dynamics (the sliding phase) and adjusting the control law in such a way that a sliding condition is achieved (the reaching phase). Sliding mode control (SMC) for integer order systems was used for a long time until its application to fractional systems began [21,23]. In [22], the control of a fractional system with input delay time and state was investigated using the SMC method. In [24], the synchronization of the uncertain, fractional Duffing Holmes system was investigated through the SMC method. In [25], the SMC method was used to stabilize a system of invariant, fractional order with linear time. Fractional chaotic systems were investigated under SMC [26, 27]. Several improved SMC methods based on neural networks [28], Riccati's approach [29], LMI [30] and the adaptive technique [31] have been proposed.

In [32], a fuzzy adaptive controller was designed with a fractional calculus approach and sliding mode. In [33], an adaptive step back output feedback controller was proposed for a group of nonlinear fractional systems. The authors of [34] presented an adaptive sliding mode control for nonlinear systems with uncertainty. In [35], a high order sliding mode observer, combined with an adaptive fraction order sliding mode control, was proposed for diabetic patients. A fractional sliding mode control method was proposed in [36] for a group of nonlinear systems that were subject to uncertainty. The paper [37] examined the design of an adaptive sliding mode controller for fractional linear systems that were exposed to uncertainty and nonlinear disturbances. The authors of [38] assessed the application of a fractional control method based on a sliding mode used to track and stabilize a group of nonlinear fractional systems subject to uncertainty and drew upon a second order sliding mode approach in combination with a PI based design for the single input mode.

A sliding mode control law was proposed in [39], based on a new backstepping technique for nonlinear fractional systems subject to mismatched disturbances. In [40], a fractional sliding mode control was designed to use the particle swarm optimization (PSO) algorithm for a specific class of nonlinear fractional systems. The paper [41] studied the sliding mode control of output feedback for nonlinear fractional systems and proposed a necessary and sufficient condition for the existence of a sliding surface in the LMI form. The authors of [42] used a sliding mode control for a group of fractional systems with matched and mismatched disturbances and offered a fractional disturbance observer to approximate both matched and mismatched disturbances. In addition, the sliding surface was constructed based on the oberver's output. Based on the LMI method, [43] proposed a sliding mode control method for a group of fractional systems in which the upper bound of the known uncertainty was given. In [44], the stabilization problem was investigated by adaptive sliding mode control for a group of uncertain, nonlinear fractional Hopfield neural networks. In [45] a conformable fractional order sliding mode control method is studied for a class of fractional order chaotic systems in the presence of uncertainties and disturbances and stability analysis of the controller is derived by Lyapunov method with conformable fractional order operators.

The authors of [46] presented a second order, LQR- based sliding mode controller for a group of nonlinear fractional systems. A sliding mode active disturbance rejection control was proposed in [47] for nonlinear fractional systems with uncertainty and unknown disturbances. In [48], the asymptotic stability of a group of nonlinear fractional systems with bounded inputs, in the presence of uncertainty and external disturbances, was studied by adaptive constrained sliding mode control. By using sliding mode controllers, the stabilization of chaotic dynamic systems of nonlinear, fractional order was investigated in [49]. Building on the composite learning sliding mode control to control fractional nonlinear systems with actuator faults, [50] introduced a fractional integral sliding surface. Recently, in [51] a new adaptive fuzzy fractional order fast terminal sliding mode control method used for a class of nonlinear systems in the presence of uncertainties and external disturbances.

In the last two or three years, the use of sliding mode control in combination with fractional order calculations or in order to stabilize fractional order systems has attracted the attention of many researchers [52]. Fractional order sliding mode control law has been recently used for stabilizing different systems such as image encryption [53], Power Grids [54], cryptography [55], complex systems [56] and so on.

Despite its significance among advanced control methods resulting from high resistance to uncertainties and high ability to control indeterminate conditions, sliding mode control has a major drawback; the chattering phenomenon. The chattering phenomenon can lead to unwanted fluctuations in the control system and derail the behavior of the system from what is desirable [57, 58]. There are some ways to prevent chattering. One of the most common methods is to use a fuzzy algorithm to obtain the amplitude of a narrow boundary layer around the sliding surface. Nonetheless, this is a slow process [59]. Another method is to apply high order sliding mode control, what prevents chattering without reducing accuracy. However, in addition to the diff iculty of implementation, this method has the problem of chattering when high order sliding dynamics increase the relative degree of the system [60]. The continuous ap proximation method is more useful for the reduction of chattering than the other methods mentioned here. This method designs the control signal discontinuity by substituting a continuous function and creating a narrow boundary layer around the sliding surface; no report has been published concerning this method as yet [61].

Although many studies have been conducted in the field of control of nonlinear fractional systems in the presence of uncertainties and disturbances, most of them have some drawbacks, to the best of our knowledge, that we now briefly describe. First, the studies do not solve the problem of stabilizing for the general form of nonlinear, aff ine fractional systems. Also, in most cases, the upper bound of uncertainties and external disturbances is considered to be known. This is in contrast to what happens in real applications, where the limits of model uncertainties and external disturbances are unknown in practical systems. This issue has been addressed and discussed in our present research by designing a new robust adaptive fractional sliding mode controller (RAFSMC) which is proposed in next Section.

The contribution and innovations of this paper can then be summarized as follows:

•Considering a more general form of nonlinear fractional order systems as aff ine nonlinear systems.

•Designing a new robust-adaptive fractional order sliding mode controller by proposing a new fractional order sliding surface.

•Stabilizing the system in the presence of both model uncertainties and external disturbances; where the upper band of uncertainties and disturbances is unknown.

•Estimating the upper bound of uncertainties by designing stable adaptive rules.

•Solving the problem of glucose level regulation by considering the nonlinear fractional order model of Glucose-insulin Systems. as applications to the proposed methodology.

This paper is organized as follows. In Section 2, the problem is formulated and the model of nonlinear, fractional aff ine systems in the presence of model uncertainties and disturbances is presented. The robust adaptive fractional sliding mode controller is designed in Section 3. In Section 4, simulation results are given to confirm the analytical results. Finally, the conclusion closes the paper in Section 5.

II. Formulation of the problem

A. Model of the system

The system under consideration is a class of nonlinear, aff ine fractional systems that can describe a comprehensive range of dynamic systems:

$$ {}_{0}^{C}D_{t}^{\alpha}x(t) = f(x) + g(x)u(t), \quad x(0) = x_0. \tag{1}$$

Here, $f: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is an arbitrary nonlinear function, piecewise continuous and local Lipschitz with Lipschitz constant in the area enclosing the origin. Also, $x(t) \in \mathbb{R}^n$ denotes pseudo-state, $g: \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is an arbitrary nonlinear function and $u(t) \in \mathbb{R}^m$ is the input of the system. It is assumed that $0 < \alpha < 1$ and ${}_{0}^{C}D_{t}^{\alpha}$ denotes the Caputo derivative operator, which is defined by

$$ {}_{0}^{C}D_{t}^{\alpha}x(t) = \frac{1}{(\Gamma(\lceil\alpha\rceil - \alpha)}\int_0^t \frac{x^{(\lceil\alpha\rceil)}(\tau)}{(t-\tau)^{\alpha-\lceil\alpha\rceil+1}}d\tau \tag{2}$$

where $\Gamma(.)$ denotes the Gamma function and $\lceil\alpha\rceil$ is the least integer that is greater than or equal to α [62].

B. A system with model uncertainty and external disturbance

Considering the external disturbances and model uncertainties, the nonlinear fractional system in (1) can be expressed as

$$ {}_{0}^{C}D_{t}^{\alpha}x(t) = \Big(f\big(x(t)\big) + \delta f\big(x(t)\big)\Big) + g(x(t))u(t) + d(t) \tag{3}$$

where $d(t) \in \mathbb{R}^n$ denotes the external, bounded and unknown disturbance and $\delta f(x)$ shows additive model uncertainties. By aggregating the uncertainties and disturbances, the model of the system can be written as

$$ {}_{0}^{C}D_{t}^{\alpha}x(t) = f\big(x(t)\big) + g\big(x(t)\big)u(t) + E(x(t)), \tag{4}$$

where $E(x(t)) = \delta f(x(t)) + d(t)$. Now, consider the following assumption.

This format is actually the general form of fractional order nonlinear affine systems which are exposed to perturbations and many systems in practice can be written in this format. Robotic systems, biological systems, all the linear systems, chemical processes, etc. can be written in this affine form. As you can see, two of these real systems are mentioned in the simulation section, including the glucose-insulin system and the chaotic Lu system.

Assumption 2.1 Assume that the amount of external disturbances and uncertainties is bounded as

$$\| E(x(t)) \| \leq \gamma, \quad (5)$$

where $\gamma \in \mathbb{R}^+$ indicates the upper bound of model uncertainties and external disturbances and is considered to be unknown. The aim of this paper is to design a sliding mode control law for the NFS exposed to model uncertainties and external disturbances in (4), in such a way that in spite of Assumption 2.1 the system becomes asymptomatically stable. In addition, an adaptive law is also designed to estimate the upper bound of model uncertainties in (5). Asymptotic stability is defined as follows

Definition 2.2 ([64]). The constant vector x_{eq} is an equilibrium point of the fractional system ${}_0D_t^{\alpha}x(t) = f(x(t))$ if and only if

$$f(x_{eq}) = 0$$

We assume, without loss of generality, that the equilibrium point is equal to 0 , that is, $x_{eq} = 0$.

Definition 2.3 ((Stability of NFS) [64]). Suppose that $x_{eq} = 0$ is equilibrium point of ${}_0D_t^{\alpha}x(t) = f(x(t))$. The system is said to be stable if for any initial condition $x(0) \in R^{n\times 1}$, there exists $\delta > 0$ such that any solution $x(t)$ of ${}_0D_t^{\alpha}x(t) = f(x(t))$ satisfies $\| x(t) \| < \delta$, for all $t > 0$. Furthermore, the fractional system is said to be asymptotically stable if the system is stable and $\| x(t) \| \to 0$ as $t \to \infty$.

In the next section, we design a sliding mode stabilizing controller for the NFS in (3).

III. RAFSMC design

SMC is a robust, nonlinear Lyapunov-based control method. The sliding mode design approach consists of two steps. The first involves the design of a switching function $s = 0$, such that the sliding motion satisfies the design specifications. The second one concerns the selection of a control law which will enforce the system to converge to the sliding surface (the sliding mode).

A. Defining the fractional sliding surface

Considering the input affine NFS in (4), the m dimensional, fractional sliding surface can be defined by

$$S(t) = Q\int_0^t {}_0^cD_\tau^{\alpha}x(\tau)d\tau - Q\int_0^t (f(x(\tau)) + L(x(\tau)))d\tau, \quad (6)$$

where $S(t) = [s_1, s_2, \dots, s_m] \in \mathbb{R}^{m\times 1}$ is the sliding surface vector, $Q \in \mathbb{R}^{m\times n}$ and $L(x) \in \mathbb{R}^{n\times 1}$ are the control parameters that should satisfy the following assumption.

Assumption 3.1 We assume that the vector $L(x) \in \mathbb{R}^{n\times 1}$ and the matrix $Q \in \mathbb{R}^{m\times n}$ are chosen in such a way that they satisfy the following conditions.

1- $Q \neq 0$.

2 - $Qg(x) \in \mathbb{R}^{m\times m}$ is nonsingular.

3 - The NFS ${}_0^cD_t^{\alpha}x(t) = f(x(t)) + L(x(t))$ is asymptotically stable.

We considered the sliding mode such that, firstly, when is 0 , the desired characteristics and provided and, secondly, the system's dynamics would appear by deriving from it.

In the SMC design, the switching surface and its derivative should meet the conditions $s(t) = 0$ and $\dot{s}(t) = 0$, then based on (6) we have

$$Q{}_0^cD_t^{\alpha}x(t) - Q(f(x(t)) + L(x(t))) = 0, \quad (7)$$

where by assuming $Q \neq 0$ we find that

$${}_0^cD_t^{\alpha}x(t) = f(x(t)) + L(x(t))$$

Consequently, according to Assumption 3.1, the fractional equation in (7) implies that the closed-loop system is asymptotically stable, meaning that $x(t) \to 0$, when $t \to \infty$.

B. Selection of the reaching law

To satisfy the sliding mode condition, consider the reaching law

$$\dot{s} = -\mu s - \rho\, \mathrm{sign}(s), \quad (8)$$

where $\mu, \rho \in \mathbb{R}^+$,

and $\mathrm{sgn}(s) = [\mathrm{sgn}(s_1), \mathrm{sgn}(s_2), \dots, \mathrm{sgn}(s_m)]^T \in \mathbb{R}^m$ denotes the sign function vector [9].

Lemma 3.2 ([63]). By adopting the reaching law as in (8), the system reaches the switching surface at the finite time T^* given by

$$T^* = \frac{1}{\mu}\ln\left(\frac{\rho + \mu \max_{i=1,\dots,m} |s_i(0)|}{\rho}\right). \quad (9)$$

C. Controller design

In this subsection, we use the fractional sliding surface in (6) and consider the reaching law in (8) to design a new stabilizing, RAFSMC for the system introduced in (4) which is robust against model uncertainties and external disturbances. Besides, in this controller, the unknown upper bound of model uncertainties and disturbances is obtained via a stable adaptive law.

In the following, we present a theorem that describes the basics and fundamentals of RAFSMC design.

Theorem 3.3 Consider the NFS exposed to model uncertainties and external disturbances described in (3). By applying the control law

$$u(t) = (Qg(x))^{-1}(QL(x) - \mu s - \rho\hat{\gamma}\mathrm{sgn}(s)), \quad (10)$$

in which the estimation of upper bound $\hat{\gamma}$ is obtained from the adaptive law

$$\dot{\hat{\gamma}}(t) = k_1 \rho s^T \mathrm{sgn}(s), \quad (11)$$

where $k_1 > 0$ is the adaptive law gain and the sliding surface is defined as in (6), along with Assumptions 2.1,3.1 we find that the closed-loop system (4) is asymptotically stable if the constant, scalar values μ and ρ are chosen such that $\mu > 0$ and $\rho > \| Q \|$.

Proof. Differentiating the sliding surface defined in (6) and substituting the NFS in (3) we obtain

$$\begin{aligned} \dot{s}(t) = &\ QD^{\alpha}x(t) - Q(f(x(t)) + L(x(t)) \\ = &\ Q(f(x(t)) + \delta f(x(t)) + g(x(t))u(t) + d(t)) \\ &-Q(f(x(t)) + L(x(t)) \\ = &\ Q(g(x(t))u(t) + E(x(t)) - QL(x(t)). \end{aligned} \quad (12)$$

Now, consider the Lyapunov candidate function

$$V(t) = \frac{1}{2}s^T s + \frac{1}{2k_1}\tilde{\gamma}^2, \quad (13)$$

where $k_1 > 0$ and $\tilde{\gamma} = \hat{\gamma} - \gamma$ stands for the upper bound estimation error. Then, differentiating (13) with respect to time and substituting from (12) result in

$$\begin{aligned} \dot{V}(t) = &\ s^T\dot{s} + \frac{1}{k_1}\tilde{\gamma}\dot{\tilde{\gamma}} \\ = &\ s^T(Q(g(x(t))u(t) + E(x(t)) - QL(x(t)))) \\ &+ \frac{1}{k_1}\tilde{\gamma}\dot{\tilde{\gamma}}. \end{aligned} \quad (14)$$

Now, applying the RAFSMC in (10) to (14) yields

$$\begin{aligned} \dot{V}(t) = &\ s^T(QL(x(t)) - \mu s - \rho\hat{\gamma}\mathrm{sgn}(s) \\ &+QE(x(t)) - QL(x(t)) + \frac{1}{k_1}(\hat{\gamma} - \gamma)\dot{\gamma} \end{aligned} \quad (15)$$

where removing the similar terms and noticing $\dot{\tilde{\gamma}} = \dot{\hat{\gamma}}$ we obtain

$$\dot{V}(t) = s^T\left(-\mu s - \rho\hat{\gamma}\mathrm{sgn}(s) + QE(x(t)) + \frac{1}{k_1}(\hat{\gamma} - \gamma)\dot{\hat{\gamma}}. \quad (16)$$

Now, considering the definition of $\| s \|$ as $s^T\mathrm{sgn}(s)$, $s^T s = \| s \|^2$ and applying the adaptive law from (11) we find that

$$\dot{V}(t) = -\mu \| s \|^2 - \rho\hat{\gamma} \| s \| + s^T QE(x(t)) \quad (17)$$

$$+\rho(\hat{\gamma} - \gamma) \| s \| = -\mu \| s \|^2 + s^T QE(x(t)) - \rho\gamma \| s \|. \quad (18)$$

The upper bound of $\dot{V}(t)$ in (18) can be computed as

$$\dot{V}(t) \le -\mu \| s \|^2 + \| s \| \| Q \| \| E(x(t)) \| - \rho\gamma \| s \| \quad (19)$$

So, considering the inequality (5) in Assumption 1 we can write

$$\begin{aligned} \dot{V}(t) \le &-\mu \| s \|^2 + \| s \| \| Q \| \gamma - \rho\gamma \| s \| = \\ &-\mu \| s \|^2 + (\| Q \| - \rho)\gamma \| S \end{aligned} \quad (20)$$

As a result, by selecting $\mu > 0$ and $(\| Q \| - \rho) < 0$ (or $\rho > \| Q \|$), we find that the upper bound of the derivative of the Lyapunov function is negative semi-definite. Now, by defining $\tilde{w}(t) = \mu \| s \|^2 + Q \| - \rho)\gamma \| s \|$ and integrating the both sides of (20), we deduce that $V(0) \ge V(t) + \int_0^t \tilde{w}(\lambda)d\lambda$. From this inequality we conclude that $\infty > V(0) \ge \lim_{t\to\infty}\int_0^t \tilde{w}(\lambda)d\lambda$. Hence, according to the Barbalat lemma [65], $\tilde{w}(t)$ and thus $s(t)$, converges to 0 when $t \to \infty$ and $\tilde{\gamma}$ remains bounded. So, according to the definition of the sliding surface in (6) and the stable equation in (7) and by the convergence of s to 0, the NFS in (3) is asymptotically stable and the psudostate $x(t)$ of the system will asymptotically converge to 0 .

It is worth to mention that the RAFSMC controller proposed in this paper, is designed based on Lyapunov theory and none of the controllers designed using this method are unique, because the Lyapunov theory proposes only sufficient and not necessary condition for stability. Therefore, this controller is not necessarily unique, meaning that there may be thousands of other controllers with different structures that may asymptotically stabilize the system.

In this paper, in order to reduce the chattering similar to what is suggested in [63,66], we use a continuous function instead of the sign function. To this end, we approximate the sign function in (10) and (11) by the function

$$\rho\,\mathrm{sgn}(s) \cong \frac{\rho^2 s}{\rho \| s \| + \sigma(t)}, \quad (21)$$

where $\sigma(t) > 0$ is a positive, bounded function such that $\int_0^\infty \sigma(t)dt < \infty$. For example,

$$\sigma(t) = \frac{1}{1+t^n},\ n \ge 2. \quad (22)$$

IV. Simulation Results

We show the effectiveness of our proposed method by two examples.

Example 4.1. Consider the model that described by the Caputo fractional differential equations [67]

$$\begin{cases} {}_0^C D_t^\alpha x_1(t) = -p_1[x_1(t) - G_b] - x_1(t)x_2(t) + d(t) \\ {}_0^C D_t^\alpha x_2(t) = -p_2 x_2(t) + p_3[x_3(t) - I_b],\ 1 < \alpha < 1 \\ {}_0^C D_t^\alpha x_3(t) = -n[x_3(t) - I_b] + u(t), \end{cases}$$

$$y = Cx(t)\ \ x(0) = [380, 0.0001, 310]^T \quad (23)$$

which $x_1(t)$ represent the temporal dynamics of the blood glucose concentration at time $t, x_2(t)$ the auxiliary function representing insulin-excitable tissue glucose uptake activity, proportional to insulin concentration in a 'distant' compartment and $x_3(t)$ the blood insulin concentration at time t.

$u(t)$ defines the insulin injection rate and replaces the normal insulin regulation of the body, which acts as the control variable. Since the normal insulin regulatory system does not exist in the body of diabetic patients, this glucose absorption is considred as a disturbance for the system dynamics and $d(t)$ shows the rate at which glucose is absorbed by the blood from the intestine, following food intake. The glucose concentration in blood is considered as the output $y(t)$, where

$$y(t) = [1 \quad 0 \quad 0]x(t) \tag{24}$$

For convenience we use the following replacement,

$$\begin{cases} x_1(t) := x_1(t) - G_b \\ x_2(t) := x_2(t) \\ x_3(t) := x_3(t) - I_b \end{cases} \tag{25}$$

Then system (23) will be as follows:

$$\begin{cases} {}_0^C D_t^C x_1(t) = -p_1 x_1(t) - x_1(t)x_2(t) + d(t) \\ {}_0^C D_t^\alpha x_2(t) = -p_2 x_2(t) + p_3 x_3(t) \\ {}_0^C D_t^\alpha x_3(t) = -n x_3(t) + u(t) \end{cases} \tag{26}$$

TABLE 1
PARAMETERS AND DESCRIPTIONS IN (EX. 4.1)

Parameter	DESCRIPTION	Value
p_1	The insulin-independent constant rate of glucose uptake in muscles and liver	$0.001(1/\min)$
p_2	Therate for decrease in tissue glucose uptake ability	$0.23(1/\min)$
p_3	The insulin-dependent increase in glucose uptake ability in tissue per unit of insulin concentration above the basal level	$6.3 \times 10^{-4}((\mu U/ml)^{-1}\min^{-2})$
G_b	The basal value of glucose concentration in plasma	$80(mg/dl)$
I_b	The basal value of insulin concentration in plasma	$10(\mu U/ml)$
n	The first order decay rate for insulin in blood	$0.16(1/\min)$

Now, the nonlinear fractional order glucose-insulin model (26) with the parameter values of a diabetic patient can be rewritten as [67]

$$\begin{aligned} {}_0^C D_t^\alpha x(t) &= f(x) + g(x)u(t) + Dd(t), 0 < \alpha < 1 \\ y(t) &= Cx(t), \end{aligned} \tag{27}$$

where

$$f(x) = Ax + [-x_1(t)x_2(t), 0, 0]^T$$
$$A = \begin{bmatrix} -p_1 & 0 & 0 \\ 0 & -p_2 & p_3 \\ 0 & 0 & -n \end{bmatrix}$$
$$g(x) = [0,0,1]^T$$
$$D = [1,0,0]^T$$
$$C = [1 \quad 0 \quad 0].$$

We calculate the equilibrium point of the system (23) according to Definition 2.2 It will be obtained $[G_b, 0, I_b]$. The model uncertainties and external disturbances are considered as follows.

$$\delta f(x(t)) = 0.1 \begin{bmatrix} -p_1 & 0 & 0 \\ 0 & -p_2 & p_3 \\ 0 & 0 & -n \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, \tag{28}$$
$$d(t) = [0.1 \text{ rand}, 0, 0]^T.$$

where rand is a random value between 0 and 1 . Using Theorem 3.3 and Assumption 3.1, the parameters of the RAFSMC and the adaptive law, respectively in (10) and (11), can be selected as follows.

$$L(x(t)) = \begin{bmatrix} x_1(t)x_2(t) - 5x_1(t) \\ -p_3 x_3(t) - 4x_2(t) \\ -3x_3(t) \end{bmatrix}$$
$$Q = [0 \quad 0 \quad 1]$$
$$\mu = 40, \rho = 3, k_1 = 4. \tag{29}$$

The initial value for the adaptive law are assumed as

$$\gamma(0) = 2$$

in order to eliminate the chattering, by using (21) and (22), the sign function of the RAFSMC (12) is approximated by the function

$$\rho\, \mathrm{sgn}(s) \cong \frac{\rho^2 s}{\rho \parallel s \parallel + \sigma(t)}, \quad \sigma(t) = \frac{1}{1 + t^2}. \tag{30}$$

The result of simulating the behavior of system without controller is shown in Fig 1.

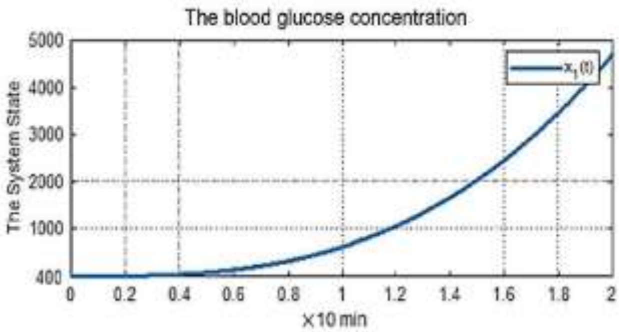

Fig. 1. The state response of the glucose concentration in system without controller (Ex. 4.1)

As it can be seen, the blood glucose concentration rises significantly in a system without a controller. Applying the RAFSMC as in (10)-(11) to the glucose-insulin system in (23), the simulation results shown in Fig 2-7 are obtained.

As it is evident from this Fig 2 the blood glucose concentration converges to the $G_b = 80mg/dl$ during about 20 minutes which shows asymptotic stability of the system in convergence to its equilibrium point in the presence of random external disturbance and uncertainty in the nonlinear model of Glucose-Insulin system.

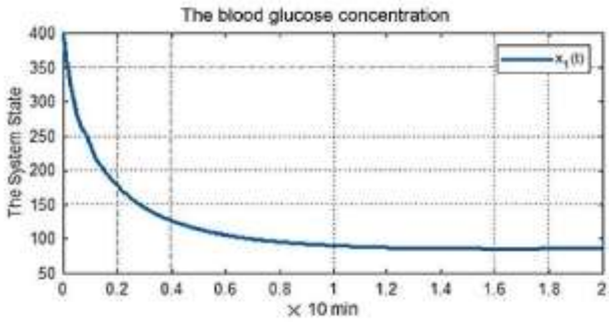

Fig. 2. The state response of the glucose concentration with controller (Ex. 4.1)

Fig 3 and Fig 4 show the trajectories x_2, x_3 that reach the equilibrium point. The insulin concentration as input controller in Fig 5 remained bounded and converges to a bounded value after stabilizing the system.

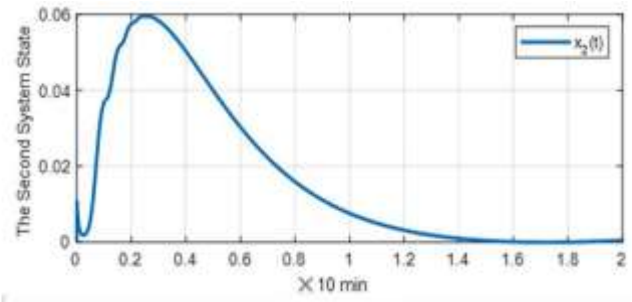

Fig. 3. The state response of the insulin-excitable tissue glucose (Ex. 4.1)

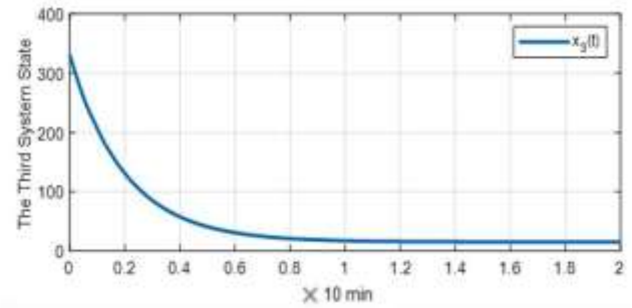

Fig. 4. The state response of the blood insulin concentration (Ex. 4.1)

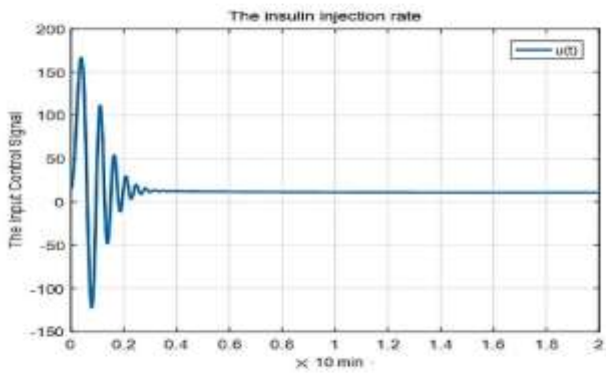

Fig. 5. The insulin concertration as input controller (Ex. 4.1)

In Fig 6, adaptive parameter is shown. These parameter is consistent with what was demonstrated in the stability of Theorem 8 and converge to a constant value. Finally, the sliding surface of the system is indicated in Fig 7. The sliding surface converges to 0 in a finite time.

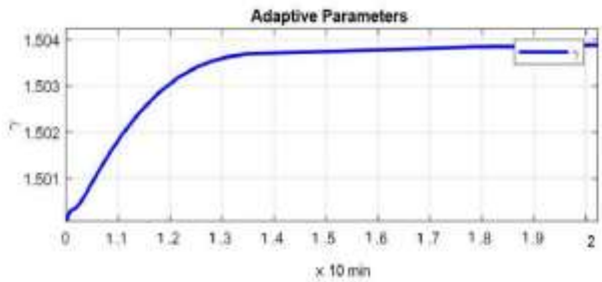

Fig. 6. Estimated adaptive parameter (Ex. 4.1)

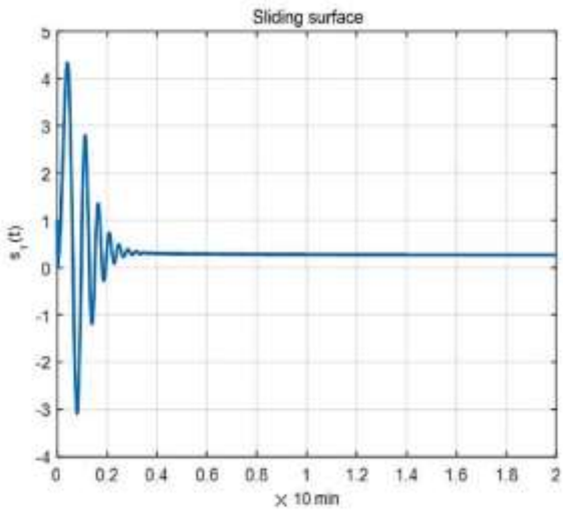

Fig. 7. The sliding surface of the glucose-insulin system (Ex. 4.1)

As a further explanation, it should be noted that Fig 2 to 4 are state variables in the glucose-insulin model. As we can see, glucose concentration, insulinexcitable tissue glucose and blood insulin concentration respectively converge to their equilibrium values. Apart from being achieved in a reasonable time, which is less than 10 minutes for the patient, this convergence has neutralized the effect of uncertainties and disturbances by applying the control law. Fig 6 and 7 are respectively the adaptive parameter and sliding surface. According to the stability proof, we expect that the adaptive parameter converges to zero in a finite constant value and the sliding surface also converges to a finite time while not having chattering. Fig 6 and 7 easily confirm these both facts.

Example 4.2. Consider the Lu fractional system. Given external disturbances and modelling uncertainties, the nonlinear fractional system model is selected from [49].

$$\begin{cases} {}^{C}D^{\alpha}x_1(t) = 36(x_2(t) - x_1(t)) + \delta f_1(t, x(t)) + d_1(t) + u_1(t), \\ {}^{C}D^{\alpha}x_2(t) = 20(x_2(t) - x_1(t)x_3(t) + \delta f_2(t, x(t)) + d_2(t) + u_2 \\ {}^{C}D^{\alpha}x_3(t) = x_1(t)x_2(t) - 3x_3(t) + \delta f_3(t, x(t)) + d_3(t) + u_3(t) \end{cases}$$
(31)

$$X(t_0) = [10, -5, 5]^T$$

where the values of uncertainty and disturbance are given by

$$\begin{cases} \delta f_1(t, x(t)) + d_1(t) = 0.2\cos(3t)x_1(t) + 0.15\sin(2t) \\ \delta f_2(t, x(t)) + d_2(t) = 0.25\sin(4t)x_2(t) + 0.2\sin(3t)i \\ \delta f_3\big(t, x(t)\big) + d_3(t) = 0.3\sin(2t)\,x_3(t) + 0.25\cos(4t) \end{cases} \tag{32}$$

This model is widely used in the electrical industry and power distribution systems.

Simulation of the behavior of the system without controller

For $\alpha = 0.98$, the result of simulating the behavior of the system without controller is shown in Fig 8.

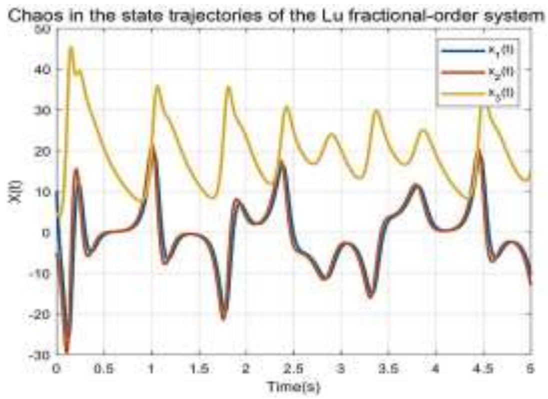

Fig. 8. Behavior of the Lu system without controller (Ex. 4.2).

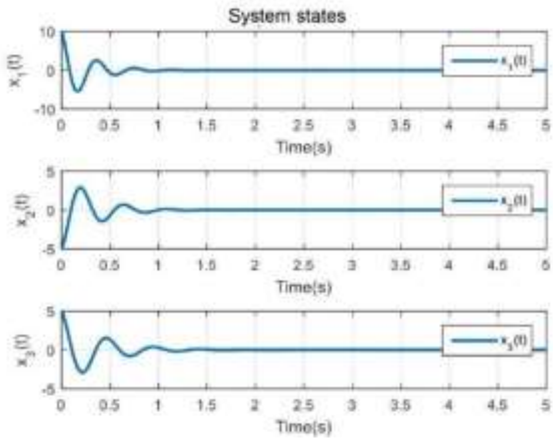

Fig. 9. System states after applying proposed controller (Ex. 4.2)

Simulation of system with controller

Using the method presented in this paper, the closed loop system is simulated in the presence of modelling uncertainties and external disturbances.Select the controller parameters to satisfy the Assumption 3.1 and Theorem 3.3, as follows.

$$L(x(t)) = \begin{bmatrix} -36x_2(t) + 30x_1(t) \\ x_1(t)x_3(t) - 25x_2(t) \\ -x_1(t)x_2(t) - x_3(t) \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{33}$$

In addition, the control gains and adaptive laws are selected as follows.

$$\mu = 40, \ \rho = 2, \ k_1 = 4. \tag{34}$$

To reduce the chattering, the sign function is approximated by

$$\rho \operatorname{sgn}(s) \cong \frac{\rho^2 s}{\rho\|s\| + \sigma(t)}, \ \sigma(t) = \frac{1}{1+t^2}. \tag{35}$$

Considering the initial adaptive law as

$$\gamma(0) = 2, \tag{36}$$

the simulation results of the RAFSMC are shown in the Fig 9, 10, 11, 12 .

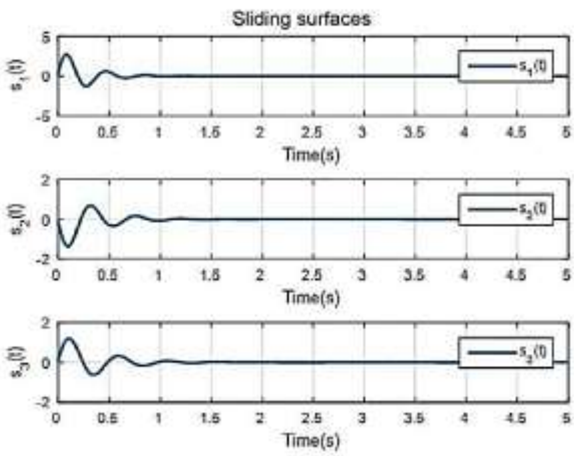

Fig. 10. Sliding surfaces converge to 0 by applying the control law (Ex. 4.2)

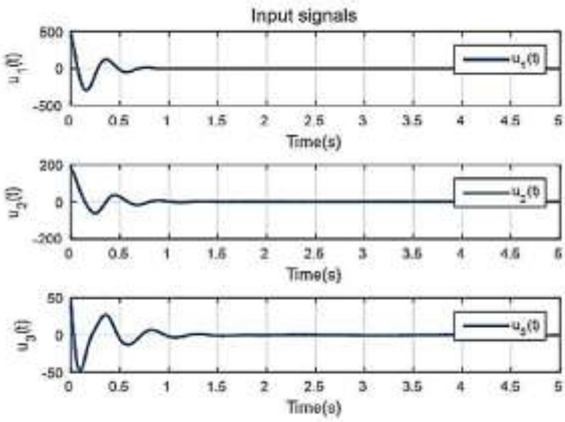

Fig. 11. Proposed RAFSMC (Ex. 4.2)

The stability of system by using the control input is shown in Fig 9, where the system states converge to 0 . Fig 10 shows the convergence of the sliding surface to zero and the control signal is shown in Fig 11. The adaptive parameter tends to a limited value in Fig 12.

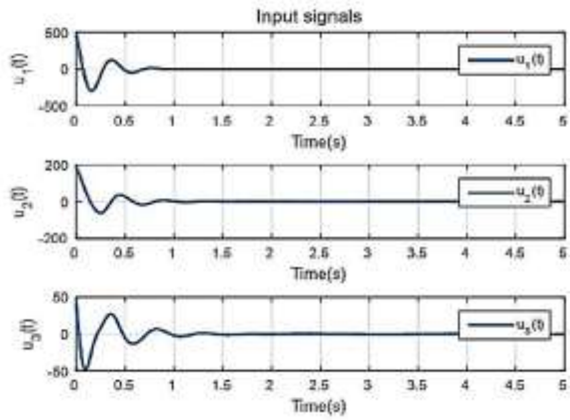

Fig. 12. Estimated adaptive parameter (Ex. 4.2)

We note that in [49], the following assumption is considered for uncertaintties and disturbances.

$$|\delta f_i(x,t)| + |d_i(t)| \leq \rho_i < \infty;\ i = 1,2,\ldots,n. \qquad (37)$$

Finally, the control law designed in the article is as follows.

$$\begin{aligned} u_i(t) = \quad & -(f_i(t,x_i(t)) + k_i|x_i(t)|^{\eta}\tanh(x_i(t)) \\ & +\rho_i + \gamma_i S_i(t) + \lambda_i |S_i(t)|^{\delta}\tanh(S_i(t)). \end{aligned} \qquad (38)$$

The outcome is that the parameter ρ_i is used in the control law. That is, the upper bound of the sum of uncertainties and disturbances in this article is assumed to be known and the designer must be aware of and include this upper bound in the control law. However, the method presented in our article does not have such a limitation.

Briefly, we know, the assumption of an unknown upper bound for uncertainties and disturbances is more realistic in practical applications. In this article, according to Equation (3), we consider a nonlinear affine fractional order system exposed to uncertainties and external disturbances with unknown upper bound. A new robust adaptive fractional sliding mode controller was designed to stabilize this system. The proposed controller provides the features and general advantage of sliding mode controllers, such as high accuracy of sliding mode part, simplicity of implementation, and resistance to external disturbances and parametric uncertainties, also we used the advantage of adaptive controller for approximating that upper bound.

The stability of the proposed controller was proved in Theorem 3.3. At the end, for checking the effectiveness of the method, in Example 4.1, we showed that the state variables converge to the equilibrium point faster and in a shorter period compared with the controller used for the stability of the same system in reference [67].

Also, in Example 4.2, the designed controller in reference [49], (with known upper bound of uncertainties and disturbances) was compared with our proposed controller (with unknown upper bound), which is a significant strength and can lead to a more effective solution in real applications.

V. Conclusion

In this paper, stability of a class of NFSs in the presence of uncertainty and external disturbance was considered. To solve the problem, a RAFSMC with a new fractional sliding surface was designed. Since in real life, the upper bound of uncertainties and external disturbances are unknown, we use an adaptive law to approximate it, however in most articles the upper bound is considered to be known. Finally, to evaluate the efficiency of the proposed method, the fractional models of the glucose-insulin system and the Lu system were considered and simulation confirm analytical results. Also rapid convergence to the equilibrium point is another benefit of the method .Extending this work to situations in which the Fractional order nonlinear system is subject to delay (especially in the case of biological systems such as glucose-insulin which are subject to real delay), the design of an adaptive sliding mode control law where the control input is saturated due to the limited range of the input Control in practice, generalizing the method for incommensurate fractional order systems or variable order fractional nonlinear problems and finally, optimization of controller coefficients with methods based on computational intelligence or meta-heuristic algorithms are topics for further work.

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