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Investigating Muskingum-Cunge Method Application of Different Schemes in Flood Routing

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ABSTRACT

Wave flood is the one kind of gradually varied flow. Flood prediction and flood control are very significant; therefore, one of the essential factors in design and evaluation of hydraulic structures and water resources planning is the prediction of flow discharge in rivers which is done by flood routing operation. Muskingum cunge is classified under hydraulic routing categories. This method has been used for modeling unsteady flows. Since this technique has sufficient accuracy, it does not need recorded data. In this study, in addition to the description of constant and variable parameters Muskingum-cunge method, the results were compared by numerical solution included kinematic wave using HEC-RAS software and dynamic model using Mike_11. Results represent that Muskingum-cunge method surely can be used for flood flow routing in rivers.

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1. Introduction

One of the important issues in the area of designing and evaluating hydraulic structures, planning water resources and forecasting flow and the maximum water level in the river is that of surviving floods by different return periods. Most rivers do not have measuring stations; therefore, there is a need for a method that enables us to estimate discharge and water level in a certain location. Hence the flood routing is used to evaluate the flood wave profile along the river. We can say that flood routing which helps us understand the flood-flow characteristics is the alphabet of hydraulic structure design. Flood routing is the operation through which the downstream flow hydrograph is determined by the upstream flow hydrograph. There are different flood routing methods including: Muskingum-cunge, Kinematic Wave and Dynamic wave model. Muskingum-cunge method has sufficient accuracy and does not require frequent calibration and cross section and solution based on recorded data. Thus the parameters of Muskingum -cunge method are calculated based on physical characteristics of the river [1, 2, 3 and 4]. This method differs from Muskingum method in that the basis for the latter is changed by determining the parameters in a specific way by the Cunge et. al. based on diffusion and the possibility of taking into account the lateral flow. On the other hand, according to the parameters set by measurable Hydraulic Data in the river, Muskingum -cunge method can easily be used for rivers whose discharge has not been measured [3, 5]. Studies carried out on Muskingum -Cunge model include those by Garbrecht, Brunner, Perumal, Ponce, and Lugo. In this study, Muskingum-cunge method with

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constant parameter or constant parameter of Muskingum cunge and a four-point variable are used. An alternative method is kinematic wave model which use local acceleration, momentum transfer and convective acceleration, but ignores the term pressure in the momentum equation. This type of wave dominates the flood flow if the effects of inertia and pressure gradient compared with the bottom slope is negligible. Among the reasons for using this method is that it is easy and does not require downstream boundary condition [3]. Ponce, Hromadk, Davoodi and Chow have carried out studies to advance Kinematic wave model. In this paper the nonlinear solution of kinematic wave model is used by HEC-RAS software. The dynamic wave model uses a fully numerical solution for Saint-venant equation to carry out Flood Flow Routing and has the highest accuracy among the Flood Routing methods. In this study, the dynamic wave model in High order method is used by Mike 11 software. It is a basis for comparison with the Muskingum -Cunge method with fixed and variable parameters as well as with kinematic wave method.

2.Materials and Methods

2.1. Saint-Venant equations

Saint -Venant is a set of one-dimensional equations of continuity and momentum first used by Saint -Venant in 1871. The paper which was published in the Journal of French Academy of Sciences, described the unsteady and no uniform flows in waterways [3]. For Flood Flow Hydraulic Routing, the equations can be stated in the following no conservation form, irrespective of the lateral flow, wind shear stress or Eddy loss:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=\frac{\partial Q}{\partial x}+\left(\frac{\partial A}{\partial Q}\right)\frac{\partial Q}{\partial t}=\frac{\partial Q}{\partial t}+\left(\frac{\partial Q}{\partial A}\right)\frac{\partial Q}{\partial x}=0 \quad (1)$$

$$\frac{1}{A}\frac{\partial Q}{\partial t}+\frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+g\frac{\partial y}{\partial x}-g\left(S_0-S_f\right)=0 \quad (2)$$

where Q is flow discharge, A is the waterway average cross-sectional area; G is the acceleration of gravity, S_0 is bottom slope and S_f is the friction slopes resulting from flow resistance. Based on simplifications done in the above equations, the hydraulic flood routing models are classified. Note that solving these equations by analytical method is not possible except in special cases. Instead, they can be solved by various appropriate numerical methods. The best Numerical Method in Fluid hydraulic is finite difference method.

2.2. Consant Parameter Muskingum-Cunge Model

In most natural waterways inertial and acceleration terms in momentum equation are negligible in comparison with bottom slope [1, 3]. In the absence of lateral flow, equations of continuity and momentum in the unsteady flow for prismatic sections can be reduced as follows:

$$\frac{\partial Q}{\partial t}+C_k\frac{\partial Q}{\partial x}=D\frac{\partial^2 Q}{\partial x^2} \quad (3)$$

where Ck is Kinematic Wave Celerity which is stated as relationship (4) when used in the Manning equation. D = Q / (2BS0) is coefficient of Diffusion, where B is the upper width of the stream and S0 is the bed slope.

$$C_k=\frac{dQ}{dA}=\left(\frac{5}{3}-\frac{2}{3}\frac{A}{BP}\frac{dP}{dy}\right)V \quad (4)$$

where P is wet environment, y is water depth and V is velocity. For rectangular channels, Ck = (5/3) V is considered. For other sections, Table 1 can be used. If both the inertial and pressure forces are ignored, Saint-venant equations are simplified as the known kinematic wave equation:

$$\frac{\partial Q}{\partial t}+C_k\frac{\partial Q}{\partial x}=0 \quad (5)$$

Cunge proved that the conventional Muskingum equation is similar to the transfer-diffusive equation expressed in equation 3. He obtained relationship 6 via approximate and comparison method from kinematic wave equation. This was done by replacing Partial Derivation with standard finite difference approximations in equation 5; using a box model; selecting the place weighting factor of X and time weighting factor of θ (equal to constant value of 0.5) as well as adaptation of numerical with physical diffusion. The resulting relationship is called the Muskingum equation which is presented for two consecutive intervals according to the computational cell of Figure 1.

$$Q_{i+1}^{n+1}=C_0Q_i^{n+1}+C_1Q_i^n+C_2Q_{i+1}^n \quad (6)$$

Where n is the time index and i is the space index. The coefficients of this relationship can be defined as follows [9, 10]:

$$C_0=\frac{-KX+0.5\Delta t}{K(1-X)+0.5\Delta t},\quad C_1=\frac{KX+0.5\Delta t}{K(1-X)+0.5\Delta t},\quad C_2=\frac{K(1-X)-0.5\Delta t}{K(1-X)+0.5\Delta t}$$

(7)

In these relationships, K and X are routing parameters and are defined as follows:

$$K=\frac{\Delta x}{C_k} \quad (8)$$

$$X=\frac{1}{2}\left[1-\frac{Q_r}{BS_0C_k\,\Delta x}\right] \quad (9)$$

where Δx and is the place interval and Δt is the interval. In contrast to Muskingum method, Muskingum-Cunge method does not consider parameter X as a weighting factor. Therefore, in this method, parameter X can take a negative value. In these relationships, Ck is ave Celerity, Q is reference discharge, B is the average width and S0 is the bottom slope. Consider that the parameters of Muskingum-Cunge method, K and X can be calculated as constant or variable depending on the Reference Discharge rate. In Constant Parameter Muskingum Cunge (CPMC) the Reference Discharge is constant in all computational cells; therefore, the routing coefficients do not change at any stage. To get the reference discharge for constant parameter method, relationships 10 to 12 can be used.

$$\text{CPMC1}: \quad Q_r = Q_b + 0.5\left(Q_{pi} - Q_b\right),\ C_r = f\left(Q_r\right) \qquad (10)$$

$$\text{CPMC2}: \quad Q_r = 0.5\left(Q_{pi} - Q_b\right),\ C_r = f\left(Q_r\right) \qquad (11)$$

$$\text{CPMC3}: \quad Q_r = \sum_{n=1}^{M} Q_{in} / M,\ C_r = f\left(Q_r\right) \qquad (12)$$

where Qr and Cr are reference discharge and wave velocity for determining routing parameters. Qpi and Qb are minimum discharge and maximum input hydrograph respectively; Qin is the inflow hydrograph discharge values at the time stage of n and finally, M represents the number of inflow hydrograph data.

Table 1. Estimates of wave velocity in different channels

Channel form	Manning relationship	Chezy relationship
Wide rectangular	5/3	3/2
Triangular	4/3	5/4
Parabolic	11/9	7/6

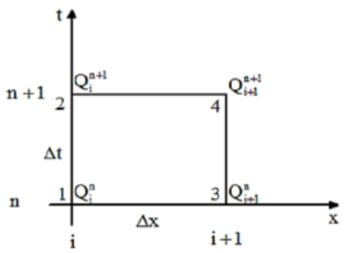

Fig.1. Computational cell of Muskingum -Cunge method

2.3. Variable parameter muskingum-cunge model

In variable parameters Muskingum cunge method, parameters of Muskingum method are obtained directly from Saint-venant equations for flood routing in prismatic channels of all forms and using any of the Manning or Chezy friction rules. Also, in this method, as in dynamic solution of Saint-venant equations, depth routing is possible simultaneous with discharge. This method is based on the assumption of steady flow between the depth in the Reach and Discharge at a distance L downstream. The interval used in this method is shown in Figure 2 [4, 5, 6, 7 and 8].

In Variable Parameter Muskingum Cunge (VPMC), the coefficients are determined based on non-linear changes of discharge reference rate in each computational cell. This method can be done in three- or four- point formats. In the three-point method, to determine the routing coefficients in the unknown node of computational cell, discharge in the known network nodes, (Qin+1 ،Qin و Qi+1n), is used. While in the four-point method, in addition to this discharge, the discharge in the unknown node of computational cell, (Qi+1n+1), is also used. Therefore, in the latter method, a process of trial and error is used for calculations. To determine the initial guess, the three-point method values are used in the four-point method. The schematic representation of these two methods can be seen in relationship (14) [5, 6 and 7].

$$\text{VPMC3}: Q_r = \left(\sum_{j=1}^{3} Q_j\right)/3 \quad , \quad C_r = \left(\sum_{j=1}^{3} C_j\right)/3 = \left(\sum_{j=1}^{3} f\left(Q_j\right)\right)/3$$

(13)

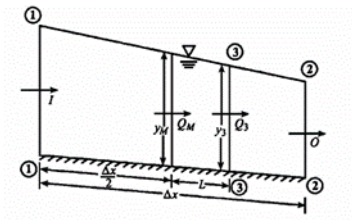

Fig 2. Interval used in the variable parameters Muskingum method

$$\text{MVPMC4}: \quad Q_r = \left(\sum_{j=1}^{4} Q_j\right)/4,\ C_r = f\left(Q_r\right) \qquad (14)$$

where numbers 3 and 4 show the three- and four-point schemes. Places of nodes j = 1,2,3,4 are shown in Figure 1.

2.4. Select Interval and Length Reach

Values of the time and place intervals are selected based on accuracy and stability. First, time intervals are examined based on the following three criteria and the smallest of them is selected: 1 - user-defined computational grid, 2 - Input hydrograph time of peak, and 3 – translation time in the desired channel range. After choosing the time interval, the place interval can be calculated by multiplying the time interval by the wave velocity. This can also do done with the following criterion which is presented for model consistency by Ponce [3].

$$\Delta x \prec \frac{1}{2}\left[C_k \Delta t + \frac{Q}{B S_0 C_k}\right] \qquad (15)$$

2.5. Applicability of Muskingum -cunge model

Muskingum-cunge method is applied to diffusion waves where dynamic terms like back water are weak. The following criterion is used to determine the scope of this application as stated by Ponce (the application criterion for wave diffusion).

$$T_r S_0 \sqrt{\frac{g}{d_0}} \succ 30 \quad (16)$$

where Tr is the input hydrograph rising time, g is acceleration of gravity, S0 is the bed slope and d0 is the average flow depth.

2.6. Kinematic wave model by HEC-RAS software

HEC-RAS software was developed in 1998 by the U.S. Army Hydrologic Engineering Centre. In the software, each of the routing methods are done by solving energy and continuity equations, but it includes various simplifying assumptions based on different flows for solving equations. Thus, appropriate method must be selected taking into account the assumptions. Kinematic wave method is used in this study where the inertial and pressure Gradient sentences are omitted from the momentum equation. The simplified equation for the Kinematic wave routing is stated as equation (17). Applicability of kinematics wave equation is checked by equation (18) as stated by Ponce (1989). This criterion states that high acuity of bed and the long time needed for rising inlet hydrograph increases the applicability of kinematics wave model. Because the pressure gradient and inertia terms of momentum equation have been removed in the kinematic wave equation, subsidence of kinematics wave will not happen. Since subsidence of flood waves in every river is evident, determining the degree of importance the slope of bed has relative to other Saint - Venant momentum equation sentences is necessary. Determining the magnitude of sentences, we can find that in a river with high gradient, S0 is more important than other sentences, while this is not true in low gradient rivers. So this method is appropriate for high slope rivers.

$$S_0 = S_f \quad (17)$$

$$(T_r V_0 S_0)/y_0 > 85 \quad (18)$$

Where V_0 and y_0 are the average flow depth and velocity, respectively and T_r is the rise time for the inlet hydrograph [3, 11 and 12].

The continuity equation (1) can be stated in the form of finite difference equations:

$$\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \frac{A_{i+1}^{j+1} - A_i^{j+1}}{\Delta t} = 0 \quad (19)$$

By inserting equation (4) in the above equation, we have:

$$\frac{\Delta t}{\Delta x} Q_{i+1}^{j+1} + \alpha (Q_{i+1}^{j+1})^{\beta} = \frac{\Delta t}{\Delta x} Q_i^{j+1} + \alpha (Q_{i+1}^{j})^{\beta} \quad (20)$$

This equation is simplified so that the unknown discharge Q_{i+1}^{j+1} is on the left and all known values are on the right side. This relationship is nonlinear relative to Q_{i+1}^{j+1}; therefore, a numerical method such as Newton method is needed.

2.7. Dynamic wave model by MIKE 11 software

The software has been developed by Danish Hydraulic Institute (DHI) and recently the Japanese CTI has improved it. In this study, the software Hydrodynamic module is used for one-dimensional simulation of the river flow.

The hydrodynamic module (HD) represents the heart of Mike 11 and contains all the core functionality for simulating hydrodynamic processes of the model.

The HD Module uses an implicit, finite difference solver that calculates water level and flow for rivers and estuaries. Mike 11 even allows you to integrate your river and floodplain modeling with watershed processes, sewer systems and coastal processes.

The HD Module includes:

1. Fully dynamic solutions for open channel flow (nonlinear St. Venant equations)
2. High-order fully dynamic, Diffusive wave, Kinematic wave, Quasi-steady state, and Kinematic routing(Muskingum, Muskingum-cunge)
3. Automatic adapting of subcritical and supercritical flows
4. A variety of structures, including weirs, culverts, bridges, pumps, regulating and control structures, dam-break structures, etc.

2.8. Initial and Boundary Condition

The flow initial conditions in Muskingum -Cunge, Kinematic wave and Dynamic wave models has been considered steady. In other words, at the start, flood is flowing in all parts of the Base Discharge interval. For upstream boundary condition the inlet flood hydrograph is used and for the downstream boundary condition in the dynamic wave model Discharge-Depth relationship according to the Manning equation is used [13].

2.9. The Model's Assumptions

To model Muskingum–Cunge, Kinematic wave and dynamic wave methods, a wide rectangular channel of 42 km length and the following data were used: Δx, Δt as place and time intervals equal 900 m and 180 seconds respectively; B as width of the upper stream equals 90 m; S_0

as the bed slope equals 0.000078, Q_{max} as peak inlet discharge equals 685 cubic meters per second; n as Manning roughness coefficient equals 0.033, A_p as surface area equivalent to the peak inlet discharge equals 84.15 square meters. Also, the base discharge is assumed equal to 35 cubic meters per second.

2.10. Model evaluation criteria

2.10.1. Amount of attenuation of peak discharge (ε):

$$\varepsilon(\%) = \left(1 - \frac{Q_{PO,E}}{Q_{PI}}\right) * 100 \qquad (21)$$

In case there is not another inlet input in the middle of the period, the downstream Hydrograph peak discharge is lower than the upstream Hydrograph peak discharge. This is called attenuation or subsidence of peak discharge. The above relationship shows the percent of this flood subsidence (ε) where values of flood subsidence for estimation hydrographs are compared. In this regard Qpo is the outlet hydrograph peak discharge as observed or computed and Qpi is the inlet hydrograph peak discharge peak.

2.10.2. The amount of lag between the upstream and downstream hydrographs (ξ):

$$\xi(\%) = \left(1 - \frac{T_{PI}}{T_{PO,E}}\right) * 100 \qquad (22)$$

The time for the downstream hydrograph to reach the peak is longer than that for the upstream hydrograph. The above relationship shows the amount of this lag (ξ). In this case, the flood lag is compared for each of the estimated hydrographs. In this regard, Tpo is the occurrence time of outlet hydrograph peak discharge and Tpi is the occurrence time of inflow hydrograph peak discharge.

3. Results and discussions

The results of comparing models are presented below. Table 2 shows the flood flow routing done by different methods at kilometers +20 and +42 in the upstream. Table 3 shows the amounts of attenuation and lag for upstream and downstream hydrographs. Figure 3 shows the translation for flood hydrograph at +42 km. Figure 4 is a good basis for the discharges routed by variable and fixed parameters Muskingum -Cunge methods and Kinematic wave model. Finally, prediction of water depth in channels at different places and times by Dynamic wave method resulting from MIKE 11 software output is presented in Figure 5. The most important results of model evaluations are:

1. According to Tables 2 and 3 and figures 3 and 4, variable parameters Muskingum -Cunge method has the greatest harmony with results of the dynamic wave method. Performance evaluation for VPMC schema with dynamic wave model shows that this method has done well in estimating the subsidence percentage. Comparing lag percentage in this method with that in dynamic wave shows a high harmony between these two methods.
2. Kinematic wave method where the inertial terms and gradient pressure are not considered has shorter modeling and the execution time, but faces some problems in terms of forecast accuracy, especially in cases of predicting flood peak subsidence. In contrast, for Muskingum -Cunge method where the simplified Saint-venant equations are used, more consistent results are observed especially for the schema VPM with dynamic wave method (Table and Figure 3).
3. The Kinematic wave method of software HEC-RAS shows the highest variance from Dynamic wave method results. Kinematic wave method has acuter hydrograph and higher peak discharge than other hydraulic methods. It is predictable due to ignoring the inertial and gradient pressure terms in Saint-Venant equations (Table 3).
4. The hydrograph of fixed parameters Muskingum -Cunge method in both periods rises and falls earlier than those of variable parameters and dynamical wave methods. In other words, this lag in the upstream hydrograph compared to downstream hydrograph has not been modeled properly, but peak discharge subsidence has been predicted well (Figure 3).
5. The overall comparison between the Muskingum -Cunge method and Dynamic wave method shows that although in both methods, routing of the whole path is possible, i.e. the flood hydrograph can be routed at any location, the dynamic wave method allows for direct routing of water depth simultaneous with discharge. This is shown in figure (5).

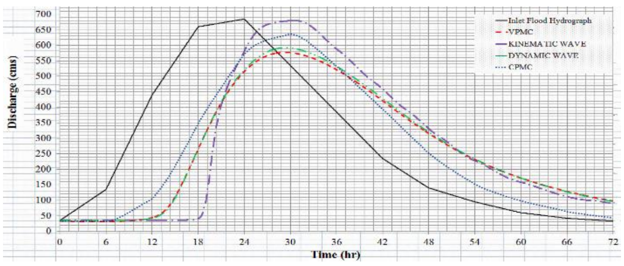

Fig 3. Graphical display of the routed inlet flood hydrograph in km +42

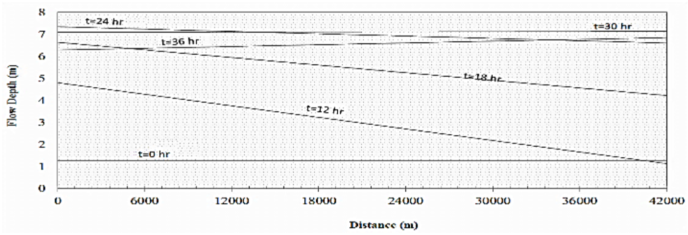

Fig 4. Hydrograph discharge for dynamic wave method using software Mike 11 as opposed to Hydrograph discharge for other methods at km +20

Table 2. Flood routing results from different methods in +20 and +42 kilometers of upstream

Time (hr)	CPMC		VPMC-4Points		Kinematic Wave by HEC-RAS		Dynamic Wave by MIKE11	
	+20Km	+42Km	+20Km	+42Km	+20Km	+42Km	+20Km	+42Km
0	35	35	35	35	35	35	35	35
6	116.02	34.23	46.3	31.54	36.23	36.21	43.90	34.38
12	382.19	110.41	218.65	44.21	117.99	36.21	218.07	41.92
18	618.19	371.53	510.10	264.93	507.62	41.04	511.47	265.02
24	680.21	610.32	616.57	515.16	675.05	581.53	624.17	520.35
30	563.32	679.55	563.69	576.52	632.80	679.69	575.18	590.94
36	413.44	568.51	457.69	521.57	490.61	588.70	464.63	532.24
42	263.44	419.69	333.05	421.44	354.75	459.48	337.91	428.63
48	158.13	269.22	230.87	315.94	229.64	331.65	232.84	319.89
54	103.32	161.96	163.39	231.72	151.82	228.19	162.79	232.94
60	66.73	105.500	116.87	171.90	103.69	158.52	115.62	171.49
66	45.42	68.05	84.88	128.61	74.54	112.03	82.76	127.32
72	34.47	46.23	64.58	97.97	52.07	89.25	61.62	95.87

Table 3. The results of model assessment criteria at kilometers +20 and +42

METHOD	+20 KM		+42 KM	
	ε (%)	ξ (%)	ε (%)	ξ (%)
MUS-CUNGE. CP	4.26	7.89	8.72	19.92
MUS-CUNGE. VP	9.79	2.04	15.72	18.08
KINEMATIC. W	0.48	10.67	0.71	21.31
DYNAMIC. W	7.94	3.87	13.61	18.20

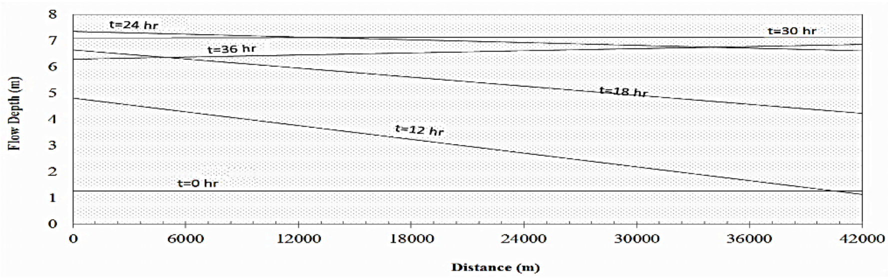

Fig.5. Prediction of channels' water depth at different times and places for dynamic wave using software MIKE 11**.**

4. Conclusion

In this paper, Constant and variable Muskingum-cunge Method has been emphasized. Assumed data with wide rectangular cross section were used to run the models. The dynamic and kinematic wave routing as well as criteria for amount of attenuation and lag in the upstream and downstream hydrographs were used to compare the results. According to the results, Muskingum -Cunge method, which has high accuracy and does not require recorded information prior to the flood, can be used as a secure method for flood routing especially in rivers without measuring stations.

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