Iranian Journal of Fuzzy Systems
Volume 20, Number 3, (2023), pp. 177-189
Original paper

Probabilistic fuzzy argumentation frameworks with finite fuzzy statuses

J. C. Wu[1], H. F. Li[2] and X. Y. Liu[3]

[1] *School of Mathematics and Statistics, Shandong Normal University, Jinan 250358, China*
[2] *School of Computer Science and Technology, Shandong Jianzhu Yniversity, Jinan 250101, China*
[1,3] *Business School, Shandong Normal University, Jinan 250014, China*

wujiachao@sdnu.edu.cn, hengfei2014@outlook.com, xyliu@sdnu.edu.cn

Abstract

Randomness and fuzziness of argumentation have attracted the interest of many researchers. However, though each of these two properties is discussed in the past, seldom literature considers both of them. The purpose of this paper aims to explore semantics of the argumentation frameworks with these two attributes at the same time. Firstly, we introduce probabilistic-fuzzy matrices to describe the arguments with randomness and fuzziness, and define the mathematical form of the probabilistic-fuzzy argumentation frameworks. In these frameworks, an argument has finite fuzzy states and each fuzzy state has a probability. This provides a mathematical foundation for the follow-up work. Then, we introduce a method of modifying the probabilities of the fuzzy states, which proposes a feasible way to revise the probabilities. Formally, it is the revision of the probabilistic-fuzzy matrices of arguments. Finally, based on this process, we set up an extension semantics system for probabilistic-fuzzy frameworks. The semantics enriches the theory of argumentation, and propose a way to check the probabilities.

Keywords: Argumentation, argumentation frameworks, fuzzy argumentation, probabilistic argumentation, semantics.

1 Introduction

Argumentation has been proven to be a useful tool in many fields, such as multi-agent systems, decision making, law, and so on. Dung's theory of argumentation frameworks (AF) [1, 4] introduces extension semantics as sets of acceptable arguments, and connects the theory to non-monotonic reasoning, games, defeasible logic. The AF theory has been extended to various forms, for example, bipolar AFs, extended AFs, multi-valued AFs, etc. Particularly, in order to deal with the uncertainty in AFs, random theory and fuzzy theory were employed to the AFs as two common techniques for uncertainty. For example, probabilities are associated to arguments and/or attacks to capture randomness [5, 6, 7, 8, 11]. Fuzzy sets are applied to represent the fuzzy degrees in AFs [2, 9, 10, 16].

The combination of fuzziness and randomness has been well discussed in decision-making systems [12, 13, 17]. Similarly, these two co-exist in a dialectical process [3] in argumentation. In some cases, information expressed by an argument can be both random and fuzzy. Consider the following argument:

Ar: The weather forecast says it will rain tomorrow. The football match should be cancelled.

On the one hand, this argument is fuzzy because the rain can be heavy, moderate or light, which will have different effects on preventing the match from being played. For example, most people can play a football match in light rain; some can play in moderate rain; and few may continue in heavy rain. Thus argument **Ar** has different fuzzy statuses. On the other hand, this argument is stochastic because there are chances of heavy, moderate and light rain. Hence, this argument is both fuzzy and stochastic. More specifically, this argument have different fuzzy statuses, and meanwhile there are chances of the argument being in different statues.

Corresponding Author: J. C. Wu
Received: March 2022; Revised: October 2022; Accepted: February 2023.
https://doi.org/10.22111/IJFS.2023.7646

There have been works concerning such arguments in the literature. For example, Dondio introduces multi-valued argumentation frameworks (MVAFs) [3]. An MVAF is divided into many fuzzy AFs. Each fuzzy AF is given a probability value, and the fuzzy degree of each argument is calculated in the fuzzy AF. The probability of an argument's fuzzy degree x is then combined by the probabilities of the fuzzy AFs where the argument's fuzzy degree is x. In Dondio's theory, the calculated probability of each fuzzy status of an argument is unique. But there is no semantics that produces extensions in a way similar to Dung's AF semantics.

So, here are the questions: Can the semantics be calculated by other methods, for example, without dividing layers? If so, how to calculate the probabilities? In order to answer these questions, this paper aims to propose a new method to establish semantics for AFs with randomness and fuzziness at the same time. In the new approach, every argument is permitted to have finitely many fuzzy statuses, and each of its fuzzy states is arranged a probability. By adjusting the probabilities, semantics is established.[1]

In this paper, we firstly introduce the mathematics tool of *probabilistic-fuzzy matrices (PF-matrices)* together with an order between them. Based on the PF-matrices, a probabilistic-fuzzy argumentation framework (PFAF) is formally established. Then we design an algorithm for modification of PF-matrices according to the attack relation between arguments. It provides new probabilities for the fuzzy statuses. Finally, by define conflict-freeness and acceptability, a semantics system for PFAFs are established in Dung's way, including the general extensions, for example, admissible extensions, complete extensions, grounded extensions, preferred extensions, etc.

There are three contributions in this work. Firstly, the PF-matrices and a formal definition of PFAFs are introduced. These have laid a foundation for further study of PFAF semantics. Secondly, a new way is proposed to modify the probabilities of fuzzy statuses. This method outputs probability directly, avoiding the process of obtaining fuzzy semantics in the decomposed fuzzy AFs. Finally, an extended semantic system for PFAFs is established. This enriches the semantic theory of PFAFs.

The contents are arranged as follows: The next section introduces PF-matrices for describing fuzzy and stochastic arguments with finitely many fuzzy statuses. It then presents our probabilistic-fuzzy argumentation frameworks. Section 3 introduces an algorithm to revising PF-matrices. Section 4 studies conflict-freeness, acceptability of arguments in PFAFs, and define extension semantics of PFAFs. At last, Section 5 concludes this article.

2 A formal definition for PFAFs

A formal description for fuzziness and randomness of arguments is necessary in the study of PFAFs. This section discusses the formal definition of PFAFs where the arguments are of finite fuzzy statuses.

Given an argument A, suppose it has finitely many fuzzy statuses and each status has a probability. Each fuzzy status is represented by a fuzzy degree μ which is a number in the range $[0, 1]$, and the probability of the fuzzy degree μ is denoted by p_μ. Then the fuzzy degrees and the probabilities of them can be represented by a matrix.

Finite fuzzy statuses can be expressed by finite degrees of fuzziness. The fuzzy degrees are all assumed in the set $\{\mu_1 = 0,\ \mu_2, ...,\ \mu_{n-1},\ \mu_n = 1\}$, where $n \geq 2$. Without loss of generality, we always suppose $\mu_1 < \mu_2 < ... < \mu_{n-1} < \mu_n$. As we know, when considering the revision of an argument's fuzzy statuses without probabilities, such as $A \to B$, if A is of a fuzzy degree μ_A, then people, such as [2, 3, 14, 18], usually revise B to the degree $1 - \mu_A$. Similarly in PFAF, we assume that A's fuzzy degree μ_A with a probability revises B to the degree $1 - \mu_A$ with a certain probability. Then both μ_A and $1 - \mu_A$ should be in the set of fuzzy degrees. Therefore, we assume the fuzzy status set large enough, so that μ_i and $1 - \mu_i$, $\forall i \leq n$, are both in the set. Then we have the following definition.

Definition 2.1. *For an argument in a PFAF, the probabilities of its fuzzy statuses can be shown by a matrix*

$$\mathbb{M} = \begin{bmatrix} \mu_n & p_{\mu_n} \\ \mu_{n-1} & p_{\mu_{n-1}} \\ ... & ... \\ \mu_1 & p_{\mu_1} \end{bmatrix}, \tag{1}$$

where $p_{\mu_1} + ... + p_{\mu_n} = 1$. *Such matrices are called probabilistic-fuzzy matrices (PF-matrices).*

Remark 2.2. *If $n = 2$, there are only two fuzzy statuses 1 and 0. The two fuzzy statuses are commonly recognized as in and out in AFs. Then the PFAF comes back to a probabilistic AF without fuzziness.*

[1]This article is an extended version of the conference paper [15]. In this edition, an argument is allowed to possess finite many statuses rather than three fuzzy statuses in [15].

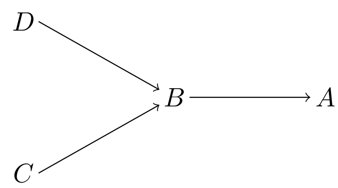

Figure 1: Argumentation graph of Example 2.6

Remark 2.3. *For all $i = 1, 2, ..., n$, since $\mu_i < \mu_{i+1}$ and μ_i, $1 - \mu_i$ are both in the fuzzy status set, we can easily obtain that $\mu_{n+1-i} = 1 - \mu_i$, i.e., $\mu_i + \mu_{n+1-i} = 1$. If $n \geq 3$ is odd, then $\frac{n+1}{2}$ is an integer. Because $\mu_{\frac{n+1}{2}} + \mu_{n+1-\frac{n+1}{2}} = 1$, we have $\mu_{\frac{n+1}{2}} = 0.5$. In other words, if there are odd fuzzy statuses, the middle fuzzy status is $\mu = 0.5$. Particularly, for $n = 3$, the PFAF comes back to an PFAF with 3 fuzzy statuses.*

Remark 2.4. *The numbers in the first column are not redundant, though it seems useless for the PFAF with 3 fuzzy statuses. For example, let n=4. Then the first column can be {1, 0.8, 0.2, 0} or {1, 0.7, 0.3, 0}. These two cases are different, even if the second columns of them are the same.*

Remark 2.5. *The restriction of $\mu_i + \mu_{n+1-i} = 1$ means that if the fuzzy degree μ_i is considered then $1 - \mu_i$ must also be considered. This restriction seems too strong such that the PF-matrix can describe few AFs with fuzziness and probabilities. However, if we allow some of the fuzzy statuses has probability 0, our PF-matrices can represent arguments with any finite fuzzy statuses. Let's see an example.*

Example 2.6. *A conference is beginning in 45 minutes. The organizers are discussing whether or not to wait for Jim.*

A: *Jim is driving here. But he is still 60 miles away. He may arrive very late. I don't think we should wait for him.*

B: *The speed limit of the high way is 80 mph. He can drive fast and arrive nearly on time. I think we'd better wait for him.*

C: *In general, the speed limit is 80 mph. But it rains now. And the forecast says the rain is going heavier soon. Jim can't drive so fast. His speed may decrease to 60 mph or lower.*

B: *How heavy the rain will be?*

C: *By the forecast, there is a 20% chance of a heavy rain, a 30% chance of a moderate rain, a 30% chance of a light rain, and a 20% of no rain.*

D: *By the way, I heard that some lanes of the high way is closed for roadworks. It may also decrease Jim's speed.*

B: *Are you sure? How serious?*

D: *I don't think it is serious. I think there is a roughly 40% chance that the work is already finished, a 30% chance that one lane is still closed and a 30% chance that two lanes are still closed.*

From the above discussion, we can extract four arguments as follows. These arguments and their attack relation form a Dung's AF shown in Figure 1 where capital letters stands for arguments and arrows denote attacks.

A: Jim will arrive very late. We should not wait for him.

B: On the high way, Jim drives very fast. He will not arrive late very much.

C: The rain goes heavier. Jim cannot drive so fast in the rain.

D: The roadworks drop Jim's speed down.

There are four fuzzy statuses of the argument C. According to the effects on the driving speed, heavy rain drops the speed very much, moderate rain drops the speed fairly, little rain drops the speed slightly, and no-rain does not drop the speed. We assume that their statuses are described by fuzzy degrees 0, 0.4, 0.6 and 1 respectively and the probabilities of their statuses are 20%, 30%, 30% and 20%. We then have the PF-matrix of argument C as follows:

$$\mathbb{M}_C = \begin{bmatrix} 1 & 0.2 \\ 0.6 & 0.3 \\ 0.4 & 0.3 \\ 0 & 0.2 \end{bmatrix}.$$

On the other hand, if we recognize that the little rain drops the speed slightly, then the fuzzy degrees of C are supposed to be 0, 0.2, 0.6 and 1. These degrees do not includes 1-0.8, and cannot be described by our PF-matrices. But we can inject them into 0, 0.2, 0.4, 0.6, 0.8 and 1, such that 0.4 and 0.8 are of probabilities 0. And the PF-matrix of C becomes

$$\mathbb{M}'_C = \begin{bmatrix} 1 & 0.2 \\ 0.8 & 0 \\ 0.6 & 0.3 \\ 0.4 & 0 \\ 0.2 & 0.3 \\ 0 & 0.2 \end{bmatrix}.$$

For convenience, in the remainder of this paper, we only list the fuzzy degrees whose probabilities are not 0. For instance, the PF-matrix $\mathbb{M}'_C$ will be written as

$$\mathbb{M}'_C = \begin{bmatrix} 1 & 0.2 \\ 0.6 & 0.3 \\ 0.2 & 0.3 \\ 0 & 0.2 \end{bmatrix}.$$

When there are more than two PF-matrices, the $(i, 1)$ element in a PF-matrix $\mathbb{M}$ is denoted by $\mu_{i,\mathbb{M}}$, which is the fuzzy status μ_i; and the $(i, 2)$ element in a PF-matrix $\mathbb{M}$ is denoted by $p_{\mu_i,\mathbb{M}}$, which is the probability of the fuzzy status μ_i.

In order to characterize the semantics of PFAFs, we introduce an order between the PF-matrices.

Example 2.7. *Suppose there are two forecasts of the rain. One says that there is a 30% chance of heavy rain and a 70% chance of moderate rain. The PF-matrix is* $\mathbb{M}_{C1} = \begin{bmatrix} 1 & 0.3 \\ 0.6 & 0.7 \end{bmatrix}$*. The other forecast predicts a 40% chance of heavy rain and a 60% chance of moderate rain, and the PF-matrix is* $\mathbb{M}_{C1} = \begin{bmatrix} 1 & 0.4 \\ 0.6 & 0.6 \end{bmatrix}$*. We would then ask which forecast predicts a heavier rain? Naturally, the second forecast indicates a heavier rain, and we denote* $\mathbb{M}_1 \preceq \mathbb{M}_2$*. If we compare them according to their PF-matrices, we have* $p_{0.6,\mathbb{M}_1} \geq p_{0.6,\mathbb{M}_2}$ *or* $p_{1,\mathbb{M}_1} \leq p_{1,\mathbb{M}_2}$

The relation $\preceq$ between PF-matrices can be extended to common PF-matrices in the next definition.

Definition 2.8. *Given two PF-matrices* $\mathbb{M}_1$ *and* $\mathbb{M}_2$*,* $\mathbb{M}_1 \preceq \mathbb{M}_2$ *if and only if for any* $i = 1, 2, ..., n$*,* $\mu_{i,\mathbb{M}_1} = \mu_{i,\mathbb{M}_2}$*, and*

$$\sum_{j=1}^{i} p_{\mu_j,\mathbb{M}_1} \geq \sum_{j=1}^{i} p_{\mu_j,\mathbb{M}_2}, \text{ or } \sum_{j=i+1}^{n} p_{\mu_j,\mathbb{M}_1} \leq \sum_{j=i+1}^{n} p_{\mu_j,\mathbb{M}_2}.$$

Note, the two inequalities are equivalent, because $\sum_{j=1}^{i} p_{\mu_j,\mathbb{M}_1} = 1 - \sum_{j=i+1}^{n} p_{\mu_j,\mathbb{M}_1}$. Hence, when checking $\mathbb{M}_1 \preceq \mathbb{M}_2$, we only need to check one of the two inequalities.

Particularly, $\mathbb{M}_1 \prec \mathbb{M}_2$ if and only if $\mathbb{M}_1 \preceq \mathbb{M}_2$ and $\mathbb{M}_2 \npreceq \mathbb{M}_1$. $\mathbb{M}_1 = \mathbb{M}_2$ iff $\mathbb{M}_1 \preceq \mathbb{M}_2$ and $\mathbb{M}_2 \preceq \mathbb{M}_1$. It is the same as the common "=" between matrices in linear algebra. We denote the set of all PF-matrices by $\mathcal{M}$.

Given a set of arguments $Args$, we use a function $S\colon Args \to \mathcal{M}$ to assign each argument a PF-matrix to represent its fuzzy statuses and probabilities. We call the function S a *PF-set* on $Args$, for convenience.

When we revise the PF-matrices of the arguments, an argument can have more than one PF-matrices, for example, the initial one and the revised one. In order to characterize different PF-matrices, we introduce the set inclusion relation between PF-sets. Given two PF-sets S_1 and S_2 on $Args$, we say that S_1 is included in S_2, denoted by $S_1 \subseteq S_2$, iff $S_1(A) \preceq S_2(A)$, for all $A \in Args$.

The empty set $\emptyset$ stands for the PF-set S, s.t. for all $A \in Args$, in the PF-matrix $S(A)$, $P_{0,S(A)} = 1$. Obviously, $\emptyset$ is the least element of all the PF-sets on $Args$ w.r.t. the set inclusion $\subseteq$.

If the set $Ar = \{A \in Args\colon 1 - p_{0,S(A)} > 0\} = \{A \in Args\colon S(A) \neq \emptyset\}$[2] is finite, the PF-set S can be represented in the form $S = \{(A, \mathbb{M}_A)\colon A \in Ar, S(A) \neq \emptyset\}$.

With the PF-set, PFAFs can be formally introduced.

[2] Ar is similar to the support set in fuzzy sets.

Definition 2.9. *Let $(Args, Atts)$ be a Dung's AF. A PFAF is a tuple $(\mathbb{A}, Atts)$, where $\mathbb{A}\colon Args \to \mathcal{M}$ is a PF-set on $Args$.*

Example 2.10. [Continue to Example 2.6] *Suppose the argument C's PF-matrix is*

$$\mathbb{A}(C) = \mathbb{M}_C = \begin{bmatrix} 1 & 0.2 \\ 0.6 & 0.3 \\ 0.4 & 0.3 \\ 0 & 0.2 \end{bmatrix}.$$

Similarly, suppose the three fuzzy degrees of D are 0, 0.5 and 1, then the PF-matrix of D is

$$\mathbb{A}(D) = \mathbb{M}_D = \begin{bmatrix} 1 & 0.3 \\ 0.5 & 0.3 \\ 0 & 0.4 \end{bmatrix}.$$

For argument B, its probabilities are determined by Jim's driving habit. However, there is no such information. In this case, we suppose that Jim will drive as quickly as possible. In other words, the initial probability of "driving fast and late for not long" is 1, i.e., in the PF-matrix of B, $P_{1,B} = 1$. It follows that $P_{\mu,B} = 0$ for the other fuzzy degrees μ of B. Similarly, we suppose $P_{1,A} = 1$ and $P_{\mu,A} = 0$ for $\mu < 1$ initially.

Then the function $\mathbb{A}\colon Args \to \mathcal{M}$ in the PFAF $(\mathbb{A}, Atts)$ can be shown as follows:

$$\mathbb{A}(C) = \mathbb{M}_C = \begin{bmatrix} 1 & 0.2 \\ 0.6 & 0.3 \\ 0.4 & 0.3 \\ 0 & 0.2 \end{bmatrix}, \ \mathbb{A}(D) = \mathbb{M}_D = \begin{bmatrix} 1 & 0.3 \\ 0.5 & 0.3 \\ 0 & 0.4 \end{bmatrix},$$

$$\mathbb{A}(B) = \mathbb{M}_B = \begin{bmatrix} 1 & 1 \end{bmatrix}, \ \mathbb{A}(A) = \mathbb{M}_A = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

3 Revision of PF-matrices

In the scenario in Example 2.6, if the probabilities of the fuzzy statuses of the arguments C (rain) and D (roadworks) are already known, then the probabilities of the fuzzy statuses of the arguments B and A should be revised or adopted according to $\mathbb{M}_C$ and $\mathbb{M}_D$. Decisions can be made by the revised probabilities (of A). In this part, we will concentrate on the algorithm to calculate the probabilities. The process of making decisions will not be discussed here.

Firstly, let's only consider the effect of C on B. The probabilities of Jim's speed and arrival time are influenced by the probabilities of the rain. For example, if there is no rain, Jim can drive around 80 mph. But in the rain, his speed will drop to 60 mph or less for safety, and the heavier the rain is, the lower the speed is. As a result, the probability that Jim drives very fast is 20%, which is the probability of no rain; the probability that Jim drives at fast speed is 30%, which is the probability of light rain; the probability that Jim drives at a moderate speed is 30%, which is the probability of moderate rain; and the probability that Jim drives very slowly is 20%, which is the probability of heavy rain. Then the PF-matrix of B should be revised to the following:

$$\mathbb{M}'_B = \begin{bmatrix} 1 & 0.2 \\ 0.6 & 0.3 \\ 0.4 & 0.3 \\ 0 & 0.2 \end{bmatrix}.$$

Now, consider the arguments D in the scenario together. Because B is attacked by C and D, the probabilities of the statuses of B should be influenced by the probabilities of C and D. According to the fuzzy degrees of C and D, the fuzzy degree of B may be 0, 0.4, 0.5, 0.6 and 1. Let's check the probabilities of the status of B. Suppose C and D are independent.

$\mu_B = 0$**:** The fuzzy degree of B is 0, i.e., Jim drives very slowly and arrives very late. In general, his speed is so slow, either because the rain is heavy or because the roadworks limit the speed seriously. Hence, either C or D is of degree 1. It equals that it's impossible that C, D are of other degrees at the same time, i.e. $P(\mu_C = 1 \text{ or } \mu_D = 1) = 1 - P(\mu_C \neq 1 \text{ and } \mu_D \neq 1)$. Denote the revised probability of $p_{0,B}$ by $p'_{0,B}$. From the independence of the rain and the roadworks, we have:

$$p'_{0,B} = 1 - (p_{0,C} + p_{0.4,C} + p_{0.6,C}) \times (p_{0,D} + p_{0.5,D}).$$

$\mu_B = 0.4$**:** The fuzzy degree of argument B is 0.4. Necessarily, the rain is moderate and the road closes no more than one lanes. Then we have

$$\begin{aligned} p'_{0.4,B} &= p_{0.6,C} \times (p_{0,D} + p_{0.5,D}) \\ &= (p_{0,C} + p_{0.4,C} + p_{0.6,C}) \times (p_{0,D} + p_{0.5,D}) - (p_{0,C} + p_{0.4,C}) \times (p_{0,D} + p_{0.5,D}). \end{aligned}$$

Other degrees: Similarly, for the other fuzzy degrees of B, the probabilities can be calculated as follows:

$$\begin{aligned} p'_{0.5,B} &= p_{0.5,D} \times (p_{0,C} + p_{0.4,C}) \\ &= (p_{0,D} + p_{0.5,D}) \times (p_{0,C} + p_{0.4,C}) - p_{0,D} \times (p_{0,C} + p_{0.4,C}), \\ p'_{0.6,B} &= p_{0.4,C} \times p_{0,D} = (p_{0,C} + p_{0.4,C}) \times p_{0,D} - p_{0,C} \times p_{0,D}, \text{ and} \\ p'_{1,B} &= p_{0,C} \times p_{0,D}. \end{aligned}$$

Putting these values into a PF-matrix, we then obtain the revised PF-matrix of B. This illustrates our idea to revise the PF-matrix of an argument. In the revision process, we make two assumptions.

- **Independence** All arguments are independent. With this assumption, the probabilities of an argument can be calculated by the product of the probabilities of its attackers.
- **Maximum** The attack relation cannot increase the probability of an argument. In other words, the revised PF-matrix of an argument is no more than its initial PF-matrix w.r.t. the order between PF-matrices.

In order to characterize the revision of PF-matrices in a PFAF, in the following part of this section, we suppose that all the arguments in a PFAF have the same fuzzy statuses (fuzzy degrees). For example, in the PFAF in Example 2.10, the fuzzy degrees of the arguments are different. In this part, we write them in the following form.

$$\mathbb{A}(C) = \mathbb{M}_C = \begin{bmatrix} 1 & 0.2 \\ 0.6 & 0.3 \\ 0.5 & 0 \\ 0.4 & 0.3 \\ 0 & 0.2 \end{bmatrix}, \ \mathbb{A}(D) = \mathbb{M}_D = \begin{bmatrix} 1 & 0.3 \\ 0.6 & 0 \\ 0.5 & 0.3 \\ 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$\mathbb{A}(B) = \mathbb{M}_B = \begin{bmatrix} 1 & 1 \\ 0.6 & 0 \\ 0.5 & 0 \\ 0.4 & 0 \\ 0 & 0 \end{bmatrix}, \ \mathbb{A}(A) = \mathbb{M}_A = \begin{bmatrix} 1 & 1 \\ 0.6 & 0 \\ 0.5 & 0 \\ 0.4 & 0 \\ 0 & 0 \end{bmatrix}.$$

Next, let's consider a procedure of the revision of the PF-matrices in PFAFs. Suppose an argument B with a PF-matrix $\mathbb{M}_B$ is attacked by a finite set of arguments $\{A_k\colon k = 1, 2, ..., m\}$, and for each argument A_k, $k = 1, 2, ..., m$, its PF-matrix is $\mathbb{M}_{A_k}$. Let's consider how the probabilities of B's fuzzy statuses change.

If B's fuzzy status is μ_i, three conditions are necessary: (a) B's original fuzzy status should be no less than μ_i; (b) all of the A_j's fuzzy statuses are no more than $1 - \mu_i = \mu_{n+1-i}$; and (c) there exists some A_j, whose fuzzy status is $1 - \mu_i = \mu_{n+1-i}$.

Suppose the revised PF-matrix of B is $\mathbb{M}'_B$. From the conditions (b) and (c), the probability $p_{\mu_i,\mathbb{M}'_B}$, for $i < n$, is no more than

$$\prod_{k=1}^{m}(\sum_{j=1}^{n+1-i} p_{\mu_j,\mathbb{M}_{A_k}}) - \prod_{k=1}^{m}(\sum_{j=1}^{n-i} p_{\mu_j,\mathbb{M}_{A_k}})\}, \tag{2}$$

where the former product is the probability that all the A_k's fuzzy statuses are no more than μ_{n+1-i}, and the latter product is the probability that all the A_k's fuzzy statuses are strictly less than μ_{n+1-i}. For $i = n$, each A_k should be the fuzzy status 0. The probability is $\prod_{k=1}^{m} p_{0,\mathbb{M}_{A_k}}$. Let $\prod_{k=1}^{m}(\sum_{j=1}^{n-n} p_{\mu_j,\mathbb{M}_{A_k}}) = 0$,[3] then Equation (2) is also valid for $i = n$.

From the condition (a), for $i < n$, $p_{\mu_i,\mathbb{M}'_B}$ is no more than

$$\sum_{j=i}^{n} p_{\mu_j,\mathbb{M}_B} - \sum_{j=i+1}^{n} p_{\mu_j,\mathbb{M}'_B}. \tag{3}$$

[3]For convenience, we suppose $\prod_{k=1}^{m}(\sum_{j=1}^{n-n} p_{\mu_j,\mathbb{M}_{A_k}}) = 0$ and $\sum_{j=n+1}^{n} p_{\mu_j,\mathbb{M}'_B} = 0$ in the following part of this paper.

For $i = n$, $p_{\mu_n,\mathbb{M}_B} \geq p_{\mu_i,\mathbb{M}'_B}$. It also coincides Equation (3), under the assumption $\sum_{j=n+1}^{n} p_{\mu_j,\mathbb{M}'_B} = 0$.

Then $p_{\mu_i,\mathbb{M}'_B}$ can be the minimum of the Equations (2) and (3). And we introduce the next definition for the revision of B's PF-matrix.

Definition 3.1. *Suppose an argument B with a PF-matrix $\mathbb{M}_B$ is attacked by a finite set of arguments $\{A_k \colon k = 1, 2, ..., m\}$, and for each argument A_k, $k = 1, 2, ..., m$, its PF-matrix is $\mathbb{M}_{A_k}$. We say that the PF-set $\{(A_k, \mathbb{M}_{A_k}) \colon k = 1, 2, ..., m\}$ revises B's PF-matrix $\mathbb{M}_B$ to $\mathbb{M}'_B$, where for all $i = 1, ..., n$,*

$$p'_{\mu_i,B} = \min\{\sum_{j=i}^{n} p_{\mu_j,B} - \sum_{j=i+1}^{n} p'_{\mu_j,B},\ \prod_{k=1}^{m}(\sum_{j=1}^{n+1-i} p_{\mu_j,A_k}) - \prod_{k=1}^{m}(\sum_{j=1}^{n-i} p_{\mu_j,A_k})\}. \tag{4}$$

In this definition, if we want to calculate $p'_{\mu_i,B}$, we should calculate $p'_{\mu_{i+1},B}$ first. Hence, this definition is actually a recursive algorithm, and the first step is to calculate $p'_{1,B}$.

Example 3.2. *Consider the PFAF in Example 2.6. The probabilities in $\mathbb{M}'_B$ are calculated as follows:*

$$\begin{aligned}
p'_{1,B} &= \min\{1, P_{0,C} \times P_{0,D}\} = 0.2 \times 0.4 = 0.08,\\
p'_{0.6,B} &= \min\{1 - p'_{1,B}, (p_{0,C} + p_{0.4,C}) \times p_{0,D} - p_{0,C} \times p_{0,D}\} = 0.12,\\
p'_{0.5,B} &= \min\{1 - (p'_{1,B} + p'_{0.6,B}), (p_{0,D} + p_{0.5,D}) \times (p_{0,C} + p_{0.4,C}) - p_{0,D} \times (p_{0,C} + p_{0.4,C})\} = 0.15,\\
p'_{0.4,B} &= \min\{1 - (p'_{1,B} + p'_{0.6,B} + p'_{0.5,B}), (p_{0,C} + p_{0.4,C} + p_{0.6,C}) \times (p_{0,D} + p_{0.5,D}) - (p_{0,C} + p_{0.4,C}) \times (p_{0,D} + p_{0.5,D})\}\\
&= 0.21,\\
p'_{0,B} &= \min\{1 - (p'_{1,B} + p'_{0.6,B} + p'_{0.5,B} + p'_{0.4,B}), 1 - (p_{0,C} + p_{0.4,C} + p_{0.6,C}) \times (p_{0,D} + p_{0.5,D})\} = 0.44.
\end{aligned}$$

Hence, the PF-matrix of B is revised to:

$$\mathbb{M}'_B = \begin{bmatrix} 1 & 0.08 \\ 0.6 & 0.12 \\ 0.5 & 0.15 \\ 0.4 & 0.21 \\ 0 & 0.44 \end{bmatrix}.$$

Similarly, the PF-matrix of A is revised by $(B, \mathbb{M}'_B)$ to

$$\mathbb{M}'_A = \begin{bmatrix} 1 & 0.44 \\ 0.5 & 0.21 \\ 0.5 & 0.15 \\ 0.5 & 0.12 \\ 0 & 0.08 \end{bmatrix}.$$

Then the decision can be made according to the probabilities in this PF-matrix.

Given a PF-set $S \subseteq \mathbb{A}$ in a PFAF $(\mathbb{A}, Atts)$, let's consider the revision of A's PF-matrix by S. Obviously, A is only influenced by the arguments B which attacks A. Then we have the following definition.

Definition 3.3. *Suppose $A \in Args$ is an argument and $S \subseteq \mathbb{A}$ is a PF-set on Args in a PFAF $(\mathbb{A}, Atts)$. We say S revises A's PF-matrix $\mathbb{M}_A$ to $\mathbb{M}'_A$, if and only if the PF-set $\{(B, S(B)) \colon (B, A) \in Atts\}$ revises $\mathbb{M}_A$ to $\mathbb{M}'_A$.*

Next, let's consider some propositions about the revision of the PF-matrices.

Proposition 3.4. *In a PFAF, if the PF-matrix $\mathbb{M}_B$ is revised to $\mathbb{M}'_B$, then $\mathbb{M}'_B \preceq \mathbb{M}_B$.*

Proof. By Equation (4), for each i, $p'_{\mu_i,B} \leq \sum_{j=i}^{n} p_{\mu_j,B} - \sum_{j=i+1}^{n} p'_{\mu_j,B}$. Moving $p'_{\mu_j,B}$ to the left, we obtain that $\sum_{j=i}^{n} p'_{\mu_j,B} \leq \sum_{j=i}^{n} p_{\mu_j,B}$. Hence, $\mathbb{M}'_B \preceq \mathbb{M}_B$. □

Next proposition shows the monotonicity of the revision, which follows obviously from Definition 3.1.

Proposition 3.5. *Let $S \subseteq S'$ be two PF-sets and $\mathbb{M}_A \preceq \mathbb{M}'_A$ be two PF-matrices of an argument A.*

1. *If S revises $\mathbb{M}_A$ to $\mathbb{M}_{A,1}$ and S' revises $\mathbb{M}_A$ to $\mathbb{M}_{A,2}$, then $\mathbb{M}_{A,2} \preceq \mathbb{M}_{A,1}$.*
2. *If S revises $\mathbb{M}_A$ to $\mathbb{M}_{A,1}$ and S revises $\mathbb{M}'_A$ to $\mathbb{M}_{A,2}$, then $\mathbb{M}_{A,1} \preceq \mathbb{M}_{A,2}$.*

Next definition introduces the revision between PF-sets.

Definition 3.6. *Let S_1 and S_2 be two PF-sets in a PFAF. S_1 revises S_2 to S'_2, iff for each $A \in Args$, S_1 revises the PF-matrix $S_2(A)$ to the PF-matrix $S'_2(A)$.*

Particularly, if S_2 is a single point set $\{(A, \mathbb{M}_A)\}$, we simply say that S_1 revises $(A, \mathbb{M}_A)$ to $(A, \mathbb{M}'_A)$.

The following proposition follows from Proposition 3.4.

Proposition 3.7. *If S is revised to S', then $\forall A \in Args$, $S'(A) \preceq S(A)$, i.e., $S' \subseteq S$.*

4 Extension semantics of PFAFs

In this section, we will establish an extension semantics system along Dung's way [4]. It shows the various levels of consistency of the probabilities for the arguments in a PFAF.

4.1 Conflict-free semantics

Consider arguments B and C in the PFAF of the scenario, and their associated PF-matrices $\mathbb{M}_B$ and $\mathbb{M}_C$. Since $P_{1,C} = 0.2$, there is a 20% chance that it will rain heavily; and since $P_{1,B} = 1$ it is certain (a 100% chance) that Jim will drive very fast and will arrive nearly on time. They are obviously in contradiction. Thus we can say that the two arguments with such PF-matrices are conflict with each other.

On the other hand, suppose $\mathbb{M}_B$ is revised to

$$\mathbb{M}'_B = \begin{bmatrix} 1 & 0.08 \\ 0.6 & 0.12 \\ 0.5 & 0.15 \\ 0.4 & 0.21 \\ 0 & 0.44 \end{bmatrix}.$$

From this matrix and argument C, we obtain that there is:

- 20% chance that it doesn't rain and Jim drives very fast of 8% chance;[4]
- 30% chance that it rains lightly, and Jim drives fast of 12% chance;
- 30% chance that it rains moderately and Jim drives at a moderate speed of 21% chance;
- 20% chance that it rains lightly, and Jim drives slowly of 44% chance.

Now we can conclude that $(B, \mathbb{M}'_B)$ is no longer conflict with $(C, \mathbb{M}_C)$.

Similarly, in PFAFs if a PF-set S revises $\mathbb{M}_A$ to $\mathbb{M}'_A$, S is consistent with $\mathbb{M}'_A$. This is similar to the conflict-free semantics in AFs.

Definition 4.1. *In a PFAF $(\mathbb{A}, Atts)$, a PF-set $S\colon Args \to \mathcal{M}$ is conflict-free, iff for any $A \in Args$, S revises $S(A)$ to itself, i.e. S revises S to itself.*

Example 4.2. *Consider the PFAF in the scenario. The PF-set S is conflict-free, where S is defined as follows:*

$$S(D) = \mathbb{M}_D = \begin{bmatrix} 1 & 0.3 \\ 0.5 & 0.3 \\ 0 & 0.4 \end{bmatrix},\ S(C) = \mathbb{M}_C = \begin{bmatrix} 1 & 0.2 \\ 0.6 & 0.3 \\ 0.4 & 0.3 \\ 0 & 0.2 \end{bmatrix},$$

$$S(B) = \mathbb{M}'_B = \begin{bmatrix} 1 & 0.08 \\ 0.6 & 0.12 \\ 0.5 & 0.15 \\ 0.4 & 0.21 \\ 0 & 0.44 \end{bmatrix},\ S(A) = \mathbb{M}'_A = \begin{bmatrix} 1 & 0.44 \\ 0.6 & 0.21 \\ 0.5 & 0.15 \\ 0.4 & 0.12 \\ 0 & 0.08 \end{bmatrix}.$$

[4]The percentage 20%-8%=12% shows the influence of the roadworks.

According to the conflict-free sets, the attacks from a PF-set to an argument can be identified into two types: sufficient attacks and tolerable attacks.

Definition 4.3. *In a PFAF, let S be a PF-set and A be an argument with a PF-matrix $\mathbb{M}_A$. Suppose S revises $\mathbb{M}_A$ to $\mathbb{M}'_A$. If $\mathbb{M}'_A \prec \mathbb{M}_A$, then we say S sufficiently attacks $(A, \mathbb{M}_A)$ or $\mathbb{M}_A$. Otherwise, i.e., $\mathbb{M}'_A = \mathbb{M}_A$, we say S tolerably attacks $(A, \mathbb{M}_A)$ or $\mathbb{M}_A$.*

Then the conflict-free sets can be represented by the tolerable attacks.

Proposition 4.4. *Let S be a PF-set in a PFAF. S is conflict-free, iff $\forall A \in Args$, S tolerably attack $(A, S(A))$.*

Proof. ($\Rightarrow$) Suppose S is conflict-free. Then S revises $S(A)$ to itself, i.e., S tolerably attack $(A, S(A))$.

($\Leftarrow$) Suppose for all $A \in Args$, S tolerably attack $(A, S(A))$. Then we have S revises $S(A)$ to itself, i.e., S is conflict-free. □

4.2 Acceptability

Following Dung's methodology, we first define the acceptability of arguments in PFAFs.

Consider the PFAF in the scenario. Since the rain and the roadworks drop Jim's speed down, Jim arrives late. When only the fuzzy degrees or the probabilities are considered, we have:

Fuzziness If the rain is heavier and the roadworks occupy more lanes, then Jim's speed will be lower. It follows that Jim will arrive later.

Probability If the probabilities of the rain and the roadworks are higher, the probability that Jim drives fast is lower. Then the probability that Jim arrives late will be higher.

When both the fuzziness and the randomness are considered together, we will discuss the consistency between the probabilities of the fuzzy statuses of the arguments. For example, if the probability of heavy rain is 20%, the probability of closing two or more lanes is 30%, then the probability that Jim's speed is very low is 44%. Consequently, we can get the probability that Jim is late for a long time is 44%. It can be read as that the two probabilities (20% of heavy rain and 30% of closing two or more lanes) defend the probability 44% of Jim's late arrival in some sense. Because the probability of no rain is 20% and the probability that the roadworks have been finished is 40%, there is a 8% chance that Jim will drive very fast and will arrive nearly on time. Similarly, for the other cases. These can be represented by the PF-matrices:

The arguments C, D with the PF-matrices $\mathbb{M}_C$ and $\mathbb{M}_D$ defend the argument A with a PF-matrix

$$\mathbb{M}_A = \begin{bmatrix} 1 & 0.44 \\ 0.6 & 0.21 \\ 0.5 & 0.15 \\ 0.4 & 0.12 \\ 0 & 0.08 \end{bmatrix},$$

by revising the PF-matrix $\mathbb{M}_B$ of B.

Following the above idea, the defence relation in PFAFs can be defined.

Definition 4.5. *In a PFAF $(\mathbb{A}, Atts)$, an argument A with a PF-matrix $\mathbb{M}_A \preceq \mathbb{A}(A)$ is acceptable to a PF-set $S \subseteq \mathbb{A}$, iff $\forall B$ with $(B, A) \in Atts$ and $\mathbb{M}_B \preceq \mathbb{A}(B)$, S revises $\mathbb{M}_B$ to $\mathbb{M}'_B$, such that the PF-set $\{(B, \mathbb{M}'_B) \colon (B, A) \in Atts\}$ revises $\mathbb{M}_A$ to itself.*

Simply, we say $(A, \mathbb{M}_A)$ is acceptable to (or defended by) S.

Note that the function $\mathbb{A}$ shows the initial probabilities of the arguments, which are also the highest probabilities of the arguments. In a semantics, for any argument, the probabilities should not be higher than it. Therefore, in Definition 4.5, we require that $\mathbb{M}_A \preceq \mathbb{A}(A)$, $\mathbb{M}_B \preceq \mathbb{A}(B)$ and $S \subseteq \mathbb{A}$.

Because of $\mathbb{M}_B \preceq \mathbb{A}(B)$ from the second part of Proposition 3.5, we have the following proposition.

Proposition 4.6. *In a PFAF, $(A, \mathbb{M}_A)$ is acceptable to (or defended by) S, iff $\forall B$ with $(B, A) \in Atts$, S revises $\mathbb{A}(B)$ to $\mathbb{M}'_B$, such that the PF-set $\{(B, \mathbb{M}'_B) \colon (B, A) \in Atts\}$ revises $\mathbb{M}_A$ to itself.*

This proposition helps us to calculate the acceptability easily.

Example 4.7. *Consider the conflict-free PF-set S in Example 4.2. It revises $\mathbb{A}(B)$ to*

$$S(B) = \mathbb{M}'_B = \begin{bmatrix} 1 & 0.08 \\ 0.6 & 0.12 \\ 0.5 & 0.15 \\ 0.4 & 0.21 \\ 0 & 0.44 \end{bmatrix}.$$

Because $\mathbb{M}'_B$ revises

$$S(A) = \mathbb{M}'_A = \begin{bmatrix} 1 & 0.44 \\ 0.6 & 0.21 \\ 0.5 & 0.15 \\ 0.4 & 0.12 \\ 0 & 0.08 \end{bmatrix}.$$

to itself, S defends $(A, \mathbb{M}'_A)$. On the other hand, $\mathbb{M}'_B$ revises $\mathbb{A}(A) = \mathbb{M}_A = \begin{bmatrix} 1 & 1 \end{bmatrix}$ to

$$\mathbb{M}'_A = \begin{bmatrix} 1 & 0.44 \\ 0.6 & 0.21 \\ 0.5 & 0.15 \\ 0.4 & 0.12 \\ 0 & 0.08 \end{bmatrix} \prec \mathbb{M}_A.$$

Hence, S does not defend $(A, \mathbb{M}_A)$.

Example 4.8. *Consider the empty set $\emptyset$ in the PFAF in Example 2.6. It revises $\mathbb{A}(C)$ and $\mathbb{A}(D)$ to themselves. Because $\{(C, \mathbb{A}(C)), (D, \mathbb{A}(D))\}$ revises*

$$\mathbb{M}'_B = \begin{bmatrix} 1 & 0.08 \\ 0.6 & 0.12 \\ 0.5 & 0.15 \\ 0.4 & 0.21 \\ 0 & 0.44 \end{bmatrix},$$

to itself, we get that $\emptyset$ defends $(B, \mathbb{M}'_B)$. But $\emptyset$ does not defend $(A, \mathbb{M}'_A)$, where

$$\mathbb{M}'_A = \begin{bmatrix} 1 & 0.44 \\ 0.6 & 0.21 \\ 0.5 & 0.15 \\ 0.4 & 0.12 \\ 0 & 0.08 \end{bmatrix}.$$

Because $\emptyset$ revises $\mathbb{A}(B)$ to itself, which revises $\mathbb{M}'_A$ to $\begin{bmatrix} 0 & 1 \end{bmatrix}$.

The monotonicity of the acceptability follows obviously from the first part of Proposition 3.5.

Proposition 4.9. *Let $S_1 \subseteq S_2$ be two PF-sets in a PFAF. If S_1 defends $(A, \mathbb{M}_A)$, then S_2 defends it.*

Proof. For any B with $(B, A) \in \mathit{Atts}$, suppose S_1 revises $\mathbb{M}_B$ to $\mathbb{M}_{B,1}$ and S_2 revises $\mathbb{M}_B$ to $\mathbb{M}_{B,2}$. Because $S_1 \subseteq S_2$, we have $\mathbb{M}_{B,2} \preceq \mathbb{M}_{B,1}$.

Suppose the PF-set $\{(B, \mathbb{M}_{B,2} \colon (B, A) \in \mathit{Atts}\}$ revises $\mathbb{M}_A$ to $\mathbb{M}'_A$. Because the PF-set $\{(B, \mathbb{M}_{B,1} \colon (B, A) \in \mathit{Atts}\}$ revises $\mathbb{M}_A$ to itself, from Proposition 3.5 we have the $\mathbb{M}_A \preceq \mathbb{M}'_A$. Together with $\mathbb{M}'_A \preceq \mathbb{M}_A$, we have $\mathbb{M}_A = \mathbb{M}'_A$. It ends the proof. □

Given a PFAF $(\mathbb{A}, \mathit{Atts})$, let $\mathcal{S}$ be the set of all the PF-sets in it, i.e., $\mathcal{S} = \{S \subseteq \mathbb{A} \mid S \colon \mathit{Args} \to \mathcal{M}\}$.

Definition 4.10. *Let $(\mathbb{A}, \mathit{Atts})$ be a PFAF. Suppose the function $F \colon \mathcal{S} \to \mathcal{S}$ is defined as follows:*

$$F(S) = \{(A, \mathbb{M}_A) \in \mathbb{A} \colon (A, \mathbb{M}_A) \text{ is defended by } S\}, \ \forall S \in \mathcal{S}.$$

Then F is called the characteristic function of the PFAF.

It follows from Proposition 3.5 that F is monotone.

Proposition 4.11. *Let F be the characteristic function of a PFAF. Suppose $S_1 \subseteq S_2 \subseteq \mathbb{A}$. Then $F(S_1) \subseteq F(S_2)$.*

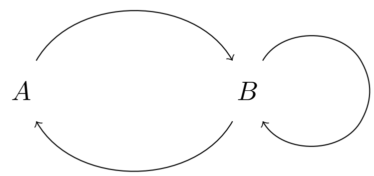

Figure 2: A simple PFAF

4.3 Extension-based semantics

In this part, we establish the semantics of PFAFs similar to Dung's extensions.

Definition 4.12. *In a PFAF* $(\mathbb{A}, Atts)$*,* $S \subseteq \mathbb{A}$ *is a conflict-free PF-set. A PF-set* S *is*

admissible *if it defends itself, i.e.,* $S \subseteq F(S)$*.*

complete *if it defends itself and does not defend any* $(A, \mathbb{M}_A)$ *with* $S(A) \prec \mathbb{M}_A$*, i.e.,* $F(S) = S$*.*

preferred *if it is a maximal admissible PF-set w.r.t. set inclusion.*

grounded *if it is the least complete PF-set.*

stable *if for any* $(A, \mathbb{M}_A)$ *with* $S(A) \prec \mathbb{M}_A$*,* S *revises* $\mathbb{M}_A$ *to* $S(A)$*.*

Example 4.13. *Consider the PFAF in Example 2.6. The PF-set* S *is admissible, complete, preferred, grounded and stable.*

The empty set $\emptyset$ *is admissible. From Example 4.8, it is not complete. Thus it is not preferred or grounded.*

Example 4.14. *Consider the PFAF* $(\mathbb{A}, Atts)$*, where*

$$\mathbb{A}(A) = \begin{bmatrix} 1 & 1 \end{bmatrix},\ \mathbb{A}(B) = \begin{bmatrix} 1 & 1 \end{bmatrix},$$

and $Atts = \{(A, B),\ (B, A)\}$*. It is the graph* $A \rightleftarrows B$*.*

The grounded extension is the empty PF-set $\emptyset$*. It is also complete, but not preferred.*

The following two PF-sets are preferred, which are also complete and stable.

S_1**:** $S_1(A) = \begin{bmatrix} 1 & 1 \end{bmatrix}$ *and* $S_1(B) = \begin{bmatrix} 0 & 1 \end{bmatrix}$.

S_2**:** $S_2(A) = \begin{bmatrix} 1 & 0.7 \\ 0 & 0.3 \end{bmatrix}$ *and* $S_2(B) = \begin{bmatrix} 1 & 0.3 \\ 0 & 0.7 \end{bmatrix}$.

For these two PF-sets, S_1 *is a preferred extension in Dung's theory;* S_2 *is a special semantics of crisp probabilistic AFs.*

Example 4.15. *Consider the PFAF in Fig. 2, where the initial PF-matrix of each argument is* $\begin{bmatrix} 1 & 1 \end{bmatrix}$.

The PF-set S *is preferred but not stable, where*

$$S(A) = \begin{bmatrix} 1 & 0.75 \\ 0 & 0.25 \end{bmatrix} \text{ and } S(B) = \begin{bmatrix} 1 & 0.2 \\ 0 & 0.8 \end{bmatrix}.$$

Obviously, S *is admissible and maximal w.r.t. set inclusion. But* S *revises*

$$\mathbb{M}_A = \begin{bmatrix} 1 & 0.8 \\ 0 & 0.2 \end{bmatrix},$$

to itself, which does not equal to $S(A)$*.*

5 Conclusion

It is interesting to allow both fuzziness and randomness in an AF. This paper introduces a model of AFs including the two. Firstly, we introduced the concept PF-matrices and applied it to define probabilistic-fuzzy argumentation frameworks. Secondly, we designed an algorithm for moderating PF-matrices of arguments in a PFAF. Then the probabilities of arguments' fuzzy statuses can be computed without layer decomposition. This new method provides a feasible way for modifying the probabilities of arguments' fuzzy statuses, especially in acyclic PFAFs. Moreover, it modifies the PF-matrices in finite steps, no more than the number of arguments. Finally, based on this algorithm, we established an extension-based semantic system of PFAFs. This provides a way for general PFAFs to check whether the probabilities are suitable for a certain semantics. The length of this process does not exceed the number of arguments in PFAF either.

This paper contributes on the following three aspects: First, we introduced PF-matrices that can describe both fuzzy and stochastic arguments with finitely many fuzzy statuses. Based on PF-matrices, we formally defined PFAFs that allow for such arguments in abstract argumentation. Second, we designed a procedure for revising PF-matrices. The revision of PF-matrices enables us to detect inconsistency between fuzzy and stochastic arguments. Third, using inconsistent arguments as conflicts between arguments, we established extension semantics for PFAFs that parallels to Dung's AF semantics.

One of the limitations of this work is that arguments have discrete fuzzy statuses. Clearly continuous fuzzy statuses would make PFAFs more capable; Another weakness is that arguments in PFAFs are assumed to be independent. In real-world situations, arguments can be dependent on other arguments.Third, in the current revision algorithm, the revised PF-matrix of an attacked argument is determined by the PF-matrix of the attacking argument only, and the information in the original PF-matrix of the attacked argument is somewhat neglected (apart from providing an upper bound of the revised PF-matrix). Further investigation on the problems all gives us directions for future work.

Acknowledgement

The authors wish to express their appreciation for several excellent suggestions for improvements in this paper made by the referees.

References

[1] P. Baroni, M. Caminada, M. Giacomin, *An introduction to argumentation semantics*, The Knowledge Engineering Review, **26**(04) (2011), 365-410.

[2] C. da Costa Pereira, A. G. Tettamanzi, S. Villata, *Changing onei's mind: Erase or rewind? Possibilistic belief revision with fuzzy argumentation based on trust*, Proceedings of the 22nd International Joint Conference on Artificial Intelligence, Barcelona, (2011), 164-171.

[3] P. Dondio, *Multi-valued and probabilistic argumentation frameworks*, Proceedings of Computational Models of Argument, Pitlochry, (2014), 253-260.

[4] P. M. Dung, *On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games*, Artificial Intelligence, **77**(2) (1995), 321-357.

[5] P. M. Dung, P. M. Thang, *Towards (probabilistic) argumentation for jury-based dispute resolution*, Proceedings of Computational Models of Argument, Amsterdam, (2010), 171-182.

[6] A. Hunter, *A probabilistic approach to modeling uncertain logical arguments*, International Journal of Approximate Reasoning, **54**(1) (2013), 47-81.

[7] A. Hunter, *Probabilistic qualification of attack in abstract argumentation*, International Journal of Approximate Reasoning, **55**(2) (2014), 607-638.

[8] A. Hunter, M. Thimm, *Probabilistic reasoning with abstract argumentation frameworks*, Journal of Artificial Intelligence Research, **59** (2017), 565-611.

[9] J. Janssen, M. De Cock, D. Vermeir, *Fuzzy argumentation frameworks*, In Information Processing and Management of Uncertainty in Knowledge-based Systems, Torremolinos, (2008), 513-520.

[10] S. Kaci, C. Labreuche, *Argumentation framework with fuzzy preference relations*, In International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, Dortmund, (2010), 554-563.

[11] H. Li, N. Oren, T. J. Norman, *Probabilistic argumentation frameworks*, In International Workshop on Theories and Applications of Formal Argumentation, Barcelona, (2011), 1-16.

[12] J. Mo, H. L. Huang, *Continuous probability-interval valued fuzzy preference relations and its application in group decision making*, Iranian Journal of Fuzzy Systems, **18**(4) (2021), 185-200.

[13] A. Saha, A. R. Mishra, P. Rani, T. Senapati, R. R. Yager, *A dual probabilistic linguistic MARCOS approach based on generalized Dombi operator for decision-making*, Iranian Journal of Fuzzy Systems, **20**(2) (2022), 83-102.

[14] J. Wu, *A Boolean model for conflict-freeness in argumentation frameworks*, AIMS Mathematics, **8**(2) (2023), 3913-3919.

[15] J. Wu, H. Li, *Probabilistic three-valued argumentation frameworks*, Proceedings of the 3rd International Conference on Logic and Argumentation, Hangzhou, (2020), 308-323.

[16] J. Wu, L. Li, W. Sun, *Gödel semantics of fuzzy argumentation frameworks with consistency degrees*, AIMS Mathematics, **5**(4) (2020), 4045-4064.

[17] M. Yadegari, S. A. Seyedin, *Rule-based joint fuzzy and probabilistic networks*, Iranian Journal of Fuzzy Systems, **17**(3) (2020), 135-149.

[18] S. Zhao, J. Wu, *An efficient algorithm of fuzzy reinstatement labelling*, AIMS Mathematics, **7**(6) (2022), 11165-11187.