

## Optimized linear model Muskingum parameters

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### ABSTRACT

Flood routing in matters relating to engineering projects, has many applications, and is designer to identify the effects of flood flows in the river helps. Muskingum method including flood routing methods that are using the recorded hydrograph based on flood characteristics and continuity equation to be provided routing unsteady flows. In the present study, in addition to the principles explaining the Muskingum methods, different techniques have been used to estimate the parameters of this method. The results for two different data sets Bvanlu River, superior method of least squares and correlation coefficients are shown.

### 1. Introduction

As an Importance unsteady flow refers to a situation in which the river flow and the water level rise unexpectedly. Flood flow and other words, such as wave motion, it is an example of gradually varied flow is unsteady, so that by changing your location in water way, the discharge and flow depth of the section and the other is changing from time to time. Flood routing, operational set by them, the hydrograph downstream by the flow hydrograph upstream unknown, is determined. Flood routing done by mathematical methods, to design engineers in understanding the implications of the river and its surroundings will help.

Can be including the important applications calculating spillways dimension in different kinds of dams, flood forecasting, determining rivers frontage, evaluating flood control systems, calculating levees height, urbanizing and estimating protection features for buildings that are exposed to flooding, refers be. Hydrological and hydraulically flood routing techniques are divided. In the hydrological routing, using only one-dimension of continuity equation and the momentum equation is neglected. In case both equations of continuity and momentum in the hydraulic routing calculations are entered. Muskingum method, include hydrological methods that are widely used in the flood routing. In the Muskingum method, inflow will be considered as a factor on the slope of water surface and thus, storage the Out flow range desired affects, so in this way, the equation of continuity and storage used. Hence, know the inflow and outflow samples from the corresponding need

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to be another flood to routing. To realizing this need, the parameters (X and K) in Muskingum equation coefficient to be determined properly. In the present study, various techniques to estimate the parameters for Muskingum method with two different data sets Bvanlu River, is taken into consideration. Also with the help of software, HEC-HMS, the parameters have been estimated.

## 2. Materials and Methods

### 2.1. Method principles Muskingum

The method by McCarty, a group of U.S Army Corps of Engineers (US Army), for flood control projects in the Muskingum River Ohio State, was provided. Muskingum method is based on the continuity equation, the equation is as follows:

$$\frac{1}{\Delta t} (S_j - S_{j-1}) = \frac{1}{2} (I_j + I_{j-1}) - \frac{1}{2} (O_j + O_{j-1}) \quad (1)$$

In this regard, S, O, I, respectively, Inflow discharge, Outflow discharge and storage volume, jth index and the time of the show. In this method, the total saved includes:

- 1 – Prism storage, this is only a function of feed rate.
- 2 – Wedge storage, which depends on the difference between the Inflow and Outflow rates. This issue by equation (2) is expressed

$$S = K[XI + (1 - X)O] \quad (2)$$

In this equation, x is the weighted factor, the river is between 0 to 0.5 is considered, k storage time constant and S, I and O, respectively, stored flow rates of Inflow discharge and Outflow discharge are at the same time.

By inserting equation (2) in equation (1) and simplifying we get:

$$O_{j+1} = \left[ \frac{-(KX-0.5\Delta t)}{(K-KX+0.5\Delta t)} \right] I_{j+1} + \left[ \frac{(KX+0.5\Delta t)}{(K-KX+0.5\Delta t)} \right] I_j + \left[ \frac{(K-KX-0.5\Delta t)}{K-KX+0.5\Delta t} \right] O_j = (C_1)I_{j+1} + (C_2)I_j + (C_3)O_j \quad (3)$$

With the values of K, X, Outflow discharge rates in time first step and upstream hydrograph, using equation (3) can be achieved in downstream hydrograph. Parameter K, roughly equal the interval time that the flood wave passes of river reach, and it was the same unit time steps, and the length of water way and wave velocity of flood

is dependent. While the parameter X is dimensionless and is related section routing storage.

### 2.2 Field Evaluation Model

Bvanlu River, located in the Northern Khorasan Province, and Atrac watershed in north of Province is located. The river was Avghaz river branches, the Kopi Dash Mountains located in the north of Shirvan stems. To evaluate the parameters of the range to the 13 and 9 miles from the river, and with different conditions, the relations between storage and weighted discharge are used.

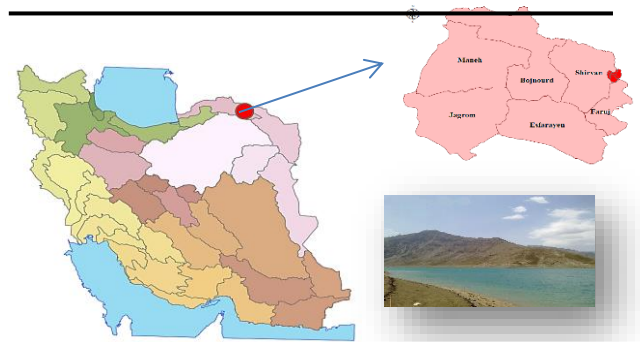


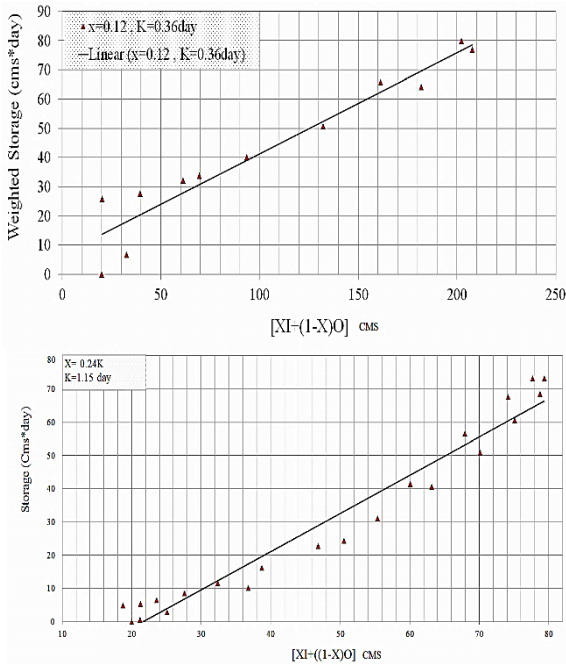
Fig.1. Geographic location, and scheme of the rivers Bvanlu Studies

### 2.3. Parameter Estimation, by Graphical Technique

Graphical approach for estimating parameters for evaluating the values K, X of the discharge hydrograph inflow (I) and outflow (Q) measured over a river reach used. According of Figure (2) when I>Q is the amount of storage volumes are added, and when it was I<Q, the volume is reduced. Therefore, the volume of water stored (S), during the period of the river, at any time, the integration of the continuity equation, is obtained:

$$S(t) = \int_0^t (I - Q)dt$$

Now, with the initial guess for different values of X, the values of [XI+ (1-X) O], for different times of the amount of stored obtained and as graphs in Figure (3) is drawn, which is mainly will loop in the form of. Among graphical diagrams, each of which was close to a straight line, that line is selected X as X is considered final. K is being able to calculate from over tangent of the line [1] and [6]. Figure (3) an example of the parameters estimating, the two data sets used in the show.



**Fig.3.**Storage -Weighted Discharge Graphs and evaluating the parameters of the graphical X, K for Data.1 and Data.2

**2.4. Parameter Evaluating, by the Least Squares Technique**

Equation (4) states that the difference between discharge and Estimated observed must be minimum and maximum correction factor of between storage and weighted discharge is the graphical approach. Can be make parameters K and X, the optimization equation (4) can be calculated. This calculation is equivalent to the maximum correction factor of between storage and discharge weight on a graphical approach.

$$\text{Minimize}_{K,X} E = \sum_{j=1}^M \{K[XI_j + (1-X)O_j] + S_1 - S_j\}^2 \quad (4)$$

In this regard, S1 initial storage, and I and O and S, respectively, storage, Inflow discharge and Outflow discharge at the same time j, by equation (1) is calculated. Aldama (1990) and Sturm (2001), the least squares method, the optimizing equation (4), the relationships (5) to (8), are presented. M, the number of Inflow data.

$$K = E + F \text{ and } X = \frac{E}{E + F} \quad (5)$$

$$Z = M \left[ (\sum I_j^2)(\sum Q_j^2) - (\sum I_j Q_j)^2 \right] + 2(\sum I_j)(\sum Q_j)(\sum I_j Q_j) - (\sum I_j)^2 (\sum Q_j)^2$$

$$- (\sum I_j^2)(\sum Q_j)^2 \quad (6)$$

$$E = \frac{1}{Z} \left\{ \left[ (\sum I_j Q_j)(\sum Q_j) - (\sum I_j)(\sum Q_j^2) \right] (\sum S_j) + \left[ M(\sum Q_j^2) - (\sum Q_j)^2 \right] (\sum I_j S_j) \right. \\ \left. \left[ (\sum I_j)(\sum Q_j) - M(\sum I_j Q_j) \right] (\sum Q_j S_j) \right\} \quad (7)$$

$$F = \frac{1}{Z} \left\{ \left[ (\sum I_j)(\sum I_j Q_j) - (\sum I_j^2)(\sum Q_j) \right] (\sum S_j) + \left[ (\sum I_j)(\sum Q_j) - M(\sum I_j Q_j) \right] (\sum I_j S_j) \right. \\ \left. + \left[ M(\sum I_j^2) - (\sum I_j)^2 \right] (\sum Q_j S_j) \right\} \quad (8)$$

**2.5. Parameter Estimation, Regression Analysis**

Linear regression analysis to the data, the relationship between weighted discharge and storage, they are linear, are suitable. There is this matter as evident in this study Data.1. For the expression this method, the first equation (3) can be rewritten as follows:

$$\frac{O_{j+1} - O_j}{I_{j+1} - I_j} = \frac{\Delta t}{K(1-X) + 0.5\Delta t} \frac{I_j - O_j}{I_{j+1} - I_j} + \frac{0.5\Delta t - kx}{k(1-x) + 0.5\Delta t} \quad (9)$$

The relation (9) can be observed linear relationship between  $(I_j - O_j)/(I_{j+1} - I_j)$ ,  $(O_{j+1} - O_j)/(I_{j+1} - I_j)$

This Thread means that the coefficients applied to linear regression analysis, can be calculated. Accordingly, we have:

$$Y_j = AX_j + B \quad ; \quad X_j = \frac{I_j - O_j}{I_{j+1} - I_j} \quad , \quad Y_j = \frac{O_{j+1} - O_j}{I_{j+1} - I_j} \quad (10)$$

$$A = \frac{M \sum X_j Y_j - \left[ \sum X_j \right] \left[ \sum Y_j \right]}{M \sum X_j^2 - \left[ \sum X_j \right]^2} \quad , \quad B = \frac{\sum Y_j - A \sum X_j}{M} \quad (11)$$

In which A and B are coefficients? By comparing relationships (10) and (11) the following parameter Muskingum methods, can be calculated as:

$$K = \frac{(1-B)\Delta t}{A} \quad , \quad X = \frac{0.5A - B}{1-B} \quad (12)$$

Since it is not necessary to present nomenclature at the beginning of the paper, each variable or symbol used in the text must be clearly defined after its first appearance in the text.

**2.6. Parameter estimating using Genetic Algorithm Optimization**

Genetic algorithm method, at first, only to solve nonlinear equations, was presented. In this way, as a non-linear search method, and the nature of the evolutionary process and genetic characteristics of organisms, is inspired. This algorithm searches the set of random solutions called population, begins. After accidentally produced the first generation of chromosomes present in the generation and evaluation based on the obtained processing of each chromosome, the population formed the next generation somehow that the optimal solution to the problem, provides guidance. In other words, individuals from populations that are less processed, the next generation will have more chances to attend. Flowchart of optimization, method genetic algorithms, in the form of (5) is presented. Here, the objective function, equation (4) and the values of K and X are determined by the somehow that the value of the objective function to be minimized. In this study, in order to accomplish this, the functions of MATLAB software, is used.

**2.7. Parameter Estimating, Based on the Correlation Coefficient**

In this method, the weighting factor is the limited range of X, the correlation coefficient and the sum of squared residuals can be, for data storage and weighted discharge can be calculated by weight, based on the highest value or lowest value of the correlation coefficient and the sum of squared residuals, the best X can be selected. The correlation coefficient of R2, based on equation (13) is calculated. Whatever the coefficients to be one closer, stronger relationship. For perform this procedure can be set initial values For the X, and with the increase of definite values of the correlation coefficient and the sum of the squared residuals, can be calculated. You can then choose the most suitable X, K can be calculated from the tangent of discharge and stronger.

$$R^2 = \frac{[\sum swf - (\sum s \sum wf)/M]^2}{[\sum S^2 - (\sum S)^2/M][\sum W_f^2 - (\sum W_f)^2/M]} \tag{13}$$

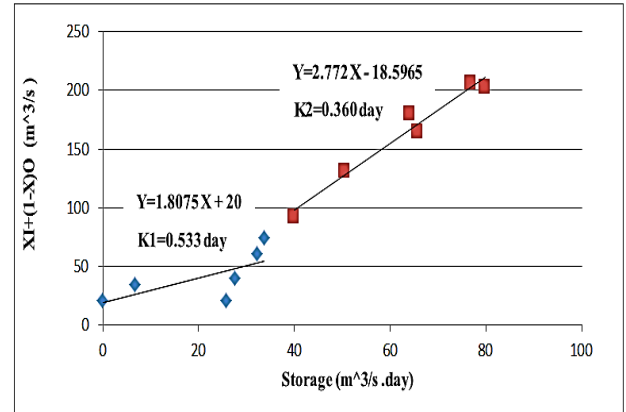
**2.8. Parameter Estimation, Segmented Regression Based on Correlation Coefficient**

This method is suitable for the data they can, the relationship between storage and weighted discharge using the equations of two lines, he expression. Thus the pair of X and Y (and [XI+ (1-X) O] are calculated from cumulative storage) are divided into two subsets. Relationships (14) and (15) represent the subject.

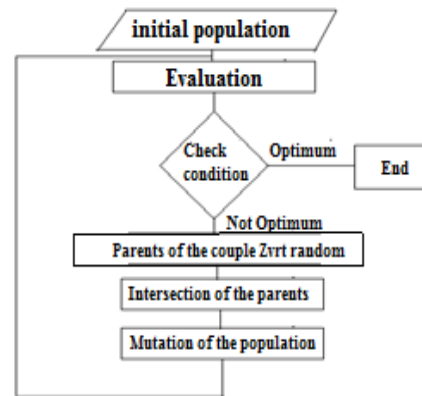
$$X_i < X^* \quad Y_i = A + BX_i \tag{14}$$

$$X_i > X^* \quad Y_i = C + DX_i \tag{15}$$

The relationships A and B and C and D are constant parameter. At one point, the two lines may be connected, or separated are considered. Based on correlation coefficient of the best lines in any of the segments, are determined separately. Figure (4), chart storage, according to weighted discharge for the Data.1, Based on this approach are presented.



**Fig.4.** Chart Storage – weighted discharge Regressions piece Data.1



**Fig.5.** Flowchart of genetic algorithm optimization

**2.9. Modeling HEC-HMS Software**

Model HEC-HMS, HEC-1 model is the advanced version, which was created in 1998 by the United States Army Center of Hydrology. Each routing methods in this software, the energy and continuity equations are Performed, but these methods for solving equations, simplifying assumptions, is considered. This software has the ability to calibrate the model parameters. Calibration is a process during which the values of the model parameters have been introduced, with the aim to reach similar results with real data, are correct. During the calibration process, sugar is the objective function optimizing, one objective function, the changes between the observed and estimated hydrograph is measured. Optimization of process

parameters by software in the form of (8) is presented. The objective function used in this study are: 1) functions weighted standard deviation of peak flows and 2) the sum of squares residual criterion, which are arranged by relationships 16) and (17) are provided. The functions of the two parameters X and K is the optimal routing.

$$Z = \sqrt{\sum_{j=1}^M [Q_{(Obs)_j} - Q_{(Out)_j}]^2 * \frac{Q_{(Obs)_j} + Q_{ave}}{2Q_{ave} * M}},$$

$$Q_{ave} = \frac{1}{M} \sum_{j=1}^M Q_{(Obs)_j} \quad (16)$$

$$Z = \sum_{j=1}^M [Q_{(Obs)_j} - Q_{(Out)_j}]^2 \quad (17)$$

In these relationships the objective function Z, Q (out), Q (abs), respectively, the observed and calculated discharge values at time step j, Q (ave) mean flow data show the number of observations and M.

### 3. Results and Discussions

Estimated parameters, using different methods, for the both data sets in Table (1) and the error percentage between routing discharge and observed discharge in the form (6) for the two data sets were used, are presented. Three criteria: 1 - residual variance between the observed discharge and estimated discharge, (M is the number of Inflow hydrograph data). 2 - The difference between the peak observed and the estimated discharge divided by the observed discharge in each method (%DPO.O =Differences Peak Outflow Observed and Estimated). 3 - Percent relative error between calculated and observed discharge in each time step (Percentage of Error) for the evaluation and comparison of different methods for estimation of parameters is used (relationships 18 to 20).

$$RV = \sum_{j=1}^M [Q_{(obs)_j}]^2 / (M - 1) \quad (18)$$

$$\%DPO.O = \left| \frac{Q_{peak_{obs}} - Q_{peak_{est}}}{Q_{peak_{obs}}} \right| \quad (19)$$

$$\% Error = \left| \frac{Q_{obs} - Q_{est}}{Q_{obs}} \right| \quad (20)$$

The main results of this study are:

1 - In all two data, the least square method and graphical method had the best fit to the observed values.

2 - To prediction the peak discharge, correlation coefficient method was robust to all two data and it had the best fit with the observed values.

3 - To prediction the time of peak Data.2, almost all procedures are properly performed, while in Data.1, the correlation technique was better than other methods.

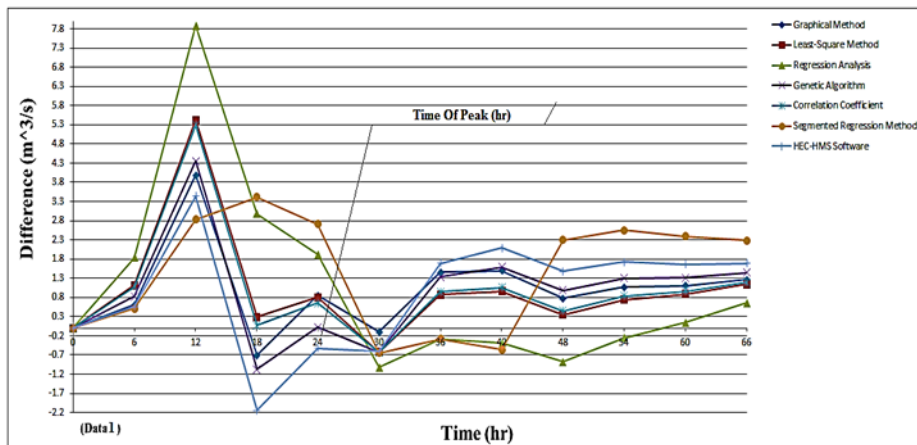
The following table (1) and error percentage were calculated For the Data.1 and Data.2 in Figure (7) is presented. Note that Figure (6) has referring the differencing between observed and estimating hydrograph. According to the results, the superiority of the method of least squares and the correlation coefficient and the predicted flood peak discharge computations, is remarkable.

### 4. Conclusion

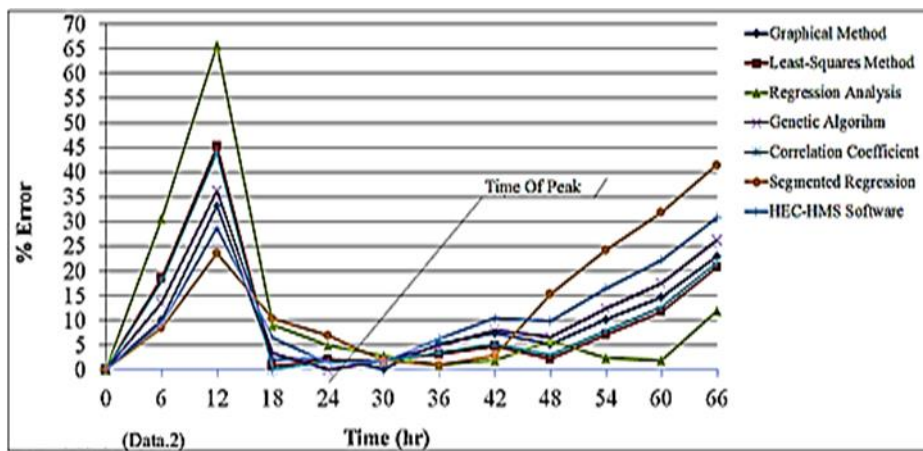
Based on the results, tables, figures, the model evaluation criteria for Data.1 &2, the error analysis were done in this study, it is recommend that whenever the relation between weighted discharge and storage is linear, it is better to use Muskingum model using least squares. Also, using the correlation coefficient method can forces peak discharge with the lowest error rates and can be expected to ensure the estimation of Muskingum model parameters (K, X) be used. Finally, it is noted that the results of the River Bvanlu harvested and spread it to other watershed, the history of past floods, and geomorphology of the region's demand.

**Table.1.**parameter estimates using different methods and evaluation criteria for the two data sets

Method	DATA.1				DATA.2					
	X	K(day)		RV	DPO. O	X	K(day)		RV	DPO. O
Graphical	0.12	0.36		205.4	4.32	0.15	0.43		2.40	2.18
Least squares	0.15	0.34		195.4	2.43	0.11	0.407		2.90	2.04
Regression Analysis	0.05	0.25		528.4	2.37	0.08	0.348		7.32	4.93
Genetic Algorithm	0.16	0.37		243.6	3.51	0.12	0.439		3.29	2.26
The correlation coefficient	0.15	0.32		229.9	0.61	0.11	0.412		3.19	1.66
Segmented Regression	0.15	0.53	0.3	327.4	3.52	0.11	0.51	0.34	4.61	6.94
HEC-HMS	0.16	0.36		278.3	3.44	0.13	0.46		3.21	2.33



**Fig.6.**Differences between observed and routing in Data 1



**Fig.7.**Percentage errors between observed discharge and routing discharge at each time step, (% Error) For the Data 1 and Data 2.

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