

# Numerical study of geometric effects on hydraulic parameters of flow over an ogee spillway

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## ABSTRACT

One way to control the height and volume of water behind the dam is the spillway. One of the most common spillways is an ogee spillway. Today, considering safety, cost and time the optimization of a given project is one of the important issues. A spillway is based on its geometry, can change the flow rate and construction cost. In this study, has tried using numerical methods such as volume of fluid (VOF) to calculate flow rates of three types of spillways. Finally, the numerical results have compared with experimental data. The comparison showed that numerical modeling can accurately predict the rate of flow over the spillway.

## 1. Introduction

One of the most important hydraulic structures in flood control, is the dam. As dams and associated structures that are very important and critical to human communities, have high importance in the study, implementation and maintenance. Any inaccuracy in these steps may cause heavy human and financial damages. During the flood, water level behind the dam rises and in this situation one safe and quick discharge is necessary. One of the most important structures that are in sync with the construction of the dam will be required and allows excess water to drain out, is spillway. One of the spillways is ogee spillway. Correct understanding of

current flowing over spillways and its modeling makes so powerful tool in the design of these structures should be available to design engineers and analysts. In this paper, a numerical method based on the volume of fluid (VOF) has been used to investigate the flow over the dam. Ogee spillway due to its excellent hydraulic characteristics, is one of hydraulic structures that have been studied more frequently. Due to ability of this spillway in discharge and measuring flow rate, has convinced the engineers to use it in wide range of flow conditions safely. Today, considering safety, cost and time optimization in project is one of important issues. Discharge and construction costs are depended on the geometry of the spillway. On the other hand, physical models are costly and time consuming. As a result, computer simulations help to save cost and time. Computer simulations can be obtained with using software and programming. The spillways are flow measurement

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structures with good hydraulic engineering characteristics. They are widely used in many cases of water fields, found to be technically sensitive hydraulic structures. Their ability to pass flooding flows efficiently, safely, properly designed, with relatively good measuring capabilities, have enabled engineers to use them in wide range of flow situations. Hydraulics of flooding flows over barrier structures have been subject of broad research works, but investigations regarding errors from measured floods (hydrometric stations in hydrologic catchments) have not been carried in coupled fashion through hydrologic -hydraulics and Computational Fluid Dynamic (CFD) analysis. People worked on different kind of spillways, shown little interest on sensitive parameters introduced errors, involved in the solution procedure and great efforts have not been made dealing with the errors precisely. Slight modification in: flow geometry, varying shape and hydraulic properties of the flow subjected to flooding, great differences, observed on the problem solution. These changes appear as errors beyond the values often required by experts to evaluate and determine the performance of the overflowing spillway working under flooding conditions. With fast developments in computational simulation for solving the governing equations of flooding flows, recently engineers have broader choice for selecting various methods of solutions in evaluating varied flow analysis. The choices of a physical model, computational model, information from the USACE<sup>1</sup> or USBR<sup>2</sup> there exist for engineers. To correlate this study with the existing USBR and USACE data, a standard ogee-crested spillway design was used. In planning of dams, it is important to determine the capacity of the spillway. This is useful for the normal operation of the dam and also for safety reasons, in case of floods. There exist design guidelines for spillways which can be applied for simpler flow situations. The design guidelines are made for standard geometries, which are fairly simple. For more complex geometries, a physical model study is needed. Most of the larger dams have a complex geometry, and therefore physical models have been used extensively.

The disadvantages with the physical models are high costs and that it can take fairly long time to get the results. In this paper we use a numeric method that is called volume of fluid (VOF) to calculate flow rates on three types of ogee spillways. The results were compared and these comparisons show good proximity between numeric results and experimental data. This paper provides information on how accurately a commercially available computational fluid dynamic CFD model can predict the spillway flow rate. This study describes a numerical model that calculates the water flow over a spillway is modeled numerically in three dimensions. The model includes

determination of the free surface. The location of the surface is used to calculate the capacity and the coefficient of discharge for the spillway.

In theory, the ogee spillway's performance attributes are due to its shape being derived from the lower surface of an aerated nappe flowing over a sharp-crested weir. The ogee shape results in near-atmospheric pressure over the crest section for single given upstream head. In general, the ogee shapes have great impact on the ogee spillway's properties. The ogee shape results in excellent hydraulic performance. The pressures and discharge over an ogee-crested dam and spillway are dependent on the crest and spillway geometry, the upstream flow depth.

Harleman et al. 1963 compiled a bibliography on dam design that includes references to the change in the discharge coefficients due to dam submergence. However, there appears to be very little information outlining changes in spillway pressures due to submergence on the spillway prior to dam submergence,[1].

Bradley 1945 defines four distinct types of flow over an ogee-crested dam. Type I flow is characterized by a supercritical jet of water staying attached through the length of the spillway. Type II flow is defined as true hydraulic jump forming on the spillway. Type III flow is defined as a drowned jump. A drowned jump is when the jet of water stays attached to the spillway face and does so for a considerable distance under the tail water. This occurs when the tail water depth is too great to allow a good hydraulic jump to form. Type IV flow occurs when the jet breaks up and the dam acts as a broad crested weir. The dam in this case, is generally under a high degree of submergence and the down-stream depth becomes a significant variable in controlling the discharge,[2].

Considerable research has been done to determine the shape of the crest of an overflow spillway, and different methods are available that depend on the relative height and upstream face slope of the spillway.

An early attempt of modeling spillway flows was completed by Cassidy. By using potential flow theory and mapping into the complex potential plane, he was able to solve free surfaces and crest pressure head and found good agreement with experimental data for a limited number of solutions. The close agreement let Cassidy to conclude that viscosity had a negligible influence on the location of free surface. He also concluded that the point of minimum pressure for a given head was dependent on the boundary configuration.

<sup>1</sup>- U.S. Army Corps of Engineers

<sup>2</sup>- U.S. Bureau of Reclamation

Guo et al expanded on the potential flow theory by applying the analytical functional boundary value theory with the substitution of variable to derive nonsingular boundary integral equations. This method was applied successfully to spillway with a free drop. Further researcher used the standard ( $k-\varepsilon$ ) equations to model turbulence, included viscous effects, solved the Reynolds-averaged Navier-Stokes (RANS) equations in two and three dimensions, shown excellent agreement for water surface and discharge coefficients for a limited number of flows. Majority of the existing information, derived from extensive data, taken from physical models, are completed by the USBR and the USACE. Researchers attempted to solve similar problems with a variety of mathematical models and computational methods. The main difficulty of the problem was, flow transition from sub-critical to supercritical flow. In addition, the discharge was unknown, solved as part of the solution. This is especially critical when the velocity head at upstream end from the spillway is significant part of the total upstream head,[3][4].

## 2. Materials and Methods

### 2.1. Spillways types are used in this study

In this study three types of spillways are used. Type I has low radius of curvature; type II has medium radius of curvature and type III has high radius of curvature. Three types of spillways are selected to investigate the geometry effects on discharge flow rates. In 2001 Savege & Johnson worked on type II numerically and experimentally and in this case their results are used. Type I has low radius of curvature and needs less materials. So, type II cheaper than others. But types II have high curvature and needs more materials and will be most expensive. In table 1, three types of spillways and its height design ( $H_d$ ) is shown.

**Table 1.** Three types of ogee spillway are used

| Types | $H_d$ (cm) | Downstream profile    | Description                |
|-------|------------|-----------------------|----------------------------|
| I     | 15.351     | $y = 0.04906x^{1.85}$ | low radius of curvature    |
| II    | 30.1       | $y = 0.02768x^{1.85}$ | Medium radius of curvature |
| III   | 40         | $y = 0.02174x^{1.85}$ | High radius of curvature   |

### 2.2. Physical model

For engineers to accurately answer the question pertaining to hydrodynamic force distribution on a dam and/or spillway, they must conduct a physical hydraulic model study, use guidelines provided by the U.S. Bureau of Reclamation or the U.S. Army Corps of Engineers, or perform a numerical study and solve the equations governing fluid flow over such structures. Although site specific physical models are considered as the best analysis method, they may be costly. Guidelines are the easiest to use but are the least accurate. Numerical methods may be a suitable alternative, but questions remain as to their accuracy in obtaining force distribution on dams and spillways. In order to aid engineers in deciding which method to use, this study compares two physical models with their respective numerical results. A physical model of typical ogee spillway similar to UWRL<sup>3</sup>, as shown Figure 1, with design head  $H_d$  of 35cm was fabricated and tested. The model was constructed of Plexiglas and was fabricated to confirm to the distinctive shape of an ogee crest. The model also included a tangent section and a typical flip bucket. Plexiglas was chosen because it could be fabricated with smooth curves and easily instrumented with pressure taps. The model was 2m wide with 90cm height. The  $P/H_d$  ratio (height of crest/design head) was related as (18/7). The spillway model was placed in flume measuring approximately 2m wide by 12m long by 125cm depth. The ogee section was installed in the flume in an area with Plexiglas sliders so that flow could be observed. The flume had a flat bottom and was equipped with baffles and wave suppressors to provide a uniform approach flow.

To ensure that sidewall effects did not influence the pressure data, the main pressure taps were located at the center of the sectional model. To assess whether or not there were sidewall effects, several pressure taps were placed laterally across the model crest axis and were observed during testing. Testing showed that the pressure taps located centrally on the model were not affected by the sidewalls of the flume. However, a tap that was located approximately 31cm away from the sidewall of flume was influenced by sidewall effects. To ensure that the center taps would not interface with adjacent taps, the taps were staggered laterally. Every third tap was placed in the center of the spillway with the other two taps placed on either side. The tap spacing was approximately 15cm apart in the flow directions and staggered approximately 15cm,[5].

<sup>3</sup>- Utah Water Res. Lab.

### 2.3. Numerical model

The commercially available CFD program, Flow-3D, which solves the RANS equations using a finite volume method, was used to complete the numerical simulation. The program subdivides the Cartesian computational domain into a grid of hexahedral cells. Each ogee-crested dam and spillway was imported into the flow domain. The program evaluates the location of the flow obstacles by implementing a cell porosity technique called the fractional area/volume obstacle representation of FAVOR method Hirt 1992,[6]. The free water surface was computed using a modified volume-of-fluid method Hirt and Nicholes 1981,[7]. For each cell, the program calculates average values for the flow parameters pressure, velocity at discrete times using a staggered grid technique Versteeg and Malalasekera 1996,[8]. A two-equation renormalized group theory model as outlined by Yakhot and Orszag 1986 and Yakhot and Smith 1992 was used for turbulence closure,[9][10]. Initially a simple computer program was written, compiled and run for conceptualizing the physical model data analysis. Flow-3D model, having broad application in water engineering, a suitable model for the 3D- fluids, widely used in literatures, was employed for analysis, supported three dimensional flows with free surface, complex geometry, flooding flow over spillway. The software is designed with five algorithms used in a regular grid network substituting equations in forms of finite and second order precision relations for solving the problems. Numerical testing included in the software are five turbulent models (Prandtl mixing length, One-equation transport, Two-equations ( $k-\varepsilon$ ) transport, Re- Normalized Group (RNG), Large Eddy Simulation (LES)). The LES was excluded here because of lack of available data. The software adopts two techniques, used for geometric simulation, the first scheme named as: VOF4: shows properties of flow with free surface. The second method named as; FAVOR5: which is an applied technique used for simulation of solid areas and volume changed, that is also used for boundary simulation. Model was used for solving Reynolds-Averaged Navier-Stokes (RANS) equations. The computational region is covered by Cartesian coordinate grid. This grid has variable-sized hexahedral cells. For each cell, software computes parameters of flow such as velocity and pressure. Free surface modeling divided the computational cells to five regions: Completely solid, Part solid with semi fluid, completely fluid, Part fluid and empty, and completely empty.

The general governing RANS and continuity equations for incompressible flow, including the VOF and FAVOR variables, are outlined as the following (1) and (2) equations:

$$\frac{\partial}{\partial x}(uA_x) + \frac{\partial}{\partial y}(vA_y) + \frac{\partial}{\partial z}(wA_z) = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + \frac{1}{V_F}(u_j A_j \frac{\partial u_i}{\partial x_j}) = \frac{1}{\rho} \frac{\partial p'}{\partial x_i} + g_i + f_i \quad (2)$$

Equation 1 and 2 are continuity and momentum equations. The variables  $u$ ,  $v$  and  $w$  represent the velocities in the  $x$ ,  $y$  and  $z$  directions;  $V_F$  is volume fraction of fluid in each cell;  $A_x$ ,  $A_y$  and  $A_z$  is fractional areas open to flow in subscript directions;  $\rho$  is density;  $p'$  is defined as the pressure;  $g_i$  is gravitational force in the subscript direction; and  $f_i$  represents the Reynolds stresses for which a turbulence model is required for closure. It can be seen that, in cells completely full of fluid  $V_F$  and  $A_j$  equal 1, thereby reducing the equations to the basic incompressible RANS equations,[11].

The FAVOR numerical algorithm in Flow-3D, outlined by Hirt and Sicilian (1985) and Hirt (1992), is a porosity technique used to define obstacles. The grid porosity value is zero within obstacles and one for cells without the obstacle. Cells only partially filled with an obstacle have a value between zero and 1, based on the percent volume that is solid. Therefore, the ogee crest's surface is defined by cells within the grid that have a porosity value between 1 and 0. The location of the interface in each cell is defined as first-order approximation, a straight line in two dimensions and a plane in three dimensions, determined by the points where the obstacle intersects the cell faces. The slicing plane not only defines the fractional volume that can contain fluid but also determines the fraction area ( $A_x$ ,  $A_y$  and  $A_z$ ) on each cell face through which flux (fluid flow) can occur. This method presented good performances between numerical models. Another numerical algorithm in Flow-3D, used in this study to simulate flow over ogee spillway is VOF method. To numerically solve the rapidly varying flow over ogee spillway, it is important that the free surface be accurately tracked. Tracking involves three sections: locating the free surface, defining the surface as a sharp interface between the fluid and air and applying boundary conditions at the interface. VOF method is a tool for tracking the free surface. This method is described by Hirt and Nichols in 1975, Nichols et al.

<sup>4</sup> -Volume of Fluid

<sup>5</sup> -Fractional Area/Volume Obstacle Representation

in 1980 and Hirt and Nichols in 1981. The VOF method is similar to the FAVOR method in defining cells that are empty, full, or partially filled with fluid. Therefore, empty cells assigned zero, full cells assigned one and partially filled cells are assigned between 0 and 1. The slope of free surface in the cells that partially filled is found by an algorithm that uses the surrounding cells to define a surface angle and surface location. In VOF method similar to FAVOR method, free surface definition done by series of connected chords in two dimensions or by connected planes in three dimensions, the VOF method allows for changing free surface over time and space, [12].

To investigate the geometric effects of spillway on flow rate, we use three upstream head such as low head ( $H_e / H_d = 0.51$ ), medium head ( $H_e / H_d = 0.82$ ) and high head ( $H_e / H_d = 1.2$ ) on all three ogee spillways computations. Here,  $H_e$  is effective height and  $H_d$  is design height. Discharge equation on an ogee spillway and Froude number outlined as the following (3) and (4) equations:

$$Q = \frac{2}{3} C_d \sqrt{2g} L H_d^{3/2} \quad (3)$$

$$Fr = \frac{V}{\sqrt{g \cdot D}} \quad (4)$$

In these equations, Q is flow rate (discharge),  $C_d$  is discharge coefficient, L is ogee crest length,  $H_d$  is design head, V is velocity and D is hydraulic depth ( $D = A/T$ , A is area and T is top width).

In open channel hydraulics, the Froude number is a very important nondimensional parameter. The Froude number, is the ratio of inertia force to gravity force. For open channel modeling, the Froude number of a model is made equal to the Froude number of the actual full-size device. The length ratio is set and the scale ratios for velocity and discharge are determined from the equality. However, the modeler must make sure that differences in friction loss between the model and the actual device are insignificant or accounted for in some way. Open channel flow water measurement generally requires that the Froude number, of the approach flow be less than 0.5 to prevent wave action that would hinder or possibly prevent an accurate head reading. When the Froude number is 1, the velocity is equal to the velocity of wave propagation, or celerity. When this condition is attained, downstream wave or pressure disturbances cannot travel upstream. A Froude number of 1 also defines a very special hydraulic condition. This flow condition is called critical.

### 3. Result and discussion

Flow simulations are done on three type of ogee crest spillways (I, II and III) in three conditions ( $H_e / H_d = 0.51$ ,  $H_e / H_d = 0.82$  and

$H_e / H_d = 1.2$ ). One of the most important goals is followed in this study, is upstream and downstream height and its effect on flow rate. Numerical results showed that upstream head can change the flow rates (discharges). In higher level of upstream head, we found higher flow rate. From the results it is understood that upstream level changes are more than downstream level changes. Effect of geometry of spillway on flow rates (discharges) and discharge coefficients ( $C_d$ ) is another obtained result from different flow simulations. In all of three cases ( $H_e / H_d = 0.51$ ,  $H_e / H_d = 0.82$  and

$H_e / H_d = 1.2$ ), ogee III has maximum flow rates (discharges) and has minimum discharge coefficients. Whereas, ogee I has minimum flow rates (discharges) and has maximum discharge coefficients. In following tables and figures, results about upstream and downstream height, flow rates and discharge coefficient on three types of ogee crest spillways (types I, II and III) in three conditions ( $H_e / H_d = 0.51$ ,  $H_e / H_d = 0.82$  and  $H_e / H_d = 1.2$ ) are shown.

**Table 2.** Upstream and downstream height on different ogee types and conditions

| Case(Hd/He) | Ogee I              |                       | Ogee II             |                       | Ogee III            |                       |
|-------------|---------------------|-----------------------|---------------------|-----------------------|---------------------|-----------------------|
|             | Upstream Height(Cm) | Downstream Height(Cm) | Upstream Height(Cm) | Downstream Height(Cm) | Upstream Height(Cm) | Downstream Height(Cm) |
| 0.51        | 8.91                | 4.05                  | 96.51               | 4.20                  | 101.45              | 4.82                  |
| 0.82        | 93.50               | 5.11                  | 105.00              | 6.00                  | 113.26              | 10.59                 |
| 1.2         | 99.50               | 7.43                  | 116.41              | 13.84                 | 128                 | 17.11                 |

**Table 3.** Flow rates on ogee type I in three conditions

| $H_v / H_d$ | Q (m3/s) |
|-------------|----------|
| 0.51        | 0.044    |
| 0.82        | 0.088    |
| 1.2         | 0.166    |

**Table 4.** Flow rates on ogee type II in three conditions

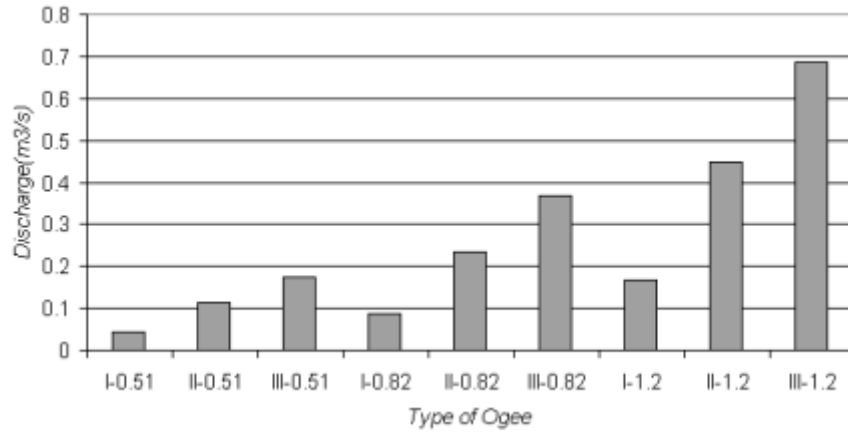
| $H_v / H_d$ | Experiment | USACE | USB   | CFD   |
|-------------|------------|-------|-------|-------|
| 0.51        | 0.126      | 0.119 | 0.117 | 0.113 |
| 0.82        | 0.268      | 0.264 | 0.254 | 0.234 |
| 1.2         | 0.488      | 0.482 | 0.463 | 0.448 |

**Table 5.** Flow rates on ogee type III in three conditions

| $H_v / H_d$ | Q(m3/s) |
|-------------|---------|
| 0.51        | 0.173   |
| 0.82        | 0.369   |
| 1.2         | 0.687   |

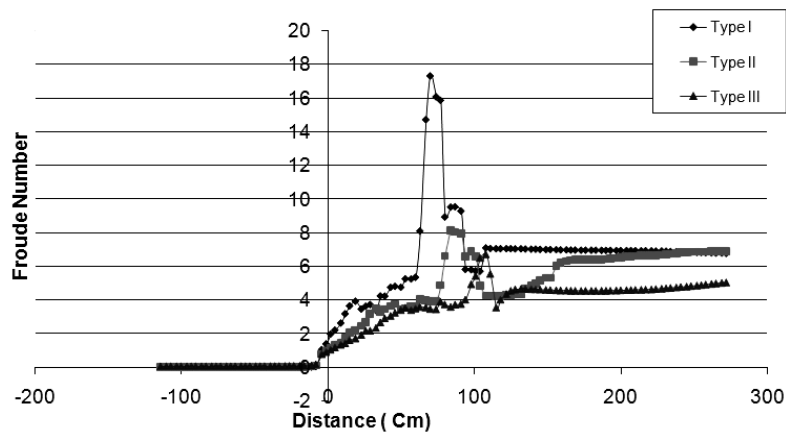
**Table 6.** Discharge coefficient (Cd) changes on three types of ogee crest in three different conditions

| Case(Hd/He) | Ogee I | Ogee II    |        |        | Ogee III  |        |
|-------------|--------|------------|--------|--------|-----------|--------|
|             |        | Experiment | USACE  | USB    | Numerical |        |
| 0.51        | 0.1354 | 0.1412     | 0.1333 | 0.1311 | 0.1266    | 0.1265 |
| 0.82        | 0.2707 | 0.3003     | 0.2958 | 0.2846 | 0.2622    | 0.2699 |
| 1.2         | 0.5107 | 0.5468     | 0.5401 | 0.5188 | 0.5020    | 0.5025 |



**Fig. 1.** Comparison of flow rates on three types of ogee crest in three different conditions

In free-surface flow, the nature of the flow (supercritical or subcritical) depends upon whether the Froude number is greater than or less than unity. In order to identify the nature of the flow (supercritical or subcritical), Froude Numbers (Fr) are calculated in different points in upstream and downstream. Froude number changes are compared on three types of ogee crest spillways (types I, II and III) in three conditions ( $H_e / H_d = 0.51$ ,  $H_e / H_d = 0.82$  and  $H_e / H_d = 1.2$ ).



**Fig.2.** Comparison of Froude Number of water flow in case  $H_e / H_d = 0.51$

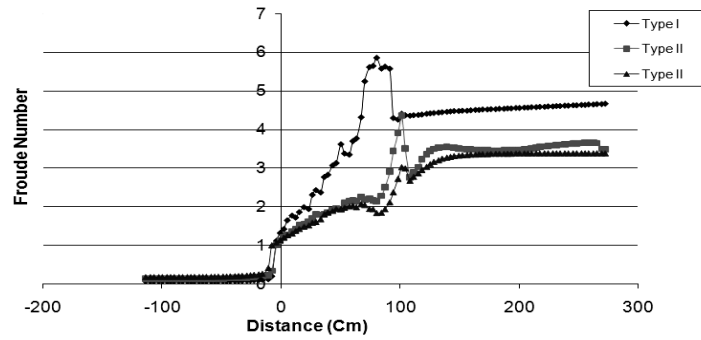


Fig. 3. Comparison of Froude Number of water flow in case  $H_e / H_d = 0.82$

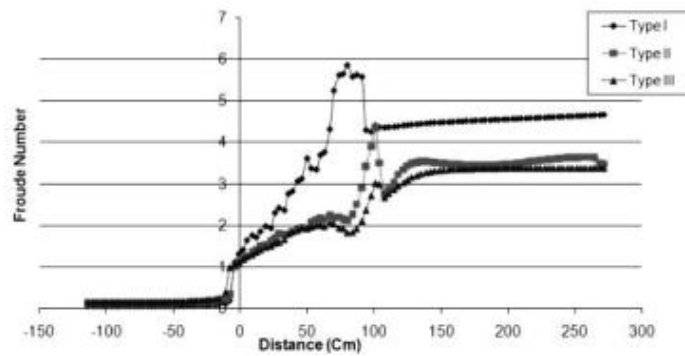


Fig. 4. Comparison of Froude Number of water flow in case  $H_e / H_d = 1.2$

According to the figures (2,3 and 4), in all cases ogee type I has maximum Froude number and ogee type III has minimum Froude number. Each ogee type, has maximum and minimum Froude number in  $H_e / H_d = 0.51$  and  $H_e / H_d = 1.2$  respectively. According to the results, the geometry of ogee profile and upstream head do not effect

on the nature of the flow sensibly. Maybe in greater and another computational field have sensible effects on the nature of the flow, but in this study not. Then, in following table, the boundaries of the nature of the flow (supercritical or subcritical) are shown.

Table 7. Boundaries of the nature of the flow on three type of ogee crest in three conditions

| Case( $H_d/H_e$ ) | Ogee I                   |                             | Ogee II                  |                            | Ogee III                 |                            |
|-------------------|--------------------------|-----------------------------|--------------------------|----------------------------|--------------------------|----------------------------|
|                   | Subcritical ( $Fr < 1$ ) | Super Critical ( $Fr > 1$ ) | Subcritical ( $Fr < 1$ ) | Supercritical ( $Fr > 1$ ) | Subcritical ( $Fr < 1$ ) | Supercritical ( $Fr > 1$ ) |
| 0.51              | $X < -4$                 | $X \geq -4$                 | $X < +2$                 | $X \geq +2$                | $X < +2$                 | $X \geq +2$                |
| 0.82              | $X < -4$                 | $X \geq -4$                 | $X < -4$                 | $X \geq -4$                | $X < -4$                 | $X \geq -4$                |
| 1.2               | $X < -4$                 | $X \geq -4$                 | $X < -4$                 | $X \geq -4$                | $X < -4$                 | $X \geq -4$                |



#### 4. Conclusion

Although physical model in hydraulic field is reliable but it has some disadvantages like time consuming and expensive. With the advancement of hardware and high-level programming languages like Fortran, C, C++ and C#, special codes and commercial software like Flow-3D, Fluent, CFX and others was created and progressed. On the other hand, problems of physical models are reduced more and more gradually but experts and engineers need physical model yet. Revolution in software and hardware helps experts and engineers to vary their experience with less time and less cost. Although numerical tools still have limitations, there are many areas where current numerical methods may offer increased accuracy over design graphs and be sufficiently accurate for the required application. Numerical models can calculate pressure data on the tangent section, thereby allowing the hydrodynamic forces on an entire spillway to be calculated. In addition, these methods may allow the analysis of 3D ogee geometries, which are sectional only in cross section, to be completed. However, numerical methods offer the potential of analyzing the entire 3D flow field, which may provide a more accurate solution.

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