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Original paper

VIKOR-based group decision-making method for software quality assessment

Z. L. Yue[1], C. Yue[2], X. H. Peng[3] and W. Wu[4]

[1,2,3,4]College of Mathematics and Computer Science, Guangdong Ocean University, Zhanjiang, Guangdong, China
[2,3]Southern Marine Science and Engineering Guangdong Laboratory (Zhanjiang), Zhanjiang, Guangdong, China

zhongliangyue@gmail.com, yuechuan-1988@163.com, pengxiaohong_gdou@163.com, wuweina_gdou@163.com

Abstract

Software quality is an important research direction in software engineering domain, which is a multi-dimen-sional evaluation problem. The VIsekriterijumska optimizacija i KOmpromisno Resenje (VIKOR) technique is a comprehensive method for handing the multi-dimensional evaluation problems. The projection is commonly used to measure the closeness between two decision objects in decision science. However, two research questions are found in this work: (1) There is no specific concrete regret matrix in current VIKOR technique; (2) The existing projection measures are not always reasonable in interval-valued intuitionistic fuzzy setting. Two research gaps are filled systematically in this paper: (1) A specific concrete regret matrix is provided in this extended VIKOR-based group decision-making method; (2) A new normalized projection measure is provided in order to measure the closeness between two decision matrices. The decision procedures are also provided in this paper. A practical application to the software quality assessment is introduced. Some experimental comparisons are provided in order to illustrate the feasibility and practicability of introduced method.

Keywords: Extended VIKOR method, normalized projection, group decision-making, interval-valued intuitionistic fuzzy number, software quality assessment.

1 Introduction

In the field of scientific management, engineering disciplines and our day-to-day life, people often face with many decision-making problems [9]. The increasing complexity of the socio-economic environment makes that many decision-making processes take place in group decision-making (GDM) settings [26, 34, 50]. In the recent decade, many researchers have made significant contributions to GDM problems [27, 35, 55].

VIsekriterijumska optimizacija i KOmpromisno Resenje (VIKOR) [5, 11, 29] technique is a comprehensive decision method. There is a regret index and measurement in the current VIKOR techniques. However, this research finds the following question:

Question 1.1. *There are no specific expressions for regret data. The regret measurement is only from a transformation of utility index.*

Question 1.1 shows a lack of independence between utility index and regret index in current VIKOR techniques.

The separation is an important measure in VIKOR technique [8, 20], which is usually based on a distance measure, such as the Euclidean measure and the Hamming distance. It is noted that the projection measure [52–54, 56, 57, 64, 65] is a separation with the angle between two decision matrices, which is a more comprehensive consideration. However, this research finds the following research question.

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Question 1.2. *The existing projection measure is not always reasonable in interval-valued intuitionistic fuzzy context (see Example 1 in Section 4.1).*

Question 1.2 shows an important flaw in GDM methods. To improve and optimize the Question 1.2, this research intends to develop a new normalized projection measure.

In fuzzy community, Atanassov and Gargov [3] introduced the concept of interval-valued intuitionistic fuzzy set, which is an extension of intuitionistic fuzzy sets [2]. And it has better flexibility and practicality in the treatment of fuzzy information and uncertainty than ordinary fuzzy sets. However, a detailed survey of the related literature shows the third research question as follows.

Question 1.3. *Software quality is a multi-dimensional evaluation problem. The existing literature paid very little attention to software quality evaluation based on GDM method.*

Question 1.3 shows a challenge that the software quality is evaluated by using GDM method in interval-valued intuitionistic fuzzy environment.

The main research motivations and research objectives of this work are as follows.

1. For Question 1.1, this work intends to provide a specific concrete group regret matrix for each evaluation alternative.

2. For the hidden flaws in existing projection measures in Question 1.2, this research intends to develop a new normalized projection measure in interval-valued intuitionistic fuzzy setting.

3. For Question 1.3, this article attempts to develop a GDM method to software quality assessment in interval-valued intuitionistic fuzzy environment.

The rest of the manuscript is organized as follows. Section 2 introduces the related work. Section 3 introduces some basic concepts used in this model. Section 4 focuses on a direct GDM method, which is based on the new projection measure developed in this work and an extended VIKOR technique. Section 5 gives a real application to software quality assessment. An experimental analysis is also provided to test the contributions of our approach in this section. Finally, some concrete conclusions are presented in Section 6.

2 Related work

The related researches are mentioned in this section. One of them is GDM methods with interval-valued intuitionistic fuzzy information. Another work is projection measures in decision science. The third is the VIKOR-based GDM methods. Their main weaknesses are also summarized.

Decision making [31, 32] is an indispensable behavior in our daily life. With the increasing complexity of real-word problems, GDM with multiple experts or decision makers is commonly used in a decision process.

GDM processes have attracted research attention in the last ten years and therefore a wide range of different methodologies were proposed [21]. For example, Kong et al. [22] proposed a decision variable-based combinatorial optimization approach for the purpose of solving the interval-valued intuitionistic fuzzy GDM problems. Chu et al. [14] proposed a GDM approach with interval-valued intuitionistic preference relations. Kong et al. [23] introduced a GDM method for solving the interval-valued intuitionistic fuzzy threat assessment problem. Cheng [13] proposed a new autocratic GDM method for hotel location selection using interval-valued intuitionistic fuzzy sets. Joshi and Kumar [19] suggested an entropy-based GDM method in interval-valued intuitionistic fuzzy setting. Hajek and Froelich [17] described an integrating TOPSIS technique with interval-valued intuitionistic fuzzy cognitive maps for effective GDM. Gupta et al. [16] addressed a GDM method based on extended TOPSIS technique under interval-valued intuitionistic fuzzy environment. Mohammadi and Makui [28] explored a GDM approach based on interval-valued intuitionistic fuzzy set and evidential reasoning methodology. Liu and Li [25] introduced a GDM method based on interval-valued intuitionistic fuzzy power Bonferroni aggregation operators. Chen and Huang [12] developed a GDM method based on interval-valued intuitionistic fuzzy values and particle swarm optimization techniques. Yue [51] developed a geometric approach for ranking interval-valued intuitionistic fuzzy numbers (IVIFNs) with an application.

The main weaknesses from this review are summarized as follows:

Question 1. The existing researches paid less attention to the bipolar information: very positive and very negative information. Researchers paid more attention to aggregation methods of individual decisions and mathematical programming methods.

Question 2. The existing researches paid less attention to software quality assessment with interval-valued intuitionistic fuzzy information.

In order to solve these questions comprehensively, this paper aims to model a direct GDM method with interval-valued intuitionistic fuzzy information. The bipolar information is used to the reference matrices in an extend VIKOR-based GDM method.

Because the classical projection models have both practical and theoretical importance, a great deal of effort has been devoted to devising efficient algorithms, in which a portion of models are applied to multi-attribute decision-making (MADM) problems. For example, Tsao and Chen [33] introduced a projection-based compromising method for MADM with interval-valued intuitionistic fuzzy information. Wei et al. [37] proposed a projection model for MADM with picture fuzzy information. Wu et al. [41] proposed a projection method for MADM with fuzzy linguistic information.

In addition, some projection models are used to GDM problems. For example, Yue [48, 49], Yue and Jia [64, 66] developed some GDM methods based on projection measure with intuitionistic fuzzy information. Yue [47], Xu and Liu [45] established two projection methods in uncertain GDM context.

Recently, the normalized projection measures are attracted by scholars. For example, Yue and Jia [65] and Yue [52–54, 56–60, 62, 63] provided some normalized projection-based GDM methods with real number and interval data. Wang et al. [36] introduced a picture fuzzy normalized projection-based VIKOR method for the risk evaluation of construction project. Ji et al. [18] suggested a normalized projection-based TODIM (Tomada de decisao interativa e multicritévio) method.

As mentioned in Introduction section, this research finds the main weaknesses on projection measures as follows:

Question 1. The existing projection values are not always belong to [0,1] in interval-valued intuitionistic fuzzy setting (see Example 1 in section 4.1).

Question 2. The existing projection measures are not always reasonable in interval-valued intuitionistic fuzzy setting (see Example 1 in section 4.1).

To fill these research gaps in projection measures, this research intends to develop a new normalized projection measure in interval-valued intuitionistic fuzzy setting.

VIKOR-based GDM methods have been attracted by many scholars [1, 30]. For example, a VIKOR-based GDM approach [42] is proposed under interval type-2 fuzzy environment. The VIKOR-based GDM methods [6, 7, 15] are introduced in intuitionistic fuzzy environment. Some VIKOR-based GDM methods with linguistic information are developed by some scholars [38, 40, 46]. The VIKOR-based GDM technologies with Pythagorean fuzzy information are shared by some scholars [10, 24, 39].

The main weaknesses on VIKOR-based GDM methods from this review are summarized as follows:

Question 1. The existing VIKOR-based GDM methods need a collective decision aggregated by all the individual decisions, in which some information resources might be canceled each other.

Question 2. There are no specific expressions for regret data. The regret measurement is only from a transformation of utility index in existing VIKOR-based GDM methods.

To fill these research gaps in VIKOR-based GDM methods, this research intends to develop an extended VIKOR-based GDM method.

3 Interval-valued intuitionistic fuzzy information and projection measure

To integrate the survey in various aspects, we divide this section into two parts: interval-valued intuitionistic fuzzy information and projection measure.

3.1 Interval-valued intuitionistic fuzzy information

Atanassov and Gargov [3] extended the intuitionistic fuzzy set [2] to interval-valued intuitionistic fuzzy set, which is characterized by a membership function and a non-membership function, whose values are intervals rather than exact numbers.

Let X is universe of discourse, an interval-valued intuitionistic fuzzy set $\tilde{A}$ in X is an object $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) | x \in X\}$, where $\mu_{\tilde{A}}(x) = [\mu^l_{\tilde{A}}(x), \mu^u_{\tilde{A}}(x)] \subseteq [0,1]$ and $\nu_{\tilde{A}}(x) = [\nu^l_{\tilde{A}}(x), \nu^u_{\tilde{A}}(x)] \subseteq [0,1]$ are intervals, $\mu^l_{\tilde{A}}(x) = \inf \mu_{\tilde{A}}(x)$, $\mu^u_{\tilde{A}}(x) = \sup \mu_{\tilde{A}}(x)$, $\nu^l_{\tilde{A}}(x) = \inf \nu_{\tilde{A}}(x)$, $\nu^u_{\tilde{A}}(x) = \sup \nu_{\tilde{A}}(x)$, and $\mu^u_{\tilde{A}}(x) + \nu^u_{\tilde{A}}(x) \leq 1$, for all $x \in X$, and $\pi_{\tilde{A}}(x) = [\pi^l_{\tilde{A}}(x), \pi^u_{\tilde{A}}(x)]$, where $\pi^l_{\tilde{A}}(x) = 1 - \mu^u_{\tilde{A}}(x_i) - \nu^u_{\tilde{A}}(x), \pi^u_{\tilde{A}}(x) = 1 - \mu^l_{\tilde{A}}(x) - \nu^l_{\tilde{A}}(x)$, for all $x \in X$.

In particular, if $\mu_{\tilde{A}}(x) = \mu^l_{\tilde{A}}(x) = \mu^u_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x) = \nu^l_{\tilde{A}}(x) = \nu^u_{\tilde{A}}(x)$, then $\tilde{A}$ is reduced to an intuitionistic fuzzy set.

Xu and Chen [43] called the interval pair

$$\tilde{\alpha} = ([\mu^l, \mu^u], [\nu^l, \nu^u]), \tag{1}$$

an IVIFN, where the $[\mu^l, \mu^u], [\nu^l, \nu^u]$, $[\pi^l, \pi^u] \subseteq [0,1], 0 \leq \mu^u + \nu^u \leq 1, \pi^l = 1 - \mu^u - \nu^u, \pi^u = 1 - \mu^l - \nu^l$.

Xu and Chen [43] introduced the following operations:

Definition 3.1. *Let $\tilde{\alpha} = ([\mu^l, \mu^u], [\nu^l, \nu^u]), \tilde{\alpha}_1 = ([\mu_1^l, \mu_1^u], [\nu_1^l, \nu_1^u]), \tilde{\alpha}_2 = ([\mu_2^l, \mu_2^u], [\nu_2^l, \nu_2^u])$ be three IVIFNs, λ be a real number, then*

1. $\lambda\tilde{\alpha} = ([1-(1-\mu^l)^\lambda, 1-(1-\mu^u)^\lambda], [(\nu^l)^\lambda, (\nu^u)^\lambda]), \lambda > 0$;
2. $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = ([\mu_1^l + \mu_2^l - \mu_1^l\mu_2^l, \mu_1^u + \mu_2^u - \mu_1^u\mu_2^u], [\nu_1^l\nu_2^l, \nu_1^u\nu_2^u])$;
3. $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = ([\mu_1^l\mu_2^l, \mu_1^u\mu_2^u], [\nu_1^l + \nu_2^l - \nu_1^l\nu_2^l, \nu_1^u + \nu_2^u - \nu_1^u\nu_2^u])$;
4. $\tilde{\alpha}_1 \vee \tilde{\alpha}_2 = ([\mu_1^l \vee \mu_2^l, \mu_1^u \vee \mu_2^u], [\nu_1^l \wedge \nu_2^l, \nu_1^u \wedge \nu_2^u])$;
5. $\tilde{\alpha}_1 \wedge \tilde{\alpha}_2 = ([\mu_1^l \wedge \mu_2^l, \mu_1^u \wedge \mu_2^u], [\nu_1^l \vee \nu_2^l, \nu_1^u \vee \nu_2^u])$;

where the $\lambda\tilde{\alpha}$ and $\tilde{\alpha}^c$, $\tilde{\alpha}_1 \vee \tilde{\alpha}_2$ and $\tilde{\alpha}_1 \wedge \tilde{\alpha}_2$ are also IVIFNs.

3.2 Projection measure

For convenience, in this work, let $M = \{1, 2, \ldots, m\}$, $N = \{1, 2, \ldots, n\}$ and $T = \{1, 2, \ldots, t\}$; $i \in M, j \in N, k \in T$.

Definition 3.2. *Let $\tilde{\alpha} = (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$ be a vector. If all $\tilde{\alpha}_j (j \in N)$ are IVIFNs, then $\tilde{\alpha}$ is called an interval-valued intuitionistic fuzzy vector (IVIFV).*

Definition 3.3. [44] *Let $\tilde{\alpha} = (([\mu_{\tilde{\alpha}}^{1l}, \mu_{\tilde{\alpha}}^{1u}], [\nu_{\tilde{\alpha}}^{1l}, \nu_{\tilde{\alpha}}^{1u}]), ([\mu_{\tilde{\alpha}}^{2l}, \mu_{\tilde{\alpha}}^{2u}], [\nu_{\tilde{\alpha}}^{2l}, \nu_{\tilde{\alpha}}^{2u}]), \ldots, ([\mu_{\tilde{\alpha}}^{nl}, \mu_{\tilde{\alpha}}^{nu}], [\nu_{\tilde{\alpha}}^{nl}, \nu_{\tilde{\alpha}}^{nu}]))$ and $\tilde{\beta} = (([\mu_{\tilde{\beta}}^{1l}, \mu_{\tilde{\beta}}^{1u}], [\nu_{\tilde{\beta}}^{1l}, \nu_{\tilde{\beta}}^{1u}]), ([\mu_{\tilde{\beta}}^{2l}, \mu_{\tilde{\beta}}^{2u}], [\nu_{\tilde{\beta}}^{2l}, \nu_{\tilde{\beta}}^{2u}]), \ldots, ([\mu_{\tilde{\beta}}^{nl}, \mu_{\tilde{\beta}}^{nu}], [\nu_{\tilde{\beta}}^{nl}, \nu_{\tilde{\beta}}^{nu}]))$ be two IVIFVs, then*

$$Proj_{\tilde{\beta}}(\tilde{\alpha}) = \frac{\tilde{\alpha}\tilde{\beta}}{|\tilde{\beta}|}, \tag{2}$$

is called the projection of $\tilde{\alpha}$ on $\tilde{\beta}$, where $\tilde{\alpha}\tilde{\beta} = \sum_{j=1}^n (\mu_{\tilde{\alpha}}^{jl}\mu_{\tilde{\beta}}^{jl} + \mu_{\tilde{\alpha}}^{ju}\mu_{\tilde{\beta}}^{ju} + \nu_{\tilde{\alpha}}^{jl}\nu_{\tilde{\beta}}^{jl} + \nu_{\tilde{\alpha}}^{ju}\nu_{\tilde{\beta}}^{ju} + \pi_{\tilde{\alpha}}^{jl}\pi_{\tilde{\beta}}^{jl} + \pi_{\tilde{\alpha}}^{ju}\pi_{\tilde{\beta}}^{ju})$ is the inner/scalar product between $\tilde{\alpha}$ and $\tilde{\beta}$; $|\tilde{\beta}| = (\sum_{j=1}^n ((\mu_{\tilde{\beta}}^{jl})^2 + (\mu_{\tilde{\beta}}^{ju})^2 + (\nu_{\tilde{\beta}}^{jl})^2 + (\nu_{\tilde{\beta}}^{ju})^2 + (\pi_{\tilde{\beta}}^{jl})^2 + (\pi_{\tilde{\beta}}^{ju})^2))^{1/2}$ is the module of $\tilde{\beta}$, where $\pi_{\tilde{\alpha}}^{jl} = 1 - \mu_{\tilde{\alpha}}^{ju} - \nu_{\tilde{\alpha}}^{ju}, \pi_{\tilde{\alpha}}^{ju} = 1 - \mu_{\tilde{\alpha}}^{jl} - \nu_{\tilde{\alpha}}^{jl}; \pi_{\tilde{\beta}}^{jl} = 1 - \mu_{\tilde{\beta}}^{ju} - \nu_{\tilde{\beta}}^{ju}, \pi_{\tilde{\beta}}^{ju} = 1 - \mu_{\tilde{\beta}}^{jl} - \nu_{\tilde{\beta}}^{jl} (j \in N)$.

The $Proj_{\tilde{\beta}}(\tilde{\alpha})$ is a measurement that the $\tilde{\alpha}$ is close to $\tilde{\beta}$ [44]. The criterion is as following.

Criterion 3.4. *The larger the value $Proj_{\tilde{\beta}}(\tilde{\alpha})$ is, the closer the $\tilde{\alpha}$ is to $\tilde{\beta}$* [44].

Definition 3.5. *Let $X = (x_{kj})_{t\times n}$ be a matrix. If all x_{kj} are IVIFNs, then X is called an interval-valued intuitionistic fuzzy matrix.*

4 An extended group decision making method

For convenience, throughout this paper, the following index sets are used in decision process.

- A set of m feasible alternatives is written as $A = \{A_i | i \in M\}$, where $M = \{1, 2, \ldots, m\}$;
- A set of attributes is written as $U = \{u_j | j \in N\}$, where $N = \{1, 2, \ldots, n\}$;
- A weight vector of attributes is written as $w = (w_1, w_2, \ldots, w_j, \ldots, w_n)$, with $0 \leq w_j \leq 1$ and $\sum_{j=1}^n w_j = 1$;
- A set of experts is written as $E = \{e_k | k \in T\}$, where $T = \{1, 2, \ldots, t\}$.

4.1 A new normalized projection measure

For the projection measure in Eq. (2), as mentioned in Criterion 1, generally, the larger the value $Proj_{\tilde{\beta}}(\tilde{\alpha})$ is, the closer the $\tilde{\alpha}$ is to the $\tilde{\beta}$. However, this research finds that Eq. (2) does not always make sense, as the following example shows.

Example 4.1. *Let* $\tilde{\alpha} = (([0.410, 0.499], [0.400, 0.500]), ([0.420, 0.498], [0.410, 0.490]), ([0.430, 0.497], [0.420, 0.491]), ([0.440, 0.496], [0.430, 0.492]))$ *and* $\tilde{\beta} = (([0.001, 0.006], [0.861, 0.983]), ([0.002, 0.007], [0.862, 0.982]), ([0.003, 0.008], [0.863, 0.981]), ([0.004, 0.009], [0.864, 0.980]))$ *be two IVIFVs. We can calculate that* $\tilde{\alpha}\tilde{\beta} = 3.4745$, $|\tilde{\alpha}| = 1.8656$ *and* $|\tilde{\beta}| = 2.6273$. *It follows that* $Proj_{\tilde{\beta}}(\tilde{\alpha}) = 1.3225$, $Proj_{\tilde{\alpha}}(\tilde{\beta}) = 1.8624$, $Proj_{\tilde{\alpha}}(\tilde{\alpha}) = 1.8656$ *and* $Proj_{\tilde{\beta}}(\tilde{\beta}) = 2.6273$ *depended on the projection in Eq. (2).*

Two questions derived from Example 4.1 are shown as follows.

1. Two projections $Proj_{\tilde{\alpha}}(\tilde{\alpha})$ and $Proj_{\tilde{\beta}}(\tilde{\beta})$ depended on Eq. (2) may be unequal. For example, as mentioned in Example 4.1, we have seen that the $Proj_{\tilde{\alpha}}(\tilde{\alpha}) = 1.8656$ and $Proj_{\tilde{\beta}}(\tilde{\beta}) = 2.6273$. Therefore we have $Proj_{\tilde{\beta}}(\tilde{\beta}) > Proj_{\tilde{\alpha}}(\tilde{\alpha})$. It is hard to believe that the $\tilde{\beta}$ is much closer to oneself than $\tilde{\alpha}$.

2. The projection depended on Eq. (2) is not a normalized measure. In other word, Eq. (2) does not always satisfy the condition $0 \le Proj_{\tilde{\beta}}(\tilde{\alpha}) \le 1$. For example, the $Proj_{\tilde{\beta}}(\tilde{\beta}) = 2.6273 > 1$ in Example 1.

To solve these questions, a new normalized projection measure is provided as follows.

Definition 4.2. *Let* $\tilde{\alpha} = (([\mu_{\tilde{\alpha}}^{1l}, \mu_{\tilde{\alpha}}^{1u}], [\nu_{\tilde{\alpha}}^{1l}, \nu_{\tilde{\alpha}}^{1u}]), ([\mu_{\tilde{\alpha}}^{2l}, \mu_{\tilde{\alpha}}^{2u}], [\nu_{\tilde{\alpha}}^{2l}, \nu_{\tilde{\alpha}}^{2u}]), \ldots, ([\mu_{\tilde{\alpha}}^{nl}, \mu_{\tilde{\alpha}}^{nu}], [\nu_{\tilde{\alpha}}^{nl}, \nu_{\tilde{\alpha}}^{nu}]))$ *and* $\tilde{\beta} = (([\mu_{\tilde{\beta}}^{1l}, \mu_{\tilde{\beta}}^{1u}], [\nu_{\tilde{\beta}}^{1l}, \nu_{\tilde{\beta}}^{1u}]), ([\mu_{\tilde{\beta}}^{2l}, \mu_{\tilde{\beta}}^{2u}], [\nu_{\tilde{\beta}}^{2l}, \nu_{\tilde{\beta}}^{2u}]), \ldots, ([\mu_{\tilde{\beta}}^{nl}, \mu_{\tilde{\beta}}^{nu}], [\nu_{\tilde{\beta}}^{nl}, \nu_{\tilde{\beta}}^{nu}]))$ *be two IVIFVs, then*

$$NProj_{\tilde{\beta}}(\tilde{\alpha}) = \frac{1 + |\tilde{\alpha}|^2}{1 + |\tilde{\alpha}|^2 + |\tilde{\alpha}\tilde{\beta} - |\tilde{\beta}|^2|}, \tag{3}$$

is called a normalized projection of the $\tilde{\alpha}$ *on* $\tilde{\beta}$, *where* $\tilde{\alpha}\tilde{\beta} = \sum_{j=1}^{n}(\mu_{\tilde{\alpha}}^{jl}\mu_{\tilde{\beta}}^{jl} + \mu_{\tilde{\alpha}}^{ju}\mu_{\tilde{\beta}}^{ju} + \nu_{\tilde{\alpha}}^{jl}\nu_{\tilde{\beta}}^{jl} + \nu_{\tilde{\alpha}}^{ju}\nu_{\tilde{\beta}}^{ju} + \pi_{\tilde{\alpha}}^{jl}\pi_{\tilde{\beta}}^{jl} + \pi_{\tilde{\alpha}}^{ju}\pi_{\tilde{\beta}}^{ju})$, $|\tilde{\alpha}|^2 = \sum_{j=1}^{n}((\mu_{\tilde{\alpha}}^{jl})^2 + (\mu_{\tilde{\alpha}}^{ju})^2 + (\nu_{\tilde{\alpha}}^{jl})^2 + (\nu_{\tilde{\alpha}}^{ju})^2 + (\pi_{\tilde{\alpha}}^{jl})^2 + (\pi_{\tilde{\alpha}}^{ju})^2)$ *is the square of module of* $\tilde{\alpha}$, $|\tilde{\beta}|^2 = \sum_{j=1}^{n}((\mu_{\tilde{\beta}}^{jl})^2 + (\mu_{\tilde{\beta}}^{ju})^2 + (\nu_{\tilde{\beta}}^{jl})^2 + (\nu_{\tilde{\beta}}^{ju})^2 + (\pi_{\tilde{\beta}}^{jl})^2 + (\pi_{\tilde{\beta}}^{ju})^2)$ *is the square of module of* $\tilde{\beta}$, $\pi_{\tilde{\alpha}}^{jl} = 1 - \mu_{\tilde{\alpha}}^{ju} - \nu_{\tilde{\alpha}}^{ju}, \pi_{\tilde{\alpha}}^{ju} = 1 - \mu_{\tilde{\alpha}}^{jl} - \nu_{\tilde{\alpha}}^{jl} (j \in N)$.

For arbitrary IVIFVs $\tilde{\alpha}$ and $\tilde{\beta}$, Eq. (3) satisfy the following conditions:

1. $0 \le NProj_{\tilde{\beta}}(\tilde{\alpha}) \le 1$;

2. $NProj_{\tilde{\alpha}}(\tilde{\alpha}) = NProj_{\tilde{\beta}}(\tilde{\beta}) = 1$.

And Eq. (3) satisfy the following Criterion.

Criterion 4.3. *The larger the value* $NProj_{\tilde{\beta}}(\tilde{\alpha})$ *in Eq. (3) is, the closer the* $\tilde{\alpha}$ *is to* $\tilde{\beta}$.

For $\tilde{\alpha}$ and $\tilde{\beta}$ in Example 1, we have $\tilde{\alpha}\tilde{\beta} = 3.4745$, $|\tilde{\alpha}|^2 = 3.4805$ and $|\tilde{\beta}|^2 = 6.9027$. It follows that $NProj_{\tilde{\beta}}(\tilde{\alpha}) = 0.5665$ and $NProj_{\tilde{\alpha}}(\tilde{\beta}) = 0.9992$ and $NProj_{\tilde{\alpha}}(\tilde{\alpha}) = NProj_{\tilde{\beta}}(\tilde{\beta}) = 1$. These results are expected.

Definition 4.4. *Let* $X = (([\mu_{kj}^l, \mu_{kj}^u], [\nu_{kj}^l, \nu_{kj}^u]))_{t\times n}$ *and* $Y = (([\xi_{kj}^l, \xi_{kj}^u], [\eta_{kj}^l, \eta_{kj}^u]))_{t\times n}$ *be two interval-valued intuitionistic fuzzy matrices, then*

$$NProj_Y(X) = \frac{1 + |X|^2}{1 + |X|^2 + |XY - |Y|^2|}, \tag{4}$$

is called the projection of X *on* Y, *where,* $|X|^2 = \sum_{k=1}^{t}\sum_{j=1}^{n}((\mu_{kj}^l)^2 + (\mu_{kj}^u)^2 + (\nu_{kj}^l)^2 + (\nu_{kj}^u)^2 + (\pi_{kj}^l)^2 + (\pi_{kj}^u)^2)$, $XY = \sum_{k=1}^{t}\sum_{j=1}^{n}(\mu_{kj}^l\xi_{kj}^l + \mu_{kj}^u\xi_{kj}^u + \nu_{kj}^l\eta_{kj}^l + \nu_{kj}^u\eta_{kj}^u + \pi_{kj}^l\rho_{kj}^l + \pi_{kj}^u\rho_{kj}^u)$, $|Y|^2 = \sum_{k=1}^{t}\sum_{j=1}^{n}((\xi_{kj}^l)^2 + (\xi_{kj}^u)^2 + (\eta_{kj}^l)^2 + (\eta_{kj}^u)^2 + (\rho_{kj}^l)^2 + (\rho_{kj}^u)^2)$, $\pi_{kj}^l = 1 - \mu_{kj}^u - \nu_{kj}^u, \pi_{kj}^u = 1 - \mu_{kj}^l - \nu_{kj}^l, \rho_{kj}^l = 1 - \xi_{kj}^u - \eta_{kj}^u$ *and* $\rho_{kj}^u = 1 - \xi_{kj}^l - \eta_{kj}^l (k \in T, j \in N)$.

Criterion 4.5. *The larger the value* $NProj_Y(X)$ *in Eq. (4) is, the closer the* X *is to* Y.

4.2 An extended VIKOR-based GDM method

In this section, an extended VIKOR-based GDM method is proposed under interval-valued intuitionistic fuzzy environment.

Suppose that

$$X_i = (\tilde{x}^i_{kj})_{t\times n}, i \in M, \tag{5}$$

is a group decision matrix of the ith alternative $A_i (i \in M)$, in which the evaluation value $\tilde{x}^i_{kj}$ with respect to jth attribute u_j, provided by kth expert d_k, is characterized by IVIFN $([\mu^{il}_{kj}, \mu^{iu}_{kj}], [\nu^{il}_{kj}, \nu^{iu}_{kj}])(i \in M, k \in T, j \in N)$.

For the weight vector $w_k = (w_1, w_2, \ldots, w_n)$ of the attributes, the weighted decision is constructed as follows:

$$Y_i = (\tilde{y}^i_{kj})_{t\times n}, i \in M, \tag{6}$$

where $\tilde{y}^i_{kj} = (w_j \tilde{x}^i_{kj})_{t\times n} = ([\tau^{il}_{kj}, \tau^{iu}_{kj}], [\upsilon^{il}_{kj}, \upsilon^{iu}_{kj}])$, by Definition 3.1, $\tau^{il}_{kj} = 1-(1-\mu^{il}_{kj})^{w_j}, \tau^{iu}_{kj} = 1-(1-\mu^{iu}_{kj})^{w_j}, \upsilon^{il}_{kj} = (\nu^{il}_{kj})^{w_j}, \upsilon^{iu}_{kj} = (\nu^{iu}_{kj})^{w_j} (i \in M, k \in T, j \in N)$.

According to the idea of VIKOR technique, the ideal decision is determined as follows:

$$Y_+ = (\tilde{y}^+_{kj})_{t\times n}, \tag{7}$$

where $\tilde{y}^+_{kj} = ([\tau^{+l}_{kj}, \tau^{+u}_{kj}], [\upsilon^{+l}_{kj}, \upsilon^{+u}_{kj}])$, $\tau^{+l}_{kj} = \max_{i\in M}\{\tau^{il}_{kj}\}, \tau^{+u}_{kj} = \max_{i\in M}\{\tau^{iu}_{kj}\}, \upsilon^{+l}_{kj} = \min_{i\in M}\{\upsilon^{il}_{kj}\}, \upsilon^{+u}_{kj} = \min_{i\in M}\{\upsilon^{iu}_{kj}\}(k \in T, j \in N)$ based on Definition 3.1.

A regret value should be the difference of an ideal value and the current value. For example, $\tilde{y}^+_{kj}$ is an ideal value, $\tilde{y}^i_{kj}$ is the current value. According to this idea, the group regret matrices can be obtained, as the following equation shows.

$$R_i = (\tilde{r}^i_{kj})_{t\times n}, i \in M, \tag{8}$$

where $\tilde{r}^i_{kj} = \tilde{y}^+_{kj} - \tilde{y}^i_{kj}$. It is noted that there is no difference $\tilde{y}^+_{kj} - \tilde{y}^i_{kj}$ provided in current literature, therefore, the $\tilde{y}^+_{kj} - \tilde{y}^i_{kj}$ are replaced by $\tilde{y}^+_{kj} \otimes \tilde{y}^i_{kj}$ in Definition 3.1. $\tilde{r}^i_{kj}$ is written as $\tilde{r}^i_{kj} = ([\iota^{il}_{kj}, \iota^{iu}_{kj}], [\kappa^{il}_{kj}, \kappa^{iu}_{kj}]) = [\tau^{+l}_{kj}, \tau^{+u}_{kj}], [\upsilon^{+l}_{kj}, \upsilon^{+u}_{kj}]) \otimes ([\tau^{il}_{kj}, \tau^{iu}_{kj}], [\upsilon^{il}_{kj}, \upsilon^{iu}_{kj}]) = ([\tau^{+l}_{kj}\tau^{il}_{kj}, \tau^{+u}_{kj}\tau^{iu}_{kj}], [\upsilon^{+l}_{kj} + \upsilon^{il}_{kj} - \upsilon^{+l}_{kj}\upsilon^{il}_{kj}, \upsilon^{+u}_{kj} + \upsilon^{iu}_{kj} - \upsilon^{+u}_{kj}\upsilon^{iu}_{kj}])$.

The maximum group regret is derived from the group regret matrices $R_i (i \in M)$ in Eq. (8), which are shown as follows:

$$R_{max} = (\tilde{r}^{max}_{kj})_{t\times n}, i \in M, \tag{9}$$

where $r^{max}_{kj} = \max_{1\le i\le m}\{r^i_{kj}\} = ([\iota^{maxl}_{kj}, \iota^{maxu}_{kj}], [\kappa^{minl}_{kj}, \kappa^{minu}_{kj}]), \iota^{maxl}_{kj} = \max_{1\le i\le m}\{\iota^{il}_{kj}\}, \iota^{maxu}_{kj} = \max_{1\le i\le m}\{\iota^{iu}_{kj}\}, \kappa^{minl}_{kj} = \min_{1\le i\le m}\{\kappa^{il}_{kj}\}, \kappa^{minu}_{kj} = \min_{1\le i\le m}\{\kappa^{iu}_{kj}\}$, and $r^i_{kj}(i \in M, k \in T, j \in N)$ is shown in Eq. (8).

The normalized projection of each group decision Y_i on the Y_+, according to Eq. (4), is calculated as follows.

$$NProj_{Y_+}(Y_i) = \frac{1+|Y_i|^2}{1+|Y_i|^2+|Y_iY_+ - |Y_+|^2|}, i \in M, \tag{10}$$

where $|Y_i|^2 = \sum_{k=1}^t \sum_{j=1}^n ((\tau^{il}_{kj})^2 + (\tau^{iu}_{kj})^2 + (\upsilon^{il}_{kj})^2 + (\upsilon^{iu}_{kj})^2 + (\pi^{il}_{kj})^2 + (\pi^{iu}_{kj})^2), |Y_+|^2 = \sum_{k=1}^t \sum_{j=1}^n ((\tau^{+l}_{kj})^2 + (\tau^{+u}_{kj})^2 + (\upsilon^{+l}_{kj})^2 + (\upsilon^{+u}_{kj})^2 + (\pi^{+l}_{kj})^2 + (\pi^{+u}_{kj})^2), Y_iY_+ = \sum_{k=1}^t \sum_{j=1}^n (\tau^{il}_{kj}\tau^{+l}_{kj} + \tau^{+u}_{kj}\tau^{+u}_{kj} + \upsilon^{il}_{kj}\upsilon^{+l}_{kj} + \upsilon^{iu}_{kj}\upsilon^{+u}_{kj} + \pi^{il}_{kj}\pi^{+l}_{kj} + \pi^{iu}_{kj}\pi^{+u}_{kj}), \tau^{il}_{kj}, \tau^{iu}_{kj}, \upsilon^{il}_{kj}, \upsilon^{iu}_{kj}$ are the same as in Eq. (6), the $\tau^{+l}_{kj}, \tau^{+u}_{kj}, \upsilon^{+l}_{kj}, \upsilon^{+u}_{kj}$ are the same as in Eq. (7), and $\pi^{il}_{kj} = 1-\tau^{iu}_{kj}-\upsilon^{iu}_{kj}, \pi^{iu}_{kj} = 1-\tau^{il}_{kj}-\upsilon^{il}_{kj}, \pi^{+l}_{kj} = 1-\tau^{+u}_{kj}-\upsilon^{+u}_{kj}$ and $\pi^{+u}_{kj} = 1-\tau^{+l}_{kj}-\upsilon^{+l}_{kj}(i \in M, j \in N)$ by Eq. (1).

Criterion 4.6. *The larger the value $NProj_{Y_+}(Y_i)$ in Eq. (10), the closer the Y_i is to Y_+ , the better the alternative A_i is.*

According to Criterion 4, a group utility measurement GU_i of alternative A_i is based on the normalized projection of Y_i on the Y_+ in Eq. (10), as the following shows.

$$GU_i = NProj_{Y_+}(Y_i), i \in M, \tag{11}$$

Inspired by literature [68], let $GU_+ = \max_{1\le i\le m}\{GU_i\}, GU_- = \min_{1\le i\le m}\{GU_i\}$. GU_+ is called the largest group utility, the GU_- is called the smallest group utility. The normalized group utility is defined as:

$$NGU_i = \begin{cases} \frac{GU_i - GU_-}{GU_+ - GU_-}, & \text{if } GU_+ \ne GU_-; \\ 1, & \text{if } GU_+ = GU_-. \end{cases} i \in M. \tag{12}$$

Criterion 4.7. *The closer the NGU_i is to 1, the better the alternative A_i is.*

The normalized projection of each group regret matrix R_i on the R_{max} is calculated as follows.

$$NProj_{R_{max}}(R_i) = \frac{1+|R_i|^2}{1+|R_i|^2+|R_iR_{max}-|R_{max}|^2|}, i \in M, \tag{13}$$

where $|R_i|^2 = \sum_{k=1}^{t}\sum_{j=1}^{n}((\iota_{kj}^{il})^2+(\iota_{kj}^{iu})^2+(\kappa_{kj}^{il})^2+(\kappa_{kj}^{iu})^2+(\rho_{kj}^{il})^2+(\rho_{kj}^{iu})^2)$, $|R_{max}|^2 = \sum_{k=1}^{t}\sum_{j=1}^{n}((\iota_{kj}^{maxl})^2+(\iota_{kj}^{maxu})^2+(\kappa_{kj}^{maxl})^2+(\kappa_{kj}^{maxu})^2+(\rho_{kj}^{maxl})^2+(\rho_{kj}^{maxu})^2)$, $R_iR_{max} = \sum_{k=1}^{t}\sum_{j=1}^{n}(\iota_{kj}^{il}\iota_{kj}^{maxl}+\iota_{kj}^{iu}\iota_{kj}^{maxu}+\kappa_{kj}^{il}\kappa_{kj}^{maxl}+\kappa_{kj}^{iu}\kappa_{kj}^{maxu}+\rho_{kj}^{il}\rho_{kj}^{maxl}+\rho_{kj}^{iu}\rho_{kj}^{maxu})$, $\iota_{kj}^{il}, \iota_{kj}^{iu}, \kappa_{kj}^{il}, \kappa_{kj}^{iu}$ are the same as in Eq. (8), the $\iota_{kj}^{maxl}, \iota_{kj}^{maxu}, \kappa_{kj}^{maxl}, \kappa_{kj}^{maxu}$ are the same as in Eq. (9), and $\rho_{kj}^{il} = 1-\iota_{kj}^{iu}-\kappa_{kj}^{iu}, \rho_{kj}^{iu} = 1-\iota_{kj}^{il}-\kappa_{kj}^{il}, \rho_{kj}^{maxl} = 1-\iota_{kj}^{maxu}-\kappa_{kj}^{maxu}$ and $\rho_{kj}^{maxu} = 1-\iota_{kj}^{maxl}-\kappa_{kj}^{maxl} (i \in M, j \in N, k \in T)$ by Eq. (1).

Criterion 4.8. *The larger the value $NProj_{R_{max}}(R_i)$ in Eq. (13), the closer the R_i is to R_{max} (the maximum group regret matrix), the worse the alternative A_i is.*

According to Criterion 6, a group regret measurement is based on the normalized projection of R_i on R_{max} in Eq. (13), as the following shows.

$$GR_i = NProj_{R_{max}}(R_i), i \in M. \tag{14}$$

Similar to Eq. (12), if we let $GR_+ = \min_{1\le i\le m}\{GR_i\}, GR_- = \max_{1\le i\le m}\{GR_i\}$. The GR_+ is called the smallest group regret, the GR_- is called the largest group regret. The normalized group regret of A_i is defined as:

$$NGR_i = \begin{cases} \frac{GR_- - GR_i}{GR_- - GR_+}, & \text{if} \quad GR_+ \neq GR_-; \\ 0, & \text{if} \quad GR_+ = GR_-. \end{cases} i \in M, \tag{15}$$

where, in order to avoid the case that denominator $GR_+ - GR_-$ is zero, we let $NGR_i = 0$ when $GR_+ = GR_-$.

Criterion 4.9. *The closer the NGR_i is to 1, the better the alternative A_i is.*

Thus, a comprehensive VIKOR measure of alternative A_i can be obtained by the following relation:

$$Q_i = \lambda NGU_i + (1-\lambda)NGR_i, i \in M, \tag{16}$$

where the λ is referred to as a compromise coefficient, and $\lambda \in [0,1]$. The values of λ and $1-\lambda$ are the weight of the normalized group utility NGU_i and the normalized group regret NGR_i, respectively. If $\lambda > 0.5$, it is indicated that experts tend to make decision based on the group utility; if $\lambda < 0.5$, it is indicated that experts tend to make decision based on the group regret; if $\lambda = 0.5$, it is indicated that experts adopt a balanced and compromised way to make decision. In general, the value $\lambda = 0.5$ is adopted.

Criterion 4.10. *The larger the value Q_i, the better the alternative A_i is.*

All the alternatives are ranked by index Q. Suppose that $Q(A_i^{(h)})$ denotes the alternative A_i ranked at hth position by Q for all $i, h, i, h = 1, 2, \cdots, m$. $Q(A_i^{(h)})$ is written as A_{i_h}. Now m alternatives compose a ranking $A_{i_1} \ge A_{i_2} \ge \cdots \ge A_{i_m}$ in descending order. For two alternatives A_{i_1} and A_{i_2}, if $Q(A_{i_1}^{(h_1)}) - Q(A_{i_2}^{(h_2)}) < 1/(m-1)$, where m is the number of alternatives, $h_2 > h_1$, then we say that the distinction between A_{i_1} and A_{i_2} are small. In this case, the alternatives $\{A_{i_1}, A_{i_2}\}$ are classified into the same grade. Beginning with A_{i_1}, the alternatives $\{A_1, A_2, \cdots, A_m\}$ are ranked in phases according to the following procedure.

Phase 1. If $Q(A_{i_1}^{(1)}) - Q(A_{i_h}^{(h)}) < 1/(m-1)$ and $Q(A_{i_1}^{(1)}) - Q(A_{i_{h+1}}^{(h+1)}) \ge 1/(m-1)$, where $h \ge 2$, then $\{A_{i_1}, A_{i_2}, \cdots, A_{i_h}\}$ are tied for the first position in the ranking list.

Phase 2. Beginning with $A_{i_{h+1}}$, if $Q(A_{i_{h+1}}^{(h+1)}) - Q(A_{i_l}^{(l)}) < 1/(m-1)$ and $Q(A_{i_{h+1}}^{(h+1)}) - Q(A_{i_{l+1}}^{(l+1)}) \ge 1/(m-1)$, where $l \ge h+1$, then $\{A_{i_{h+1}}, A_{i_{h+2}}, \cdots, A_{i_l}\}$ are tied for the second position in the ranking list. In this case, we have that $\{A_{i_1}, A_{i_2}, \cdots, A_{i_h}\} \succ \{A_{i_{h+1}}, A_{i_{h+2}}, \cdots, A_{i_l}\}$.

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Phase end. According to the thought in Phase 2, we can end the procedure until A_{i_m} is graded in the ranking list.

4.3 Presented algorithm

In sum, an extended VIKOR-based GDM approach with interval-valued intuitionistic fuzzy information is provided by the following steps.

Step 1. Establish group decision matrix.

The group decision $X_i = (\tilde{x}^i_{kj})_{t\times n}(i \in M)$ is established by Eq. (5).

Step 2. Construct weighted decisions.

For a given weight vector $w = (w_1, w_2, \ldots, w_n)$ of attributes, the weighted group decision is constructed by Eq. (6).

Step 3. Determine the ideal decision of all group decisions.

The ideal decision Y_+ of all weighted group decisions is determined by Eq. (7).

Step 4. Calculate group regret matrix.

The group regret matrix is calculated by Eq. (8).

Step 5. Determine the maximum group regret matrix.

The maximum group regret matrix is determined by Eq. (9).

Step 6. Calculate the group utility measurement.

The group utility measurement is calculated by Eqs. (10) and (11).

Step 7. Determine the normalized group utility.

The normalized group utility is determined by Eq. (12).

Step 8. Calculate the group regret measurement.

The group regret measurement of alternative A_i is established by Eqs. (13) and (14).

Step 9. Construct the normalized group regret measurement.

The normalized group regret measurement is constructed by Eq. (15).

Step 10. Construct the comprehensive VIKOR measurement.

The comprehensive VIKOR measure Q_i of alternative A_i is calculated by Eq. (16).

Step 11. Rank the preference order of alternatives.

The alternatives are ranked in descending order in accordance with the comprehensive VIKOR measure Q.

5 Experimental analysis

This section provides a real application and an experimental analysis to show the applicability and feasibility of method proposed in this paper.

5.1 Illustrative example

This subsection gives a real application to software product quality assessment to test the applicability of the approach developed in this paper.

Software project management is the process of managing the software project. Software quality is a very important aspect in software project management. Assessing software quality in the early stages of design and development is crucial as it helps reduce effort, time and money [4]. Although there are many standards developed by International Organization for Standardizations, which have specified software quality's metrics, the task is difficult since some software quality characteristics, such as maintainability, reliability and reusability, cannot be directly and objectively measured before the software product is deployed and used for a certain period of time [4].

As mentioned in the Introduction, this work finds that there are very few researches on the evaluation of software product quality from users perspectives; and there are very few researches on the evaluation of software product quality

Table 1: Group decisions of four software products.

Decision	Expert	u_1	u_2	u_3
X_1	e_1	([0.70, 0.75],[0.20, 0.25])	([0.80, 0.85],[0.10, 0.15])	([0.65, 0.70],[0.25, 0.30])
	e_2	([0.65, 0.70],[0.25, 0.30])	([0.75, 0.80],[0.15, 0.20])	([0.70, 0.75],[0.20, 0.25])
	e_3	([0.75, 0.80],[0.15, 0.20])	([0.70, 0.75],[0.20, 0.25])	([0.75, 0.80],[0.15, 0.20])
X_2	e_1	([0.10, 0.15],[0.70, 0.80])	([0.15, 0.20],[0.60, 0.75])	([0.25, 0.30],[0.60, 0.65])
	e_2	([0.15, 0.20],[0.70, 0.75])	([0.10, 0.15],[0.65, 0.80])	([0.25, 0.30],[0.65, 0.70])
	e_3	([0.10, 0.15],[0.80, 0.85])	([0.25, 0.30],[0.60, 0.65])	([0.20, 0.25],[0.70, 0.70])
X_3	e_1	([0.80, 0.85],[0.10, 0.12])	([0.75, 0.78],[0.20, 0.22])	([0.60, 0.65],[0.30, 0.35])
	e_2	([0.80, 0.85],[0.10, 0.15])	([0.80, 0.85],[0.10, 0.15])	([0.70, 0.75],[0.20, 0.25])
	e_3	([0.65, 0.70],[0.25, 0.27])	([0.60, 0.65],[0.30, 0.35])	([0.85, 0.90],[0.05, 0.10])
X_4	e_1	([0.70, 0.75],[0.13, 0.15])	([0.65, 0.70],[0.25, 0.27])	([0.60, 0.63],[0.35, 0.37])
	e_2	([0.70, 0.80],[0.18, 0.20])	([0.80, 0.85],[0.10, 0.15])	([0.65, 0.65],[0.25, 0.28])
	e_3	([0.60, 0.70],[0.25, 0.27])	([0.75, 0.75],[0.24, 0.25])	([0.60, 0.80],[0.15, 0.19])

based on GDM method. To approach these knowledge gaps systematically, in this study, the assessment of software product quality is based on user perspectives and a GDM method.

This assessment is based on a set of real data taken from the experts. Four software products are assessed here, which are used in a university, Guangdong, China. Four software products comprise a set denoted by $\{A_1, A_2, A_3, A_4\}$. The evaluation attributes of four software products comprise a set denoted by $\{u_1, u_2, u_3\}$ = {maintainability, reliability, performance efficiency}, which are based on the concerns of users. Three expert groups comprise a set denoted by $E = \{e_1, e_2, e_3\}$, where $e_k (k = 1, 2, 3)$ is the users, who are from three different colleges in this university. Specifically, e_1 is the users from college of mathematics and computer science; e_2 is the users from college of ocean and engineering; e_3 is the users from college of mechanical engineering. Each expert group is composed by teachers T, graduate students GS and undergraduate students US. That is, each expert group is divided into three class, which is written as $H = \{T, GS, US\}$. Experts provide their individual evaluations on software with respect to attributes.

Experimental data are collected from a questionnaire survey. The questions in questionnaire are answered by using some simple symbols $\{\checkmark, \times, \bigcirc\}$, where the symbols $\checkmark, \times, \bigcirc$ denote, respectively, satisfaction, dissatisfaction, hesitation or abstention. We know that an intuitionistic fuzzy number is composed of three parameters. One is the membership degree μ, which can measure the user's satisfaction (written as $\checkmark$); another is the nonmembership degree ν, which can measure the user's dissatisfaction (written as $\times$); third is a hesitation or indeterminacy index π, which can measure the user's uncertain information (written as $\bigcirc$). The nonresponse is also belonged to uncertain information. That is, the symbols $\checkmark, \times, \bigcirc$ can be synthetically expressed by an intuitionistic fuzzy number (μ, ν, π).

Let s_k^{ih} be the number of hth $(h \in H)$ group of experts, who are from kth college, and participated in the evaluation of ith alternative A_i. The symbols $\checkmark$ and $\times$ are collected from questionnaires according to attribute $u_j (j \in N)$. The number of symbols $\checkmark$ is written as y_{kj}^{ih}, and the number of symbols $\times$ is written as n_{kj}^{ih}. To obtain an IVIFN based on the numbers of symbols $\checkmark$ and $\times$, we first let

$$\xi_{kj}^{ih} = \frac{y_{kj}^{ih}}{s_k^{ih}}, \eta_{kj}^{ih} = \frac{n_{kj}^{ih}}{s_k^{ih}}, i \in M, j \in N, k \in T, h \in H. \tag{17}$$

Then an interval-valued intuitionistic fuzzy score is obtained as follows:

$$\tilde{x}_{kj}^{i} = ([\mu_{kj}^{il}, \mu_{kj}^{iu}], [\nu_{kj}^{il}, \nu_{kj}^{iu}]), i \in M, j \in N, k \in T, \tag{18}$$

where $\mu_{kj}^{il} = \xi_{kj}^{il}/\sigma_{kj}^{iu}, \mu_{kj}^{iu} = \xi_{kj}^{iu}/\sigma_{kj}^{iu}, \nu_{kj}^{il} = \eta_{kj}^{il}/\sigma_{kj}^{iu}, \nu_{kj}^{iu} = \eta_{kj}^{iu}/\sigma_{kj}^{iu}$ and $\sigma_{kj}^{iu} = \xi_{kj}^{iu} + \eta_{kj}^{iu}, \xi_{kj}^{il} = \min_{h \in H}\{\xi_{kj}^{ih}\}, \xi_{kj}^{iu} = \max_{h \in H}\{\xi_{kj}^{ih}\}, \eta_{kj}^{il} = \min_{h \in H}\{\eta_{kj}^{ih}\}, \eta_{kj}^{iu} = \max_{h \in H}\{\eta_{kj}^{ih}\}$.

It is clear that μ_{kj}^{iu} and ν_{kj}^{iu} satisfy the conditions $0 \leq \mu_{kj}^{iu} + \nu_{kj}^{iu} \leq 1 (i \in M, j \in N, k \in T)$ in Eq. (1). Reader interested in aggregation methods of an IVIFN can refer to the references in [61, 67].

The evaluation matrices of four software products, written as $X_i (i = 1, 2, 3, 4)$, are shown in Table 1, in which the evaluation values are shown in IVIFNs.

Example 5.1. *For the* 1*st alternative* A_1 *with respect to* 1*st attribute* u_1*, an IVIFN* $\tilde{x}_{11}^1 = ([0.70, 0.75], [0.20, 0.25])$ *in* X_1 *in Table 1 is elaborated in this example, where the data are collected from questionnaires, which are provided by* e_1*, who are the users from the college of mathematics and computer science in this university, Guangdong, China.*

Table 2: Weighted decisions and ideal decision of four software products.

Decision	Expert	u_1	u_2	u_3
Y_1	e_1	([0.30, 0.34],[0.62, 0.66])	([0.38, 0.43],[0.50, 0.57])	([0.34, 0.38],[0.57, 0.62])
	e_2	([0.27, 0.30],[0.66, 0.70])	([0.34, 0.38],[0.57, 0.62])	([0.38, 0.43],[0.53, 0.57])
	e_3	([0.34, 0.38],[0.57, 0.62])	([0.30, 0.34],[0.62, 0.66])	([0.43, 0.47],[0.47, 0.53])
Y_2	e_1	([0.03, 0.05],[0.90, 0.94])	([0.05, 0.06],[0.86, 0.92])	([0.11, 0.13],[0.82, 0.84])
	e_2	([0.05, 0.06],[0.90, 0.92])	([0.03, 0.05],[0.88, 0.94])	([0.11, 0.13],[0.84, 0.87])
	e_3	([0.03, 0.05],[0.94, 0.95])	([0.08, 0.10],[0.86, 0.88])	([0.09, 0.11],[0.87, 0.87])
Y_3	e_1	([0.38, 0.43],[0.50, 0.53])	([0.34, 0.37],[0.62, 0.63])	([0.31, 0.34],[0.62, 0.66])
	e_2	([0.38, 0.43],[0.50, 0.57])	([0.38, 0.43],[0.50, 0.57])	([0.38, 0.43],[0.53, 0.57])
	e_3	([0.27, 0.30],[0.66, 0.68])	([0.24, 0.27],[0.70, 0.73])	([0.53, 0.60],[0.30, 0.40])
Y_4	e_1	([0.30, 0.34],[0.54, 0.57])	([0.27, 0.30],[0.66, 0.68])	([0.31, 0.33],[0.66, 0.67])
	e_2	([0.30, 0.38],[0.60, 0.62])	([0.38, 0.43],[0.50, 0.57])	([0.34, 0.34],[0.57, 0.60])
	e_3	([0.24, 0.43],[0.50, 0.55])	([0.34, 0.34],[0.65, 0.66])	([0.31, 0.47],[0.47, 0.51])
Y_+	e_1	([0.38, 0.43],[0.50, 0.53])	([0.38, 0.43],[0.50, 0.57])	([0.34, 0.38],[0.57, 0.62])
	e_2	([0.38, 0.43],[0.50, 0.57])	([0.38, 0.43],[0.50, 0.57])	([0.38, 0.43],[0.53, 0.57])
	e_3	([0.34, 0.38],[0.57, 0.62])	([0.34, 0.34],[0.62, 0.66])	([0.53, 0.60],[0.30, 0.40])

Table 3: Group regret matrices and the maximum group regret matrix of software evaluation.

Evaluation	Expert	u_1	u_2	u_3
R_1	e_1	([0.12,0.15],[0.81,0.84])	([0.15,0.19],[0.75,0.81])	([0.12,0.15],[0.82,0.85])
	e_2	([0.10,0.13],[0.83,0.87])	([0.13,0.17],[0.78,0.83])	([0.15,0.18],[0.77,0.82])
	e_3	([0.12,0.15],[0.81,0.85])	([0.10,0.12],[0.85,0.88])	([0.23,0.29],[0.63,0.71])
R_2	e_1	([0.01,0.02],[0.95,0.97])	([0.02,0.03],[0.93,0.96])	([0.04,0.05],[0.92,0.94])
	e_2	([0.02,0.03],[0.95,0.96])	([0.01,0.02],[0.94,0.97])	([0.04,0.06],[0.92,0.94])
	e_3	([0.01,0.02],[0.97,0.98])	([0.03,0.03],[0.95,0.96])	([0.05,0.07],[0.91,0.92])
R_3	e_1	([0.15,0.19],[0.75,0.78])	([0.13,0.16],[0.81,0.84])	([0.11,0.13],[0.84,0.87])
	e_2	([0.15,0.19],[0.75,0.81])	([0.15,0.19],[0.75,0.81])	([0.15,0.18],[0.77,0.82])
	e_3	([0.09,0.12],[0.85,0.88])	([0.08,0.09],[0.88,0.91])	([0.28,0.36],[0.51,0.64])
R_4	e_1	([0.12,0.15],[0.77,0.80])	([0.10,0.13],[0.83,0.86])	([0.11,0.13],[0.85,0.87])
	e_2	([0.12,0.17],[0.80,0.83])	([0.15,0.19],[0.75,0.81])	([0.13,0.15],[0.80,0.83])
	e_3	([0.08,0.12],[0.85,0.88])	([0.12,0.12],[0.87,0.88])	([0.16,0.29],[0.63,0.71])
R_{max}	e_1	([0.15,0.19],[0.75,0.78])	([0.15,0.19],[0.75,0.81])	([0.12,0.15],[0.82,0.85])
	e_2	([0.15,0.19],[0.75,0.81])	([0.15,0.19],[0.75,0.81])	([0.15,0.18],[0.77,0.82])
	e_3	([0.12,0.15],[0.81,0.85])	([0.12,0.12],[0.85,0.88])	([0.28,0.36],[0.51,0.64])

The total number of e_1 *is 70, where the number* s_1^{11} *of teachers T is 10, the number* s_1^{12} *of graduate students GS is 20, the number* s_1^{13} *of undergraduate students US is 40. Further, the number* y_{11}^{11} *is 7; the number* n_{11}^{11} *is 2; the number* y_{11}^{12} *is 15; the number* n_{11}^{12} *is 4; the number* y_{11}^{13} *is 29; the number* n_{11}^{13} *is 10.*

By Eq. (14), we have that $\xi_{11}^{11} = y_{11}^{11}/s_1^{11} = 7/10 = 0.7, \xi_{11}^{12} = y_{11}^{12}/s_1^{12} = 15/20 = 0.75, \xi_{11}^{13} = y_{11}^{13}/s_1^{12} = 29/40 = 0.725; \eta_{11}^{11} = n_{11}^{11}/s_1^{11} = 2/10 = 0.2, \eta_{11}^{12} = n_{11}^{12}/s_1^{12} = 4/20 = 0.2, \eta_{11}^{13} = n_{11}^{13}/s_1^{13} = 10/40 = 0.25$. *Further, we have* $\xi_{11}^{1l} = \min_{1\leq h\leq 3}\{\xi_{11}^{1h}\} = \min\{\xi_{11}^{11}, \xi_{11}^{12}, \xi_{11}^{13}\} = \min\{0.7, 0.75, 0.725\} = 0.7$, $\xi_{11}^{1u} = \max_{1\leq h\leq 3}\{\xi_{11}^{1h}\} = \max\{\xi_{11}^{11}, \xi_{11}^{12}, \xi_{11}^{13}\} = \max\{0.7, 0.75, 0.725\} = 0.75$. *In addition, since* $\sigma_{11}^{1u} = \xi_{11}^{1u} + \eta_{11}^{1u} = 0.75 + 0.25 = 1$. *Therefore, by Eq. (15), we have* $\tilde{x}_{11}^{1} = ([\mu_{kj}^{il}, \mu_{kj}^{iu}], [\nu_{kj}^{il}, \nu_{kj}^{iu}]) = ([\xi_{11}^{1l}/\sigma_{11}^{1u}, \xi_{11}^{1u}/\sigma_{11}^{1u}], [\eta_{11}^{1l}/\sigma_{11}^{1u}, \eta_{11}^{1u}/\sigma_{11}^{1u}]) = ([0.70/1, 0.75/1], [0.20/1, 0.25/1]) = ([0.7, 0.75], [0.2, 0.25])$.

For the attributes' weight vector $w = (w_1, w_2, w_3) = (0.3, 0.3, 0.4)$ given by experts, the weighted group decisions are constructed by Step 2, which are shown in Table 2.

The ideal decision Y_+ of all group decisions is determined by Step 3, which is also shown in Table 2.

Based on the group decisions Y_i and the ideal decision Y_+ in Table 2, the group regret matrices are calculated by Step 4, and the maximum group regret matrix is determined by Step 5,which are shown in Table 3.

The group utility measurements $GU_i(i = 1, 2, 3, 4)$ are based on the normalized projection of each group decision Y_i on the ideal decision Y_+, which is calculated by Step 6, as the Table 4 shows.

The normalized group utilities $NGU_i(i = 1, 2, 3, 4)$ are calculated by Step 7; the group regret measurements $GR_i(i =$

Table 4: VIKOR indexes and rankings of four software products

Software	GU_i	NGU_i	Ranking	GR_i	NGR_i	Ranking	Q_i	Ranking
A_1	0.9934	0.8951	2	0.9819	0.0753	3	0.4852	3
A_2	0.9567	0.0000	4	0.9106	1.0000	1	0.5000	2
A_3	0.9866	0.7285	3	0.9877	0.0000	4	0.3642	4
A_4	0.9977	1.0000	1	0.9785	0.1195	2	0.5598	1

Table 5: VIKOR indexes and rankings of four software products based on the classical projection measure

Software	GU_i	NGU_i	Ranking	GR_i	NGR_i	Ranking	Q_i	Ranking
A_1	2.9395	0.0637	3	3.5020	0.9460	2	0.5048	2
A_2	3.1548	1.0000	1	3.9224	0.0000	4	0.5000	3
A_3	2.9631	0.1661	2	3.4780	1.0000	1	0.5831	1
A_4	2.9249	0.0000	4	3.5165	0.9134	3	0.4567	4

$1, 2, 3, 4)$ are calculated by Step 8; the normalized group regret measurements $NGU_i(i = 1, 2, 3, 4)$ are calculated by Step 9; Let $\lambda = 0.5$ in Eq. (16), the comprehensive VIKOR measurements $Q_i(i = 1, 2, 3, 4)$ are calculated by Step 10. These measurements and the rankings of software products $A_i(i = 1, 2, 3, 4)$ based on these measures are also summarized in Table 4.

Table 4 shows that the $Q(A_4^{(1)}) - Q(A_2^{(2)}) = 0.5598 - 0.5000 = 0.0598 < 1/3$, $Q(A_4^{(1)}) - Q(A_1^{(3)}) = 0.5598 - 0.4852 = 0.0746 < 1/3$, and $Q(A_4^{(1)}) - Q(A_3^{(4)}) = 0.5598 - 0.3642 = 0.1956 < 1/3$, therefore we have $\{A_1, A_2, A_3, A_4\}$ are tied for the first position in the ranking list based on comprehensive VIKOR measure Q.

5.2 Comparison with the classical projection measure

In this subsection, the normalized projection measure is compared with the classical projection measure. This comparison is based on the same data in illustrative example in Section 5.1. Other procedures are the same as the algorithm in Section 4.3, but the projection measure is replaced by the classical projection measure. Specifically,

Eq. (11) is replaced by

$$GU_i = Proj_{Y_+}(Y_i), i \in M, \tag{19}$$

where $Proj_{Y_+}(Y_i) = Y_iY_+/|Y_+|$, $Y_i, Y_+, |Y_+|$ are the same as in Eq. (10).

Eq. (14) is replaced by

$$GR_i = Proj_{R_{max}}(R_i), i \in M, \tag{20}$$

where $Proj_{R_{max}}(R_i) = R_iR_{max}/|R_{max}|$, and R_i, R_{max}, $|R_{max}|$ are the same as in Eq. (13).

The normalized group utility NGU_i is the same as in Eq. (12), but the $GU_i = Proj_{Y_+}(Y_i), GU_+ = \max_{1\le i\le m}\{GU_i\}, GU_- = \min_{1\le i\le m}\{GU_i\}$. The normalized group regret is the same as in Eq. (15), but the $GR_i = Proj_{R_{max}}(R_i)$, $GR_+ = \min_{1\le i\le m}\{GR_i\}$, $GR_- = \max_{1\le i\le m}\{GR_i\}$.

The projection-based group utility measurement GU_i is calculated by Step 6, the projection-based normalized group utility measurement NGU_i is calculated by Step 7, the projection-based group regret measurement GR_i is calculated by Step 8, the projection-based normalized group regret measurement NGR_i is calculated by Step 9. Let $\lambda = 0.5$ in Eq. (16), the comprehensive VIKOR index Q_i is calculated by Step 10. These measurements and index are shown in Table 5.

Table 5 shows that the $Q(A_3^{(1)}) - Q(A_4^{(4)}) = 0.5831 - 0.4567 = 0.1264 < 1/3$, therefore, the ranking $\{A_1, A_2, A_3, A_4\}$ are tied for the first position in the ranking list. That is, the order based on Q_i in Table 4 is supported by the classical projection measure.

5.3 Comparison with the Euclidean distance

In this subsection, the normalized projection measure is replaced by the Euclidean distance. This comparison is based on the same data in illustrative example in Section 5.1. Other procedures are the same as the algorithm in Section 4.3.

That is, Eq. (11) is replaced by

$$GU_i = D(Y_i, Y_+), i \in M, \tag{21}$$

Table 6: VIKOR indexes and rankings of four software products based on the Euclidean distance

Software	GU_i	NGU_i	Ranking	GR_i	NGR_i	Ranking	Q_i	Ranking
A_1	0.4772	0.9310	2	3.0380	0.0583	3	0.4946	2
A_2	3.5578	0.0000	4	4.4510	1.0000	1	0.5000	1
A_3	0.2488	1.0000	1	2.9506	0.0000	4	0.5000	1
A_4	0.7542	0.8473	3	3.0709	0.0802	2	0.4637	3

where $D(Y_i, Y_+) = (\sum_{i=1}^{m}\sum_{j=1}^{n}((\tau_{kj}^{il}-\tau_{kj}^{+l})^2+(\tau_{kj}^{iu}-\tau_{kj}^{+u})^2+(\upsilon_{kj}^{il}-\upsilon_{kj}^{+l})^2+(\upsilon_{kj}^{iu}-\upsilon_{kj}^{+u})^2+(\pi_{kj}^{il}-\pi_{kj}^{+l})^2+(\pi_{kj}^{iu}-\pi_{kj}^{+u})^2)^{1/2}$, and $\tau_{kj}^{il}, \tau_{kj}^{iu}, \upsilon_{kj}^{il}, \upsilon_{kj}^{iu}$ are shown in Eq. (6), $\tau_{kj}^{+l}, \tau_{kj}^{+u}, \upsilon_{kj}^{+l}, \upsilon_{kj}^{+u}$ are shown in Eq. (7), and $\pi_{kj}^{il} = 1-\tau_{kj}^{iu}-\upsilon_{kj}^{iu}, \pi_{kj}^{iu} = 1-\tau_{kj}^{il}-\upsilon_{kj}^{il}, \pi_{kj}^{+l} = 1-\tau_{kj}^{+u}-\upsilon_{kj}^{+u}$ and $\pi_{kj}^{+u} = 1-\tau_{kj}^{+l}-\upsilon_{kj}^{+l} (i \in M, j \in N)$ by Eq. (1).

Eq. (12) is replaced by.

$$NGU_i = \frac{GY_+ - GY_i}{GY_+ - GY_-}, i \in M. \tag{22}$$

where $GU_i = D(Y_i, Y_+), GU_+ = \max_{1\le i\le m}\{GU_i\}, GU_- = \min_{1\le i\le m}\{GU_i\}$.

Eq. (14) is replaced by:

$$GR_i = D(R_i, R_{max}), i \in M. \tag{23}$$

where $D(R_i, R_{max}) = (\sum_{i=1}^{m}\sum_{j=1}^{n}((\iota_{kj}^{il}-\iota_{kj}^{maxl})^2+(\iota_{kj}^{iu}-\iota_{kj}^{maxu})^2+(\kappa_{kj}^{il}-\kappa_{kj}^{maxl})^2+(\kappa_{kj}^{iu}-\kappa_{kj}^{maxu})^2+(\rho_{kj}^{il}-\rho_{kj}^{maxl})^2+(\rho_{kj}^{iu}-\rho_{kj}^{maxu})^2)^{1/2}$, the $\iota_{kj}^{il}, \iota_{kj}^{iu}, \kappa_{kj}^{il}, \kappa_{kj}^{iu}$ are the same as in Eq. (8), the $\iota_{kj}^{maxl}, \iota_{kj}^{maxu}, \kappa_{kj}^{maxl}, \kappa_{kj}^{maxu}$ are the same as in Eq. (9), and $\rho_{kj}^{il} = 1-\iota_{kj}^{iu}-\kappa_{kj}^{iu}, \rho_{kj}^{iu} = 1-\iota_{kj}^{il}-\kappa_{kj}^{il}, \rho_{kj}^{maxl} = 1-\iota_{kj}^{maxu}-\kappa_{kj}^{maxu}$ and $\rho_{kj}^{maxu} = 1-\iota_{kj}^{maxl}-\kappa_{kj}^{maxl} (i \in M, j \in N, k \in T)$ by Eq. (1).

Eq. (15) is replaced by:

$$NGR_i = \frac{GR_i - GR_{min}}{GR_{max} - GR_{min}}, i \in M. \tag{24}$$

where $GR_i = D(R_i, R_{max}), GR_{max} = \max_{1\le i\le m}\{GR_i\}$, $GR_{min} = \min_{1\le i\le m}\{GR_i\}$.

Let $\lambda = 0.5$ in Eq. (16), the comprehensive VIKOR index Q_i is calculated by Step 10, which is shown in Table 6.

Table 6 shows that the $Q(A_2^{(1)})-Q(A_4^{(3)}) = 0.5000-0.4637 = 0.0363 < 1/3$, or $Q(A_3^{(1)})-Q(A_4^{(3)}) = 0.5000-0.4637 = 0.0363 < 1/3$, therefore we have that $\{A_1, A_2, A_3, A_4\}$ are tied for the first position in the ranking list. That is, the preference relation of $\{A_1, A_2, A_3, A_4\}$ based on Q_i in Table 4 is supported by the Euclidean distance measure.

5.4 Experiments based on different measures

From Table 4, we can see that $GU(A_4^{(1)}) - GU(A_1^{(2)}) = 0.9977 - 0.9934 = 0.0043$ based on group utility measure GU, where the $GU(A_i^{(h)})$ denotes the alternative A_i is ranked at hth position by GU. It seems a bit reluctant to say that $A_4 \succ A_1$ because the difference 0.0043 is too small. We need a limitation on the difference size. According to the thought of VIKOR method, it should be greater than $1/m$, where m is the number of alternatives. Therefore we have the following criterion.

Criterion 5.2. *Let $Mea(A_{i_h}^{(h)})$ denote that the alternative A_{i_h} is ranked in hth position by measure*

$$Mea \in \{GU, NGU, GR, NGR\}.$$

Then the alternatives $\{A_{i_1}, A_{i_2}, \cdots, A_{i_m}\}$ are ranked with classification similar to the comprehensive VIKOR measure Q in Step 10.

Similar to the ranking based on the measure Q, we explore the ranking with classification based on the measure $GU, NGU, GR,$
NGR in Table 4 according to Criterion 9.

Firstly, we explore the ranking with classification based on the measure GU. From Table 4, we see that $GU(A_4^{(1)}) - GU(A_2^{(4)}) = 0.9977 - 0.9567 = 0.0410 < 1/3$, therefore we know that $\{A_1, A_2, A_3, A_4\}$ is ranked together in the first position according to Criterion 9.

Secondly, we study the ranking with classification based on the measure NGU. From Table 4, we see that $NGU(A_4^{(1)}) - NGU(A_3^{(3)}) = 1 - 0.7285 = 0.2715 < 1/3$, and $NGU(A_4^{(1)}) - NGU(A_2^{(4)}) = 1 - 0.0000 > 1/3$, therefore we can say that $\{A_1, A_2, A_4\} \succ A_3$ based on the measure NGU index.

Table 7: Rankings with classification based on different measures

Measure	A_1	A_2	A_3	A_4	Ranking with classification
GU	✓	✓	✓	✓	$\{A_1, A_2, A_3, A_4\}$
NGU	✓	✓		✓	$\{A_1, A_2, A_4\} \succ A_3$
GR	✓	✓	✓	✓	$\{A_1, A_2, A_3, A_4\}$
NGR		✓			$A_2 \succ \{A_1, A_3, A_4\}$
Q	✓	✓	✓	✓	$\{A_1, A_2, A_3, A_4\}$

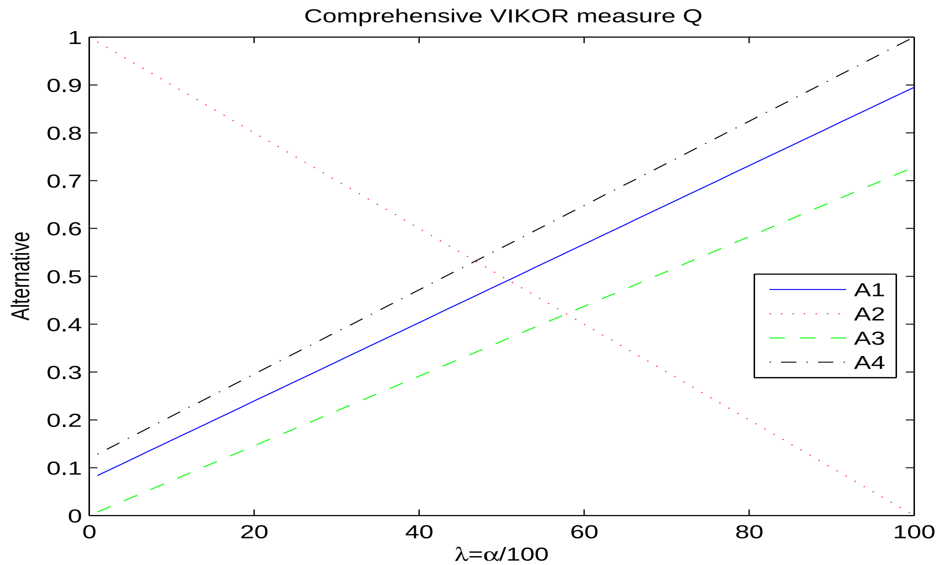

Figure 1: Rankings of four products A_1, A_2, A_3, A_4 based on Q_i in Eq. (17) with dynamic parameter λ.

Next, we study the ranking with classification based on the measure GR. Since the GR is a cost index, we determine the ranking in ascending order. From Table 4, we see that $GR(A_2^{(4)}) - GR(A_3^{(1)}) = 0.9877 - 0.9106 = 0.0771 < 1/3$, therefore we obtain that $\{A_1, A_2, A_3, A_4\}$ tied for the first in the ranking list based on GR index.

Finally, we explore the ranking with classification based on the measure NGR. From Table 4, we see that $NGR(A_2^{(1)}) - NGR(A_4^{(2)}) = 1 - 0.1195 = 0.8805 > 1/3$, and $NGR(A_4^{(2)}) - NGR(A_3^{(4)}) = 0.1195 - 0.0000 = 0.1195 < 1/3$, therefore we have that $A_2 \succ \{A_1, A_3, A_4\}$ based on NGR index.

These rankings with classification based on different five measures, including the ranking based on the comprehensive VIKOR measure Q, are also shown in Table 6.

From above experimental analysis, we can see that different model or experimental condition may lead to different results. What is an ideal result? A natural idea is that the ranking that occurs more frequently should be considered as a higher acceptance result. From Table 6 we can see that the software A_2 is the most preferred product, which appears in all the experiments. Therefore the A_2 should be believable. The following software products are A_1 and A_4. The A_3 is the worst product.

5.5 Dynamic experiments

This section provides two dynamic experiments. The experimental data are based on the illustrative example in Section 5.1.

First, we test the impact of the parameter λ on comprehensive VIKOR measure Q in Eq. (16). Let the compromise coefficient λ of Q_i as a dynamic parameter. In order to show a clear figure, let $\lambda = \alpha/100$, where $\alpha \in [0, 100]$, and let α increases from 0 to 100. The rankings of four software products A_1, A_2, A_3, A_4 based on Q_i are shown in Figure 1.

From Figure 1 we can see that the ranking is $A_2 \succ A_4 \succ A_1 \succ A_3$ when α is about in $[0, 45]$; the ranking is $A_4 \succ A_2 \succ A_1 \succ A_3$ when α is about in $[45, 50]$; the ranking is $A_4 \succ A_1 \succ A_2 \succ A_3$ when α is about in $[50, 58]$; the ranking is $A_4 \succ A_1 \succ A_3 \succ A_2$ when α is about in $[58, 100]$. Therefore we can say that A_4 is the most preferred product based on Q_i in Eq. (16) in most cases. Furthermore, the distinguishes between different alternatives are obvious. The

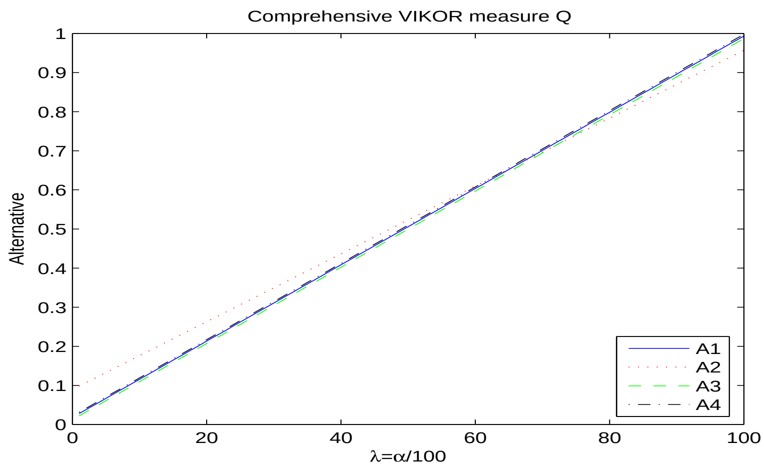

Figure 2: Rankings of four products A_1, A_2, A_3, A_4 based on Q_i in Eq. (25) with dynamic parameter λ.

good distinguishes are credited with the normalization of group utility in Eq. (12) and the normalization of group regret measurements in Eq. (15). To illustrate this, we further test the dynamic rankings of four products without normalization based on GU and GR.

If the comprehensive VIKOR measure Q_i in Eq. (16) is based on the group utility measurements GU_i and group regret measurement GR_i, then Q_i is established by:

$$Q_i = \lambda GU_i + (1-\lambda)(1-GR_i), \lambda \in [0,1], i \in M, \tag{25}$$

where the λ is the same as in Eq. (16), GU_i is shown in Eq. (11), and GR_i is shown in Eq. (14). In addition, it is noted that GR is a cost index. Here, it is transformed to a benefit by $1 - GR$.

Similar to dynamic parameter λ in Figure 1, if let α increases from 0 to 100, then the rankings of four software products A_1, A_2, A_3, A_4 based on Q_i in Eq. (25) are shown in Figure 2.

From Figure 2 we can see that the rankings of A_1, A_3, A_4 are almost indistinguishable. This is just the significance of normalization in Eqs. (12) and (15).

6 Conclusions

This paper has addressed an extended VIKOR-based GDM method under interval-valued intuitionistic fuzzy environment. The main advantages of this paper are listed as follows:

1. A new normalized projection measure has been reported under interval-valued intuitionistic fuzzy environment, which can measure the closeness between evaluation matrix and reference matrix.

2. An extended VIKOR-based GDM method has been shown in an interval-valued intuitionistic fuzzy setting, in which a specific concrete regret matrix is provided. The group regret measurement is based on the new normalized projection measure.

3. The new extended VIKOR-based GDM method is applied to software quality assessment.

Without a doubt, this model has its limitations. First, the decision information is only characterized by interval-valued intuitionistic fuzzy information. This is a limitation. Other information representations, such as intuitionistic fuzzy information, triangular fuzzy number, trapezoidal fuzzy number, are not considered in the model. The future research should continue to working on other information settings. Second, the group satisfaction information in VIKOR-based GDM method is not enough. The future work should show the specific satisfaction information. Third, the proposed method is only applied to software quality assessment. This is a limitation. The future work should continue working on other applications. Fourthly, an extended VIKOR method is used in this evaluation process of software quality, we are also interested in whether there are other effective methods for dealing with software quality. For example, a new procedure, based on a simple algebraic system of equations, called "α-Discounting Method for

Multi-Criteria Decision Making" is reported by Florentin Smarandache [32]. We try to using this method to the future work.

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Data transparency: Authors are sure that all data and materials as well as software application or custom code support their published claims and comply with field standards.

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