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A picture fuzzy distance measure and its application to pattern recognition problems

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Abstract

The picture fuzzy sets are very useful in those uncertain problems which could not be solved by fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets, fermatean fuzzy sets, and q-rung orthopair fuzzy sets. For example, medical diagnosis, personnel selection, human voting, etc. All of these problems require answers of the type no, yes, abstain, and refusal. To compare two picture fuzzy sets, the distance measures play an important role. There are a lot of studies about the distance measures of picture fuzzy sets available in the literature. However, all of these distance measures lead to unreasonable results in most of the problems. So, we in this paper suggest a new distance measure for picture fuzzy sets that is more effective than all of the available distance measures. We also demonstrate its utility in classification and diagnostic problems and contrast its performance with the available ones.

Keywords: Fuzzy set, intuitionistic fuzzy set, picture fuzzy set, distance measure, pattern analysis, medical diagnosis.

1 Introduction

The concept of the fuzzy set (FS) theory was put forward by Zadeh [50] for handling imprecise and vague information. In an FS, each element is assigned a membership value lying between 0 and 1, indicating its degree of belongingness to the set. FSs have been applied in many fields such as pattern recognition, medical diagnosis, clustering, etc. Since in an FS, the non-membership value of an element cannot be chosen independently, so Atanassov [5] introduced the concept of intuitionistic fuzzy sets (IFSs). In an intuitionistic fuzzy set (IFS), each element has a membership value and a non-membership value lying in the interval $[0, 1]$ with their sum less than or equal to one. This restriction on the sum of membership values limits the scope of IFSs and so Yager [48] proposed the concept of Pythagorean fuzzy sets (PFSs). In a PFS, each element has a membership value and a non- membership value lying in the interval $[0, 1]$ with their square sum less than or equal to one. Though PFSs are more robust than FSs and IFSs they cannot handle the situations in which the square sum of membership grades exceeds one. So Yager [49] introduced generalized orthopair fuzzy sets and termed them as q-rung orthopair fuzzy sets (q- ROPFSs). In a q-rung orthopair fuzzy set (q-ROPFS) each element has a membership value and a non-membership value lying in the interval $[0, 1]$ with their qth power sum less than or equal to one. But all of these extensions of FSs does not consider the neutrality degree of an element that plays an important role in various decision-making problems like medical diagnosis, personnel selection, human voting, etc. So, realizing this a new generalization of FSs known as picture fuzzy sets (PIFSs) were suggested by Cuong and Kreinovich [7]. In a picture fuzzy set (PIFS) each element has a membership value, a non-membership value, and a neutrality value lying in the interval $[0, 1]$ with their sum less than or equal to one. Cuong [6] suggested some basic PIFS operations, interval-valued PIFSs (IVPIFSs), and picture fuzzy soft sets (PIFSSs). Two picture fuzzy (PIF) correlation coefficients expressing only the strength of association were suggested by Singh [36]. To determine the nature of the correlation between two PIFSs, Ganie et al. [12] proposed two correlation coefficients with their utility in many decision-making problems. The applicability of PIF cross-entropy in multi-attribute decision-making (MADM) was shown by Wei [44].

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Wang et al. [43] suggested some PIF aggregation functions with their applications. Wei [46] utilized the PIF Hamacher aggregation functions in MADM problems with PIF data. The applicability of some measures of distance based on PIFSs in medical diagnosis was shown by Dutta [9]. Son [40] proposed some generalized versions of distance measures concerning PIFSs with their utility in clustering analysis. The evaluation based on distance from the average solution (EDAS) MADM method in the PIF setting was suggested by Zhang et al. [51]. The applications of a divergence measure based on PIFSs in MADM and pattern analysis were shown by Thao et al. [41]. Jana et al. [16] suggested some PIF Dombi aggregation functions and used them in MADM. The application of a PIF bidirectional projection method in multi-attribute group decision- making (MAGDM) was established by Wei et al. [47]. Based on t-norms, various PIF aggregation functions were given by Ashraf et al. [4]. A generalized measure of distance based on PIFSs was proposed by Son [39]. A novel score function and a knowledge measure for PIFSs were given by Lin et al. [24]. Some measures of comparison based on PIF theory were proposed by Joshi [17, 18]. Some new studies related to PIFSs with their utility are available in the literature [1, 2, 14, 15, 19, 20, 22, 23, 26, 27, 30, 35, 37, 42, 45, 52]. The present study is about a novel measure of distance in PIF theory that overcomes the drawbacks of the existing PIF distance measures.

Distance measures are very crucial in a range of scientific domains such as pattern recognition, decision-making, market prediction, and machine learning. Some measures of distance for PIFSs with their use in predicting flood disaster risk in the south of India were suggested by Singh et al. [38]. Four generalized distance measures based on PIFSs with their use in clustering were given by Son [40]. The applicability of four measures of distance for PIFSs in decision-making was demonstrated by Dinh and Thao [8]. Dutta [9] showed that the PIF measures of distance are very useful in medical diagnosis. A bi-parametric PIF measure of distance and its application in classification and medical diagnostics was suggested by Khan et al. [21]. A novel PIF measure of distance based on direct operations of membership grades was proposed by Ganie and Singh [11]. Some other studies concerning PIF distance measures are given in [3, 13, 25, 29, 33, 34, 52]. However, all of these PIF measures of distance have some drawbacks.

The following are the primary reasons that prompted us to conduct this research:

- Some existing PIF distance metrics don't meet all of the axiomatic criteria.
- Most of the available PIF measures of distance give unreasonable results during the computation of the distance between different PIFS.
- The existing PIF measures of distance fail to recognize an unknown pattern in pattern recognition problems.

So, taking these facts into consideration, we in this paper suggest a novel PIF measure of distance and its applicability in classification problems. The main contributions of this paper are:

- We suggest an innovative PIF measure of distance with its properties.
- Through numerical examples, we establish that the suggested measure overcomes all the drawbacks of the available distance measures.
- We show how the recommended metric can be used in pattern analysis and compare the findings to existing measures.
- We also establish its utility in medical diagnosis.

The rest of the paper is organized as: Section 2 is preliminary. All the existing PIF measures of distance are given in Section 3. A new measure of distance for PIFSs along with its properties is suggested in Section 4. The comparison of the suggested measure with the existing ones through numerical examples is also shown in Section 4. Section 5 is based on the applicability of the suggested measure in classification and diagnostic problems. The conclusion and future study are given in Section 6.

2 Preliminaries

Here we give some basic definitions concerning fuzzy and non-standard fuzzy sets. Let $PIFS(W)$ be the collection of all PIFSs on the universal set $W = \{a_1, a_2, ..., a_z\}$.

Definition 2.1. [50] *A fuzzy set (FS) A_1 in W is given by $A_1 = \{(a_k, \mu_{A_1}(a_k)), a_k \in W\}$ with $\mu_{A_1}(a_k)$ indicating the grade of membership of $a_k \in W$ in A_1 such that $0 \leq \mu_{A_1}(a_k) \leq 1$.*

Definition 2.2. [5] *An intuitionistic fuzzy set (IFS)* A_1 *in* W *is given by* $A_1 = \{(a_k, \mu_{A_1}(a_k), \nu_{A_1}(a_k)), a_k \in W\}$ *with* $\mu_{A_1}(a_k)$ *and* $\nu_{A_1}(a_k)$ *indicating the grade of membership and non-membership of* $a_k \in W$ *in* A_1 *such that* $0 \leq \mu_{A_1}(a_k) + \nu_{A_1}(a_k) \leq 1$. *Also,* $\delta_{A_1}(a_k) = \sqrt{1 - \mu_{A_1}(a_k) - \nu_{A_1}(a_k)}$ *indicates the hesitancy degree of* $a_k \in W$ *in* A_1.

Definition 2.3. [5] *For two IFSs* A_1 *and* A_2 *in* W*, some operations are given below:*

- $A_1 \subseteq A_2$ *if and only if* $\mu_{A_1}(a_k) \leq \mu_{A_2}(a_k)$ *and* $\nu_{A_1}(a_k) \geq \nu_{A_2}(a_k)$.
- $A_1 = A_2$ *if and only if* $A_1 \subseteq A_2$ *and* $A_1 \supseteq A_2$.
- $(A_1)^c = \{(a_k, \nu_{A_1}(a_k), \mu_{A_1}(a_k)), a_k \in W\}$.
- $A_1 \cup A_2 = \{(a_k, max(\mu_{A_1}(a_k), \mu_{A_2}(a_k)), min(\nu_{A_1}(a_k), \nu_{A_2}(a_k))), a_k \in W\}$.
- $A_1 \cap A_2 = \{(a_k, min(\mu_{A_1}(a_k), \mu_{A_2}(a_k)), max(\nu_{A_1}(a_k), \nu_{A_2}(a_k))), a_k \in W\}$.

Definition 2.4. [7] *A picture fuzzy set (PIFS)* $A_1 \in PIFS(W)$ *is given by* $A_1 = \{(a_k, \mu_{A_1}(a_k), \nu_{A_1}(a_k), \gamma_{A_1}(a_k)), a_k \in W\}$ *with* $\mu_{A_1}(a_k), \nu_{A_1}(a_k)$ *and* $\gamma_{A_1}(a_k)$ *indicating the grade of membership, non-membership and neutrality respectively of* $a_k \in W$ *in* A_1 *such that* $0 \leq \mu_{A_1}(a_k) + \nu_{A_1}(a_k) + \gamma_{A_1}(a_k) \leq 1$. *Also,* $\theta_{A_1}(a_k) = \sqrt{1 - \mu_{A_1}(a_k) - \nu_{A_1}(a_k) - \gamma_{A_1}(a_k)}$ *indicates the refusal degree of* $a_k \in W$ *in* A_1.

Definition 2.5. [6] *For* A_1 *and* $A_2 \in PIFS(W)$*, some operations are given below:*

- $A_1 \subseteq A_2$ *if and only if* $\mu_{A_1}(a_k) \leq \mu_{A_2}(a_k)$, $\nu_{A_1}(a_k) \geq \nu_{A_2}(a_k)$, *and* $\gamma_{A_1}(a_k) \leq \gamma_{A_2}(a_k)$
- $A_1 = A_2$ *if and only if* $A_1 \subseteq A_2$ *and* $A_1 \supseteq A_2$.
- $(A_1)^c = \{(a_k, \nu_{A_1}(a_k), \mu_{A_1}(a_k), \gamma_{A_1}(a_k)), a_k \in W\}$.
- $A_1 \cup A_2 = \{(a_k, max(\mu_{A_1}(a_k), \mu_{A_2}(a_k)), min(\nu_{A_1}(a_k), \nu_{A_2}(a_k)), min(\gamma_{A_1}(a_k), \gamma_{A_2}(a_k))), a_k \in W\}$.
- $A_1 \cap A_2 = \{(a_k, min(\mu_{A_1}(a_k), \mu_{A_2}(a_k)), max(\nu_{A_1}(a_k), \nu_{A_2}(a_k)), min(\gamma_{A_1}(a_k), \gamma_{A_2}(a_k))), a_k \in W\}$.

Definition 2.6. [38] *A PIF measure of distance is a function* $DM : PIFS(W) \times PIFS(W) \to [0, 1]$ *such that*

(DM1) $0 \leq DM(A_1, A_2) \leq 1$.

(DM2) $DM(A_1, A_2) = DM(A_2, A_1)$.

(DM3) $DM(A_1, A_2) = 0 \Leftrightarrow A_1 = A_2$.

(DM4) $DM(A_1, A_2) \leq DM(A_1, A_3)$ *and* $DM(A_2, A_3) \leq DM(A_1, A_3)$, *where* $A_1 \subseteq A_2 \subseteq A_3$.

3 The existing PIF measures of distance

Here, we list the existing PIF measures of distance available in the literature. Their limitations are discussed in Section 4.2.

Singh et al. [38]

$$DM_{SMKSS1}(A_1, A_2) = \frac{1}{4z} \sum_{k=1}^{z} \left(|\mu_{A_1}(a_k) - \mu_{A_2}(a_k)| + |\nu_{A_1}(a_k) - \nu_{A_2}(a_k)| + |\gamma_{A_1}(a_k) - \gamma_{A_2}(a_k)| + |\theta_{A_1}(a_k) - \theta_{A_2}(a_k)| \right).$$

$$DM_{SMKSS2}(A_1, A_2) = \left(\frac{1}{4z} \sum_{k=1}^{z} \left(\begin{array}{r} |\mu_{A_1}(a_k) - \mu_{A_2}(a_k)|^2 + |\nu_{A_1}(a_k) - \nu_{A_2}(a_k)|^2 + |\gamma_{A_1}(a_k) - \gamma_{A_2}(a_k)|^2 \\ + |\theta_{A_1}(a_k) - \theta_{A_2}(a_k)|^2 \end{array} \right) \right)^{\frac{1}{2}}.$$

$$DM_{SMKSS3}(A_1, A_2) = \frac{1}{4z} \sum_{k=1}^{z} max \left(|\mu_{A_1}(a_k) - \mu_{A_2}(a_k)|, |\nu_{A_1}(a_k) - \nu_{A_2}(a_k)|, |\gamma_{A_1}(a_k) - \gamma_{A_2}(a_k)|, |\theta_{A_1}(a_k) - \theta_{A_2}(a_k)| \right).$$

$$DM_{SMKSS4}(A_1, A_2) = \left(\frac{1}{4z} \sum_{k=1}^{z} max \left(\begin{array}{r} |\mu_{A_1}(a_k) - \mu_{A_2}(a_k)|^2, |\nu_{A_1}(a_k) - \nu_{A_2}(a_k)|^2, |\gamma_{A_1}(a_k) - \gamma_{A_2}(a_k)|^2, \\ |\theta_{A_1}(a_k) - \theta_{A_2}(a_k)|^2 \end{array} \right) \right)^{\frac{1}{2}}.$$

Singh et al. [38] applied these measures of distance for determining the risk of flood disaster in South India.

Son [40]

$$DM_{S1}(A_1,A_2)=\frac{\sum_{k=1}^{z}\left(\frac{\triangle\mu_k+\triangle\nu_k+\triangle\gamma_k}{3}+max(\triangle\mu_k,\triangle\nu_k,\triangle\gamma_k)\right)}{\sum_{k=1}^{z}\left(\frac{\triangle\mu_k+\triangle\nu_k+\triangle\gamma_k}{3}+max(\triangle\mu_k,\triangle\nu_k,\triangle\gamma_k)\right)+max_k(\Phi_k^{A_1},\Phi_k^{A_2})+\sum_{k=1}^{z}|\Phi_k^{A_1}-\Phi_k^{A_2}|+1}$$

$$DM_{S2}(A_1,A_2)=\frac{\left(\sum_{k=1}^{z}\left(\frac{\triangle\mu_k^2+\triangle\nu_k^2+\triangle\gamma_k^2}{3}+max(\triangle\mu_k^2,\triangle\nu_k^2,\triangle\gamma_k^2)\right)\right)^{\frac{1}{2}}}{\left(\sum_{k=1}^{z}\left(\frac{\triangle\mu_k^2+\triangle\nu_k^2+\triangle\gamma_k^2}{3}+max(\triangle\mu_k^2,\triangle\nu_k^2,\triangle\gamma_k^2)\right)\right)^{\frac{1}{2}}+\left(max_k(\Phi_k^{A_1},\Phi_k^{A_2})+\sum_{k=1}^{z}|\Phi_k^{A_1}-\Phi_k^{A_2}|^2\right)^{\frac{1}{2}}+1}$$

$$DM_{S3}(A_1,A_2)=\frac{\frac{1}{z}\sum_{k=1}^{z}\left(\frac{\triangle\mu_k+\triangle\nu_k+\triangle\gamma_k}{3}+max(\triangle\mu_k,\triangle\nu_k,\triangle\gamma_k)\right)}{\frac{1}{z}\sum_{k=1}^{z}\left(\frac{\triangle\mu_k+\triangle\nu_k+\triangle\gamma_k}{3}+max(\triangle\mu_k,\triangle\nu_k,\triangle\gamma_k)\right)+max_k(\Phi_k^{A_1},\Phi_k^{A_2})+\frac{1}{z}\sum_{k=1}^{z}|\Phi_k^{A_1}-\Phi_k^{A_2}|+1}$$

$$DM_{S4}(A_1,A_2)=\frac{\left(\frac{1}{z}\sum_{k=1}^{z}\left(\frac{\triangle\mu_k^2+\triangle\nu_k^2+\triangle\gamma_k^2}{3}+max(\triangle\mu_k^2,\triangle\nu_k^2,\triangle\gamma_k^2)\right)\right)^{\frac{1}{2}}}{\left(\frac{1}{z}\sum_{k=1}^{z}\left(\frac{\triangle\mu_k^2+\triangle\nu_k^2+\triangle\gamma_k^2}{3}+max(\triangle\mu_k^2,\triangle\nu_k^2,\triangle\gamma_k^2)\right)\right)^{\frac{1}{2}}+\left(max_k(\Phi_k^{A_1},\Phi_k^{A_2})+\frac{1}{z}\sum_{k=1}^{z}|\Phi_k^{A_1}-\Phi_k^{A_2}|^2\right)^{\frac{1}{2}}+1}$$

where, $\triangle\mu_k=|\mu_{A_1}(a_k)-\mu_{A_2}(a_k)|$, $\triangle\nu_k=|\nu_{A_1}(a_k)-\nu_{A_2}(a_k)|$, $\triangle\gamma_k=|\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k)|$, $\Phi_k^{A_1}=|\mu_{A_1}(a_k)+\nu_{A_1}(a_k)+\gamma_{A_1}(a_k)|$, $\Phi_k^{A_2}=|\mu_{A_2}(a_k)+\nu_{A_2}(a_k)+\gamma_{A_2}(a_k)|$, $k=1,2,...,z$.

Son [40] established the applicability of these measures of distance in clustering analysis.

Dinh and Thao [8]

$$DM_{DT1}(A_1,A_2)=\frac{1}{3z}\sum_{k=1}^{z}\left(|\mu_{A_1}(a_k)-\mu_{A_2}(a_k)|+|\nu_{A_1}(a_k)-\nu_{A_2}(a_k)|+|\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k)|\right).$$

$$DM_{DT2}(A_1,A_2)=\frac{1}{z}\left(\sum_{k=1}^{z}\left((\mu_{A_1}(a_k)-\mu_{A_2}(a_k))^2+(\nu_{A_1}(a_k)-\nu_{A_2}(a_k))^2+(\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k))^2\right)\right)^{\frac{1}{2}}.$$

$$DM_{DT3}(A_1,A_2)=\frac{1}{z}\sum_{k=1}^{z}max\left(|\mu_{A_1}(a_k)-\mu_{A_2}(a_k)|,|\nu_{A_1}(a_k)-\nu_{A_2}(a_k)|,|\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k)|\right).$$

$$DM_{DT4}(A_1,A_2)=\frac{1}{z}\left(\sum_{k=1}^{z}max\left(|\mu_{A_1}(a_k)-\mu_{A_2}(a_k)|^2,|\nu_{A_1}(a_k)-\nu_{A_2}(a_k)|^2,|\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k)|^2\right)\right)^{\frac{1}{2}}.$$

Dinh and Thao [8] demonstrated the use of these distance measures in pattern recognition problems.

Dutta [9]

$$DM_{D1}(A_1,A_2)=\frac{1}{2}\sum_{k=1}^{z}\left(|\mu_{A_1}(a_k)-\mu_{A_2}(a_k)|+|\nu_{A_1}(a_k)-\nu_{A_2}(a_k)|+|\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k)|+|\theta_{A_1}(a_k)-\theta_{A_2}(a_k)|\right).$$

$$DM_{D2}(A_1,A_2)=\frac{1}{2z}\sum_{k=1}^{z}\left(|\mu_{A_1}(a_k)-\mu_{A_2}(a_k)|+|\nu_{A_1}(a_k)-\nu_{A_2}(a_k)|+|\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k)|+|\theta_{A_1}(a_k)-\theta_{A_2}(a_k)|\right).$$

$$DM_{D3}(A_1,A_2)=\frac{1}{2}\sum_{k=1}^{z}\left((\mu_{A_1}(a_k)-\mu_{A_2}(a_k))^2+(\nu_{A_1}(a_k)-\nu_{A_2}(a_k))^2+(\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k))^2+(\theta_{A_1}(a_k)-\theta_{A_2}(a_k))^2\right).$$

$$DM_{D4}(A_1,A_2)=\frac{1}{2z}\sum_{k=1}^{z}\left((\mu_{A_1}(a_k)-\mu_{A_2}(a_k))^2+(\nu_{A_1}(a_k)-\nu_{A_2}(a_k))^2+(\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k))^2+(\theta_{A_1}(a_k)-\theta_{A_2}(a_k))^2\right).$$

$$DM_{D5}(A_1,A_2)=\frac{1}{2}\sum_{k=1}^{z}\left(\begin{array}{c}\left(\frac{|\mu_{A_1}(a_k)-\mu_{A_2}(a_k)|+|\nu_{A_1}(a_k)-\nu_{A_2}(a_k)|+|\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k)|+|\theta_{A_1}(a_k)-\theta_{A_2}(a_k)|}{4}\right)\\ +\left(\frac{max\left(|\mu_{A_1}(a_k)-\mu_{A_2}(a_k)|,|\nu_{A_1}(a_k)-\nu_{A_2}(a_k)|,|\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k)|,|\theta_{A_1}(a_k)-\theta_{A_2}(a_k)|\right)}{2}\right)\end{array}\right).$$

Dutta [9] applied these distance measures in medical diagnostic problems.

Khan et al. [21]

$$DM_{KKDK}(A_1,A_2)=\left[\frac{1}{3z(t+1)^p}\sum_{k=1}^{z}\left(\begin{array}{c}|t(\mu_{A_1}(a_k)-\mu_{A_2}(a_k))-(\nu_{A_1}(a_k)-\nu_{A_2}(a_k))-(\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k))|^p\\ +|t(\nu_{A_1}(a_k)-\nu_{A_2}(a_k))-(\mu_{A_1}(a_k)-\mu_{A_2}(a_k))+(\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k))|^p\\ +|t(\gamma_{A_1}(a_k)-\gamma_{A_2}(a_k))-(\mu_{A_1}(a_k)-\mu_{A_2}(a_k))+(\nu_{A_1}(a_k)-\nu_{A_2}(a_k))|^p\end{array}\right)\right]^{\frac{1}{p}},$$

with $p=1,2,3,...$ as l_p norm and $t=3,4,...$ as uncertainty level.

Khan et al. [21] suggested its use in classification problems.

4 New distance measure for picture fuzzy sets

Here we suggest a new PIF measure of distance that overcomes the shortcomings of the PIF measures of distance given in Section 3.

4.1 A novel PIF measure of distance

Let $A_1,A_2\in PIFS(W)$, then a PIF measure of distance is given by

$$DM_G(A_1,A_2)=\frac{1}{4z}\sum_{k=1}^{z}\left(\begin{array}{c}|tan^{-1}\mu_{A_1}(a_k)-tan^{-1}\mu_{A_2}(a_k)|+|tan^{-1}\nu_{A_1}(a_k)-tan^{-1}\nu_{A_2}(a_k)|\\ +|tan^{-1}\gamma_{A_1}(a_k)-tan^{-1}\gamma_{A_2}(a_k)|+|tan^{-1}\theta_{A_1}(a_k)-tan^{-1}\theta_{A_2}(a_k)|\end{array}\right). \quad (1)$$

Theorem 4.1. *The PIF measure of distance DM_G given in Eq. (1) is a valid measure of distance for PIFSs.*

Proof. To prove this result, we show that the suggested measure DM_G satisfies (DM1)-(DM4) of Definition 2.6.

(DM1) Since $0\leq\mu_{A_1}(a_k),\mu_{A_2}(a_k)\leq 1,\forall k=1,2,...,z.$, so we have $0\leq tan^{-1}\mu_{A_1}(a_k),tan^{-1}\mu_{A_2}(a_k)\leq 1$, and thus $0\leq|tan^{-1}\mu_{A_1}(a_k)-tan^{-1}\mu_{A_2}(a_k)|\leq 1$. Similarly, we have $0\leq|tan^{-1}\nu_{A_1}(a_k)-tan^{-1}\nu_{A_2}(a_k)|\leq 1$, $0\leq|tan^{-1}\gamma_{A_1}(a_k)-tan^{-1}\gamma_{A_2}(a_k)|\leq 1$, and $0\leq|tan^{-1}\theta_{A_1}(a_k)-tan^{-1}\theta_{A_2}(a_k)|\leq 1$. Therefore, we get $0\leq DM_G(A_1,A_2)\leq 1$.

(DM2)

$$\begin{aligned}DM_G(A_1,A_2)&=\frac{1}{4z}\sum_{k=1}^{z}\left(\begin{array}{c}|tan^{-1}\mu_{A_1}(a_k)-tan^{-1}\mu_{A_2}(a_k)|+|tan^{-1}\nu_{A_1}(a_k)-tan^{-1}\nu_{A_2}(a_k)|\\ +|tan^{-1}\gamma_{A_1}(a_k)-tan^{-1}\gamma_{A_2}(a_k)|+|tan^{-1}\theta_{A_1}(a_k)-tan^{-1}\theta_{A_2}(a_k)|\end{array}\right)\\ &=\frac{1}{4z}\sum_{k=1}^{z}\left(\begin{array}{c}|tan^{-1}\mu_{A_2}(a_k)-tan^{-1}\mu_{A_1}(a_k)|+|tan^{-1}\nu_{A_2}(a_k)-tan^{-1}\nu_{A_1}(a_k)|\\ +|tan^{-1}\gamma_{A_2}(a_k)-tan^{-1}\gamma_{A_1}(a_k)|+|tan^{-1}\theta_{A_2}(a_k)-tan^{-1}\theta_{A_1}(a_k)|\end{array}\right)\\ &=DM_G(A_2,A_1).\end{aligned}$$

(DM3) Let $A_1=A_2$, then $\mu_{A_1}(a_k)=\mu_{A_2}(a_k),\nu_{A_1}(a_k)=\nu_{A_2}(a_k),\gamma_{A_1}(a_k)=\gamma_{A_2}(a_k)$ and $\theta_{A_1}(a_k)=\theta_{A_1}(a_k)\ \forall\ k=1,2,...,z$. So, we have

$$\begin{aligned}DM_G(A_1,A_2)&=\frac{1}{4z}\sum_{k=1}^{z}\left(\begin{array}{c}|tan^{-1}\mu_{A_2}(a_k)-tan^{-1}\mu_{A_2}(a_k)|+|tan^{-1}\nu_{A_2}(a_k)-tan^{-1}\nu_{A_2}(a_k)|\\ +|tan^{-1}\gamma_{A_2}(a_k)-tan^{-1}\gamma_{A_2}(a_k)|+|tan^{-1}\theta_{A_2}(a_k)-tan^{-1}\theta_{A_2}(a_k)|\end{array}\right)\\ &=0.\end{aligned}$$

(DM4) Let $A_1\subseteq A_2\subseteq A_3$, then $\mu_{A_1}(a_k)\leq\mu_{A_2}(a_k)\leq\mu_{A_3}(a_k),\nu_{A_1}(a_k)\geq\nu_{A_2}(a_k)\geq\nu_{A_3}(a_k)$, and $\gamma_{A_1}(a_k)\leq\gamma_{A_2}(a_k)\leq\gamma_{A_3}(a_k)$. So, we have $|tan^{-1}\mu_{A_1}(a_k)-tan^{-1}\mu_{A_2}(a_k)|\leq|tan^{-1}\mu_{A_1}(a_k)-tan^{-1}\mu_{A_3}(a_k)|$, $|tan^{-1}\nu_{A_1}(a_k)-tan^{-1}\nu_{A_2}(a_k)|\leq|tan^{-1}\nu_{A_1}(a_k)-tan^{-1}\nu_{A_3}(a_k)|$, $|tan^{-1}\gamma_{A_1}(a_k)-tan^{-1}\gamma_{A_2}(a_k)|\leq|tan^{-1}\gamma_{A_1}(a_k)-tan^{-1}\gamma_{A_3}(a_k)|$, and

$|tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_2}(a_k)| \leq |tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_3}(a_k)|.$

So,

$$\begin{aligned}&\frac{1}{4z}\sum_{k=1}^{z}\begin{pmatrix}|tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\mu_{A_2}(a_k)| + |tan^{-1}\nu_{A_1}(a_k) - tan^{-1}\nu_{A_2}(a_k)| \\ + |tan^{-1}\gamma_{A_1}(a_k) - tan^{-1}\gamma_{A_2}(a_k)| + |tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_2}(a_k)|\end{pmatrix} \\ \leq &\frac{1}{4z}\sum_{k=1}^{z}\begin{pmatrix}|tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\mu_{A_3}(a_k)| + |tan^{-1}\nu_{A_1}(a_k) - tan^{-1}\nu_{A_3}(a_k)| \\ + |tan^{-1}\gamma_{A_1}(a_k) - tan^{-1}\gamma_{A_3}(a_k)| + |tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_3}(a_k)|\end{pmatrix}.\end{aligned}$$

Thus, $DM_G(A_1, A_2) \leq DM_G(A_1, A_3)$. Similarly, we have $DM_G(A_2, A_3) \leq DM_G(A_1, A_3)$. Hence, DM_G given in Eq. (1) is a valid PIF measure of distance. □

Theorem 4.2. *The PIF measure of distance DM_G given in Eq. (1) has the following properties.*

1. $DM_G(A_1^c, A_2^c) = DM_G(A_1, A_2)\ \forall\ A_1, A_2 \in PIFS(W)$.
2. $DM_G(A_1, A_2^c) = DM_G(A_1^c, A_2)\ \forall\ A_1, A_2 \in PIFS(W)$.
3. $DM_G(A_1, A_1^c) = 0$ *if and only if* $\mu_{A_1}(a_k) = \nu_{A_1}(a_k)\ \forall\ 1 \leq k \leq z$.

Proof. 1. We have

$$\begin{aligned}DM_G(A_1^c, A_2^c) &= \frac{1}{4z}\sum_{k=1}^{z}\begin{pmatrix}|tan^{-1}\nu_{A_1}(a_k) - tan^{-1}\nu_{A_2}(a_k)| + |tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\mu_{A_2}(a_k)| \\ + |tan^{-1}\gamma_{A_1}(a_k) - tan^{-1}\gamma_{A_2}(a_k)| + |tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_2}(a_k)|\end{pmatrix} \\ &= \frac{1}{4z}\sum_{k=1}^{z}\begin{pmatrix}|tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\mu_{A_2}(a_k)| + |tan^{-1}\nu_{A_1}(a_k) - tan^{-1}\nu_{A_2}(a_k)| \\ + |tan^{-1}\gamma_{A_1}(a_k) - tan^{-1}\gamma_{A_2}(a_k)| + |tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_2}(a_k)|\end{pmatrix} \\ &= DM_G(A_1, A_2).\end{aligned}$$

2.

$$\begin{aligned}DM_G(A_1, A_2^c) &= \frac{1}{4z}\sum_{k=1}^{z}\begin{pmatrix}|tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\nu_{A_2}(a_k)| + |tan^{-1}\nu_{A_1}(a_k) - tan^{-1}\mu_{A_2}(a_k)| \\ + |tan^{-1}\gamma_{A_1}(a_k) - tan^{-1}\gamma_{A_2}(a_k)| + |tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_2}(a_k)|\end{pmatrix} \\ &= \frac{1}{4z}\sum_{k=1}^{z}\begin{pmatrix}|tan^{-1}\nu_{A_1}(a_k) - tan^{-1}\mu_{A_2}(a_k)| + |tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\nu_{A_2}(a_k)| \\ + |tan^{-1}\gamma_{A_1}(a_k) - tan^{-1}\gamma_{A_2}(a_k)| + |tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_2}(a_k)|\end{pmatrix} \\ &= DM_G(A_1^c, A_2).\end{aligned}$$

3.

$$\begin{aligned}DM_G(A_1, A_1^c) = 0 &\Leftrightarrow \frac{1}{4z}\sum_{k=1}^{z}\begin{pmatrix}|tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\nu_{A_1}(a_k)| + |tan^{-1}\nu_{A_1}(a_k) - tan^{-1}\mu_{A_1}(a_k)| \\ + |tan^{-1}\gamma_{A_1}(a_k) - tan^{-1}\gamma_{A_1}(a_k)| + |tan^{-1}\theta_{A_1}(a_k) - tan^{-1}\theta_{A_1}(a_k)|\end{pmatrix} = 0 \\ &\Leftrightarrow |tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\nu_{A_1}(a_k)| = 0\ \forall\ k \\ &\Leftrightarrow tan^{-1}\mu_{A_1}(a_k) - tan^{-1}\nu_{A_1}(a_k) = 0\ \forall\ k \\ &\Leftrightarrow \mu_{A_1}(a_k) = \nu_{A_1}(a_k)\ \forall\ k.\end{aligned}$$

□

4.2 Comparative study

Here, we compare the suggested measure with the existing ones through some numerical examples related to the computation of the distance between different PIFSs.

Example 4.3. *Consider six different cases of PIFSs with each case consisting of two different PIFSs as shown below.*
Case I: $\{A_1 = \{(a_1, 0.4, 0.4, 0.2)\}, A_2 = \{(a_1, 0.2, 0.5, 0.3)\}\}$, *Case II:* $\{A_1 = \{(a_1, 0.1, 0.4, 0.5)\}, A_2 = \{(a_1, 0.2, 0.5, 0.3)\}\}$,
Case III: $\{A_1 = \{(a_1, 0.4, 0, 0.6)\}, A_2 = \{(a_1, 0, 0, 0)\}\}$, *Case IV:* $\{A_1 = \{(a_1, 0.5, 0, 0.5)\}, A_2 = \{(a_1, 0, 0, 0)\}\}$,
Case V: $\{A_1 = \{(a_1, 0.3, 0, 0.7)\}, A_2 = \{(a_1, 0.2, 0, 0.8)\}\}$, *Case VI:* $\{A_1 = \{(a_1, 0.3, 0, 0.7)\}, A_2 = \{(a_1, 0.4, 0, 0.6)\}\}$.
The computed distance values for these six cases using the available measures of distance along with the suggested one are shown in Table 1.

From Table 1, we have:

Table 1: Computed values of distance regarding Example 4.3

	Case I	Case II	Case III	Case IV	Case V	Case VI
DM_{S1}	**0.1429**	**0.1429**	0.2373	0.2174	**0.0769**	**0.0769**
DM_{S2}	**0.1091**	**0.1091**	0.2322	0.2110	**0.0606**	**0.0606**
DM_{S3}	**0.1429**	**0.1429**	0.2373	0.2174	**0.0769**	**0.0769**
DM_{S4}	**0.1091**	**0.1091**	0.1958	0.1771	**0.0606**	**0.0606**
DM_{D1}	**0.2000**	**0.2000**	**1.0000**	**1.0000**	**0.1000**	**0.1000**
DM_{D2}	**0.2000**	**0.2000**	**1.0000**	**1.0000**	**0.1000**	**0.1000**
DM_{D3}	**0.1732**	**0.1732**	0.8718	0.8660	**0.1000**	**0.1000**
DM_{D4}	**0.1732**	**0.1732**	0.8718	0.8660	**0.1000**	**0.1000**
DM_{D5}	**0.2000**	**0.2000**	**1.0000**	**1.0000**	**0.1000**	**0.1000**
DM_{DT1}	**0.1333**	**0.1333**	**0.3333**	**0.3333**	**0.0667**	**0.0667**
DM_{DT2}	**0.2449**	**0.2449**	0.7211	0.7071	**0.1414**	**0.1414**
DM_{DT3}	**0.2000**	**0.2000**	0.6000	0.5000	**0.1000**	**0.1000**
DM_{DT4}	**0.2000**	**0.2000**	0.6000	0.5000	**0.1000**	**0.1000**
DM_{SMKSS1}	**0.1000**	**0.1000**	**0.5000**	**0.5000**	**0.0500**	**0.0500**
DM_{SMKSS2}	**0.1225**	**0.1225**	**0.6164**	**0.6164**	**0.0707**	**0.0707**
DM_{SMKSS3}	**0.0500**	**0.0500**	**0.2500**	**0.2500**	**0.0250**	**0.0250**
DM_{SMKSS4}	**0.1000**	**0.1000**	**0.5000**	**0.5000**	**0.0500**	**0.0500**
DM_{KKDK}	**0.1667**	**0.0833**	0.1833	0.1667	**0.0833**	**0.0833**
DM_G	0.0901	0.0883	0.4266	0.4282	0.0395	0.0398

Bold values denote unreasonable results. $p = 1$ and $t = 3$ in D_{KKDK}.

1. *The PIF measures of distance* $DM_{S1}, DM_{S2}, DM_{S3}, DM_{S4}, DM_{D1}, DM_{D2}, DM_{D3}, DM_{D4}, DM_{D5}, DM_{DT1}, DM_{DT2}, DM_{DT3}, DM_{DT4}, DM_{SMKSS1}, DM_{SMKSS2}, DM_{SMKSS3}, DM_{SMKSS4}$*, and* DM_{KKDK} *gives the same distance for two different cases (Case I and Case II).*

2. *The PIF measures of distance* $DM_{D1}, DM_{D2}, DM_{D5}, DM_{DT1}, DM_{SMKSS1}, DM_{SMKSS2}, DM_{SMKSS3}$*, and* DM_{SMKSS4} *give the same distance for two distinct cases (Case III and Case IV).*

3. *The PIF measures of distance* $DM_{S1}, DM_{S2}, DM_{S3}, DM_{S4}, DM_{D1}, DM_{D2}, DM_{D3}, DM_{D4}, DM_{D5}, DM_{DT1}, DM_{DT2}, DM_{DT3}, DM_{DT4}, DM_{DT4}, DM_{SMKSS1}, DM_{SMKSS2}, DM_{SMKSS3}, DM_{SMKSS}$*, and* DM_{KKDK} *gives the same distance for two different cases (Case V and Case VI).*

4. *The PIF measures of distance* DM_{D1}, DM_{D2}*, and* DM_{D5} *give "1" as the distance between those PIFSs which are not complement of each other (Case III and Case IV).*

5. *The suggested PIF measure of distance* DM_G *computes the distance of all six different cases without any unreasonable results.*

Example 4.4. *Consider four different cases of PIFSs with each case consisting of two different PIFSs as shown below. Case I:* $\{A_1 = \{(a_1, 0.6, 0.27, 0.1)\}, A_2 = \{(a_1, 0.28, 0.55, 0.1)\}\}$*, Case III:* $\{A_1 = \{(a_1, 0.6, 0.4, 0)\}, A_2 = \{(a_1, 0, 0, 0)\}\}$*, Case II:* $\{A_1 = \{(a_1, 0, 0.87, 0.1)\}, A_2 = \{(a_1, 0.28, 0.55, 0.1)\}\}$*, Case IV:* $\{A_1 = \{(a_1, 0.5, 0.5, 0)\}, A_2 = \{(a_1, 0, 0, 0)\}\}$*. The computed values of distance for these four cases using the available measures of distance along with the suggested one are shown in Table 2. From Table 2, we observe the following:*

1. *The distance values for the two distinct cases (Case I and Case II) comes out to be equal by the PIF measures of distance* $DM_{S1}, DM_{S2}, DM_{S3}, DM_{S4}, DM_{D1}, DM_{D2}, DM_{D3}, DM_{D4}, DM_{D5}, DM_{DT1}, DM_{DT2}, DM_{DT3}, DM_{DT4}, DM_{SMKSS1}, DM_{SMKSS2}, DM_{SMKSS3}, DM_{SMKSS4}$*, and* DM_{KKDK}*.*

2. *The distance values for the two distinct cases (Case III and Case IV) come out to be equal by the PIF measures of distance* $DM_{D1}, DM_{D2}, DM_{D5}, DM_{DT1}, DM_{SMKSS1}, DM_{SMKSS2}, DM_{SMKSS3}$*, and* DM_{SMKSS4}*.*

3. *The PIF measures of distance* DM_{D1}, DM_{D2}*, and* DM_{D5} *give "1" as the distance between those PIFSs which are not complement of each other (Case III and Case IV).*

4. *The distance values due to the suggested measure* DM_G *for all the four cases are different.*

Table 2: Computed values of distance regarding Example 4.4

	Case I	Case II	Case III	Case IV
DM_{S1}	**0.2055**	**0.2055**	0.2373	0.2174
DM_{S2}	**0.1662**	**0.1662**	0.2322	0.2110
DM_{S3}	**0.2055**	**0.2055**	0.2373	0.2174
DM_{S4}	**0.1523**	**0.1523**	0.1958	0.1771
DM_{D1}	**0.3200**	**0.3200**	**1.0000**	**1.0000**
DM_{D2}	**0.3200**	**0.3200**	**1.0000**	**1.0000**
DM_{D3}	**0.3020**	**0.3020**	0.8718	0.8660
DM_{D4}	**0.3020**	**0.3020**	0.8718	0.8660
DM_{D5}	**0.3200**	**0.3200**	**1.0000**	**1.0000**
DM_{DT1}	**0.2000**	**0.2000**	**0.3333**	**0.3333**
DM_{DT2}	**0.4252**	**0.4252**	0.7211	0.7071
DM_{DT3}	**0.3200**	**0.3200**	0.6000	0.5000
DM_{DT4}	**0.3200**	**0.3200**	0.6000	0.5000
DM_{SMKSS1}	**0.1600**	**0.1600**	**0.5000**	**0.5000**
DM_{SMKSS2}	**0.2135**	**0.2135**	**0.6164**	**0.6164**
DM_{SMKSS3}	**0.0800**	**0.0800**	**0.2500**	**0.2500**
DM_{SMKSS4}	**0.1600**	**0.1600**	**0.5000**	**0.5000**
DM_{KKDK}	**0.2500**	**0.2500**	0.1833	0.1667
DM_G	0.1366	0.1315	0.4266	0.4282

Bold values denote unreasonable results. $p = 1$ and $t = 3$ in D_{KKDK}.

Example 4.5. *Consider five different cases of PIFSs with each case consisting of two different PIFSs as shown below:*
Case I: $\{A_1 = \{(a_1, 0.4, 0.2, 0.1)\}, A_2 = \{(a_1, 0.5, 0.2, 0.1)\}\}$, *Case II:* $\{A_1 = \{(a_1, 0.4, 0.2, 0.1)\}, A_2 = \{(a_1, 0.5, 0.1, 0.1)\}\}$,
Case III: $\{A_1 = \{(a_1, 0.5, 0.5, 0)\}, A_2 = \{(a_1, 0, 0, 0)\}\}$, *Case IV:* $\{A_1 = \{(a_1, 1, 0, 0)\}, A_2 = \{(a_1, 0, 0, 0)\}\}$,
Case V: $\{A_1 = \{(a_1, 0.3, 0.4, 0.1)\}, A_2 = \{(a_1, 0.4, 0.3, 0.1)\}\}$.
The computed values of distance of these five cases using the available measures of distance along with the suggested one are shown in Table 3.
From Table 3, we have the following:

1. *The distance between the different PIFSs (Case I and Case II) comes out to be the same by the PIF measures of distance* $DM_{D1}, DM_{D2}, DM_{D5}, DM_{DT3}, DM_{DT4}, DM_{SMKSS1}, DM_{SMKSS3}$*, and* DM_{SMKSS4}.

2. *The distance between the different PIFSs (Case III and Case IV) due to the PIF measures of distance* DM_{D1}, DM_{D2}, $DM_{D5}, DM_{DT1}, DM_{SMKSS1}, DM_{SMKSS3}$*, and* DM_{SMKSS4} *come out to be the same.*

3. *The PIF measure of distance* DM_{DT1} *gives "0.0667" as the distance between the PIFSs (Case I and Case V) and "0.3333" as the distance between PIFSs (Case III and Case IV).*

4. *We have* A_1 *(Case I)* $\subseteq A_2$ *(Case I)* $\subseteq A_2$ *(Case II). So, by the axiom (DM4) of Definition 2.6, the distance between the PIFSs in Case I must be less than the distance between the PIFSs in Case II. But we see here that this is not true for the distance measures* DM_{D3}, DM_{D4}*, and* DM_{SMKSS2}*. So, these measures do not satisfy the property (DM4) of a PIF measure of distance.*

5. *The suggested measure* DM_G *gives satisfactory results in all five cases without any counterintuitive situation.*

Thus, from Examples 4.3-4.5, we conclude that the suggested distance measure is more robust and effective than all of the available distance measures in PIF theory.

5 Pattern analysis

Pattern recognition refers to the classification of an unrecognized pattern into some familiar patterns. In the fuzzy environment, the compatibility measures such as correlation measure, similarity measure, accuracy measure, etc., are employed for performing pattern recognition. We here use the suggested measure and contrast the performance with the available measures in the following examples.

Table 3: Computed values of distance regarding Example 4.5

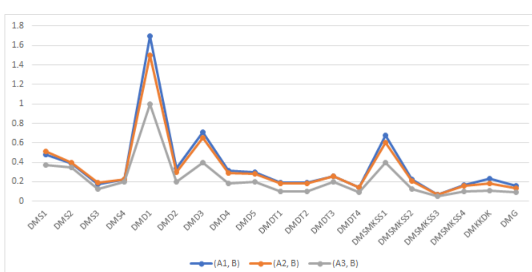
	Case I	Case II	Case III	Case IV	Case V
DM_{S1}	0.1176	0.1429	0.2174	0.3077	0.0847
DM_{S2}	0.1017	0.1108	0.2110	0.3235	0.0638
DM_{S3}	0.1176	0.1429	0.2174	0.3077	0.0847
DM_{S4}	0.0902	0.1036	0.1771	0.2779	0.0669
DM_{D1}	**0.2000**	**0.2000**	**1.0000**	**1.0000**	0.1000
DM_{D2}	**0.2000**	**0.2000**	**1.0000**	**1.0000**	0.1000
DM_{D3}	0.2000	0.1732	0.8660	1.0000	0.1000
DM_{D4}	0.2000	0.1732	0.8660	1.0000	0.1000
DM_{D5}	**0.2000**	**0.2000**	**1.0000**	**1.0000**	0.1000
DM_{DT1}	**0.0667**	0.1000	**0.3333**	**0.3333**	**0.0667**
DM_{DT2}	0.2000	0.2236	0.7071	1.0000	0.1414
DM_{DT3}	**0.2000**	**0.2000**	0.5000	1.0000	0.1000
DM_{DT4}	**0.2000**	**0.2000**	0.5000	1.0000	0.1000
DM_{SMKSS1}	**0.1000**	**0.1000**	**0.5000**	**0.5000**	0.0500
DM_{SMKSS2}	0.1414	0.1225	0.6124	0.7071	0.0707
DM_{SMKSS3}	**0.0500**	**0.0500**	**0.2500**	**0.2500**	0.0250
DM_{SMKSS4}	**0.1000**	**0.1000**	**0.5000**	**0.5000**	0.0500
DM_{KKDK}	0.0833	0.1250	0.6464	0.7500	0.0065
DM_G	0.0443	0.0452	0.4282	0.3927	0.0445

Bold values denote unreasonable results. $p = 1$ and $t = 3$ in D_{KKDK}.

Example 5.1. [12] *Let A_1, A_2, A_3, and B be four patterns given in the form of PIFSs as:*

$$A_1 = \{(a_1, 0.1, 0.3, 0.4), (a_2, 0.4, 0.3, 0.1), (a_3, 0.3, 0.4, 0.2), (a_4, 0.2, 0.5, 0.3), (a_5, 0.5, 0.3, 0.1)\},$$
$$A_2 = \{(a_1, 0.7, 0.1, 0.1), (a_2, 0.2, 0.3, 0.4), (a_3, 0.2, 0.1, 0.5), (a_4, 0.1, 0.5, 0.2), (a_5, 0.3, 0.3, 0.3)\},$$
$$A_3 = \{(a_1, 0.4, 0.3, 0.1), (a_2, 0.5, 0.3, 0.2), (a_3, 0.4, 0.3, 0), (a_4, 0.7, 0, 0.2), (a_5, 0.6, 0.1, 0.1)\},$$
$$B = \{(a_1, 0.6, 0.2, 0.1), (a_2, 0.3, 0.4, 0.2), (a_3, 0.4, 0.3, 0.2), (a_4, 0.7, 0.1, 0), (a_5, 0.4, 0.2, 0.2)\}.$$

We have to check the resemblance of B with $A_j, j = 1, 2, 3$. The pattern B belongs to $A_j, 1 \leq j \leq 3$, if the distance between B and A_j is minimum. The computed values of distance between B and $A_j, 1 \leq j \leq 3$ using the available measures are given in Table 4 and also shown in Figure 1.



n1.

Figure 1: Distance between the known and unknown patterns regarding Example 5.1

From Table 4, we observe that B is closest to A_3 as indicated by all the distance measures. Our suggested measure also shows the same result. Thus B is classified into A_3.

Table 4: Computed values of distance regarding Example 5.1

	(A_1, B)	(A_2, B)	(A_3, B)	Result
DM_{S1}	0.4755	0.5154	0.3750	A_3
DM_{S2}	0.3880	0.3951	0.3491	A_3
DM_{S3}	0.1775	0.1872	0.1250	A_3
DM_{S4}	0.2232	0.2268	0.1955	A_3
DM_{D1}	1.7000	1.5000	1.0000	A_3
DM_{D2}	0.3400	0.3000	0.2000	A_3
DM_{D3}	0.7071	0.6557	0.4000	A_3
DM_{D4}	0.3162	0.2933	0.1789	A_3
DM_{D5}	0.3000	0.2800	0.2000	A_3
DM_{DT1}	0.1933	0.1867	0.1000	A_3
DM_{DT2}	0.1929	0.1833	0.1000	A_3
DM_{DT3}	0.2600	0.2600	0.2000	A_3
DM_{DT4}	0.1456	0.1428	0.0894	A_3
DM_{SMKSS1}	0.6800	0.6000	0.4000	A_3
DM_{SMKSS2}	0.2236	0.2074	0.1265	A_3
DM_{SMKSS3}	0.0650	0.0650	0.0500	A_3
DM_{SMKSS4}	0.1628	0.1597	0.1000	A_3
DM_{KKDK}	0.2283	0.1867	0.1117	A_3
DM_G	0.1552	0.1360	0.0926	A_3

$p = 1$ and $t = 3$ in D_{KKDK}.

Example 5.2. *Let A_1, A_2, A_3, and B be four patterns given in the form of PIFSs as:*

$$\begin{aligned}
A_1 &= \{(a_1, 0.5, 0, 0.4), (a_2, 0.7, 0, 0.1), (a_3, 0.4, 0, 0.6), (a_4, 0.7, 0, 0.2)\}, \\
A_2 &= \{(a_1, 0.5, 0, 0.2), (a_2, 0.6, 0, 0.1), (a_3, 0.2, 0, 0.7), (a_4, 0.7, 0, 0.3)\}, \\
A_3 &= \{(a_1, 0.5, 0, 0.3), (a_2, 0.7, 0, 0), (a_3, 0.4, 0, 0.5), (a_4, 0.7, 0, 0.3)\}, \\
B &= \{(a_1, 0.4, 0, 0.3), (a_2, 0.7, 0, 0.1), (a_3, 0.3, 0, 0.6), (a_4, 0.7, 0, 0.3)\}.
\end{aligned}$$

We have to check the resemblance of B with $A_j, j = 1, 2, 3$. The pattern B belongs to $A_j, 1 \leq j \leq 3$, if the distance between B and A_j is minimum. The computed distance between B and $A_j, 1 \leq j \leq 3$ using the available distance measures is given in Table 5. From Table 5 we observe the following:

1. *The PIF measures of distance $DM_{D1}, DM_{D2}, DM_{D3}, DM_{D4}, DM_{D5}, DM_{SMKSS1}, DM_{SMKSS2}$, and DM_{SMKSS4} fail to recognize B as they show that B is closest to both A_2 as well as A_3.*

2. *The PIF measure of distance DM_{DT1} shows that B is closest to both A_1 as well as to A_3 and thus fails to recognize B.*

3. *The distance between B and all the three patterns $A_j, j = 1, 2, 3$ comes out to be the same by the PIF measures of distance DM_{DT3} and DM_{DT4} and thus fails to recognize B.*

4. *The PIF measures of distance $DM_{S1}, DM_{S2}, DM_{S3}, DM_{S4}, DM_{SMKSS3}, DM_{KKDK}$ and the suggested measure DM_G assigns B to one of $A_j, j = 1, 2, 3$.*

Thus, we observe from Examples 5.1 and 5.2, that the suggested measure is more effective than most of the existing distance measures.

5.1 Medical diagnosis

To know the disease with which a person is suffering is known as a medical diagnosis. The PIF measures of compatibility play a key role in medical diagnosis. The formulation of a problem related to medical diagnosis in the PIF setting is shown below.
Problem: A set of patients $P = \{P_1, P_2, ..., P_j\}$ and a set of diseases $D = \{D_1, D_2, ..., D_k\}$ together with their symptoms $S = \{S_1, S_2, ..., S_l\}$.

Table 5: Computed values of distance regarding Example 5.2

	(A_1, B)	(A_2, B)	(A_3, B)	Result
DM_{S1}	0.1529	0.1818	0.1646	A_1
DM_{S2}	0.2282	0.2336	0.2299	A_1
DM_{S3}	0.0491	0.0545	0.0502	A_1
DM_{S4}	0.1300	0.1325	0.1303	A_1
DM_{D1}	0.4000	**0.3000**	**0.3000**	Unable to classify
DM_{D2}	0.1000	**0.0750**	**0.0750**	Unable to classify
DM_{D3}	0.2236	**0.1732**	**0.1732**	Unable to classify
DM_{D4}	0.1118	**0.0866**	**0.0866**	Unable to classify
DM_{D5}	0.1000	**0.0750**	**0.0750**	Unable to classify
DM_{DT1}	**0.0333**	0.0417	**0.0333**	Unable to classify
DM_{DT2}	**0.0500**	0.0559	**0.0500**	Unable to classify
DM_{DT3}	**0.0750**	**0.0750**	**0.0750**	Unable to classify
DM_{DT4}	**0.0433**	**0.0433**	**0.0433**	Unable to classify
DM_{SMKSS1}	0.2000	**0.1500**	**0.1500**	Unable to classify
DM_{SMKSS2}	0.0791	**0.0612**	**0.0612**	Unable to classify
DM_{SMKSS3}	0.0250	0.0187	0.0188	A_2
DM_{SMKSS4}	0.0612	**0.0433**	**0.0433**	Unable to classify
DM_{KKDK}	0.0292	0.0521	0.0417	A_1
DM_G	0.0467	0.0316	0.0335	A_2

Bold values denote unreasonable results. $p = 1$ and $t = 3$ in D_{KKDK}.

Aim: Determination of the disease with which a person is suffering.
Identification principle: A person is suffering from a disease with which its distance is minimum.
Now, in the below example, we consider a medical diagnosis problem with PIF information.

Example 5.3. [12] *Let the patients and diseases symptoms be represented hypothetically in the form of PIFSs as shown in Table 6 and Table 7 respectively. Now, we compute the distance between the symptoms of patients and symptoms of*

Table 6: Symptoms of the patients

	S_1	S_2	S_3	S_4	S_5
P_1	(0.8, 0.1, 0)	(0.4, 0.2, 0.3)	(0.1, 0.2, 0.4)	(0.6, 0.2, 0.1)	(0.2, 0.3, 0.5)
P_2	(0.1, 0.6, 0.2)	(0.3, 0.2, 0.4)	(0.9, 0.1, 0)	(0.2, 0.5, 0.2)	(0.3, 0.2, 0.4)
P_3	(0.7, 0.2, 0.1)	(0.4, 0.3, 0.2)	(0.1, 0.5, 0.2)	(0.3, 0.2, 0.4)	(0.1, 0.4, 0.3)
P_4	(0.4, 0.3, 0.2)	(0.5, 0.2, 0.1)	(0.3, 0.5, 0.1)	(0.3, 0.4, 0.2)	(0.1, 0.7, 0.2)

Table 7: Symptoms of the diseases

	S_1	S_2	S_3	S_4	S_5
D_1	(0.1, 0.5, 0.3)	(0, 0.35, 0.5)	(0.2, 0.5, 0.3)	(0.2, 0.4, 0.35)	(0.8, 0.1, 0)
D_2	(0.1, 0.5, 0.3)	(0.2, 0.3, 0.4)	(0.8, 0, 0)	(0.2, 0.3, 0.4)	(0.2, 0.3, 0.35)
D_3	(0.3, 0.3, 0.4)	(0.6, 0.1, 0.2)	(0.2, 0.4, 0.3)	(0.2, 0.3, 0.35)	(0.1, 0.6, 0.2)
D_4	(0.7, 0, 0)	(0.2, 0.35, 0.4)	(0.0, 0.5, 0.4)	(0.7, 0, 0.1)	(0.1, 0.5, 0.3)
D_5	(0.4, 0, 0)	(0.3, 0.4, 0.2)	(0.1, 0.5, 0.35)	(0.4, 0.2, 0.3)	(0.1, 0.5, 0.25)

diseases with the help of the suggested distance measure DM_G as shown in Table 8 and Figure 2.

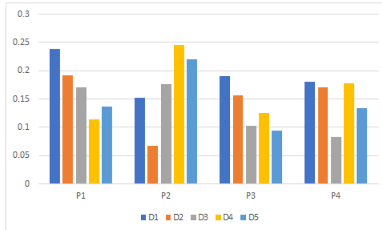
n2. 

Figure 2: Distance of the patients with the diseases using DM_G

Table 8: Computed values of distance between patients and diseases using DM_G

	D_1	D_2	D_3	D_4	D_5
P_1	0.2381	0.1919	0.1702	**0.1141**	0.1371
P_2	0.1529	**0.0675**	0.1770	0.2460	0.2198
P_3	0.1907	0.1570	0.1026	0.1253	**0.0944**
P_4	0.1807	0.1701	**0.0826**	0.1776	0.1334

Bold values indicate the disease with which the patient is suffering.

From Table 8, it is clear that patient P_1 has disease D_4 , patient P_2 has disease D_2, patient P_3 has disease D_5 , and patient P_4 has disease D_3.

6 Conclusion

The suggested PIF measure of distance is more robust than all of the available distance measures. The proposed measure has given good results in the computation of the distance between various cases of distinct PIFSs. Also, it does not lead to any counterintuitive situation while calculating the distance values of different PIFSs. The existing PIF measures of distance give unreasonable results in most of the problems related to pattern analysis. However, the suggested measure performs better than the existing measures and gives satisfactory results in pattern recognition. Also, the suggested measure has given satisfactory results in medical diagnosis.

In the future, we will extend this study to spherical fuzzy sets [28, 10], T-spherical fuzzy sets [28, 32], complex fuzzy sets [31], etc. We will also study the knowledge measures, similarity measures, dissimilarity measures, etc. for some recent generalizations of FSs.

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