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Original paper

Consistency fuzzy cross entropy based VIKOR approach for multi-criteria decision making

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Abstract

The primary aim of the notion of consistency fuzzy set (CFS) is to model the uncertain information given in a fuzzy multi environment and so to obtain meaningful data from the fuzzy multi-sets and to present this data in a compact form via some statistical tools. The data collected by fuzzy multi-sets are processed via CFSs and a sort of data science is done in the fuzzy environment. In this paper, we present a cross entropy measure and a sine entropy measure between CFSs to contribute to the processing and modeling of data obtained by statistical methods. Also, we construct a new VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method by using CFSs that is called CF-VIKOR to carry out decision process in multi-criteria decision making. Finally, we give a comparison analysis between the obtained results and the existing results in literature with the help of Spearman's, Kendall's rank correlation coefficient measures and Wang and Triantaphyllou validity test to show the efficiency and rationality of the proposed CF-VIKOR method.

Keywords: Consistency fuzzy set, fuzzy cross entropy measure, fuzzy sine entropy measure, VIKOR method, multi-criteria decision making.

1 Introduction

The notion of fuzzy set (FS) was proposed by Zadeh [68] as a strong tool to express ambiguous information in decision making. A FS is described by a membership function that is defined from a certain set to closed interval $[0,1]$. Later, this concept has been extended to some other FSs to better model vague information (see, e.g., [3, 4, 10, 71]) and many new decision making applications of these FSs have been presented (see, e.g.,[41, 63]). Also more sensitive FSs have been obtained by integrating these FSs with interval mathematics and linguistic variables (see, e.g., [16, 70]). However, decision makers (DMs) have problems in determining the belonging of an element to a set and FSs are incapable of capturing DM's hesitancy in many decision making problems. To overcome this problem, Torra [54] developed the concept of hesitant fuzzy set (HFS). The membership function is defined from the universal set to the family of all subsets of the closed interval $[0,1]$ in HFS setting. Therefore, the membership degree of an element to a set can be written as a subset of $[0,1]$ instead of an element of $[0,1]$. As a result, a HFS models uncertainty data better than the most type of the FSs, thanks to its ability to assign more than one degree of belonging to the elements of the universal set. However, repetitive information is ignored in a HFS. In order to prevent the loss of information, Yager [62] proposed the concept of fuzzy multi-set (FMS). The main characteristic of a FMS is that the membership degree of an element can be expressed as a sequence rather than an exact number in $[0,1]$. This concept not only solves the problem arising from the indecision of the DMs in decision making, but also is used in group decision making by corresponding each term of the sequence to a DM's decision. It actually increases the uncertainty in the set and makes it difficult to make

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decision, although the increase in the length of the sequence seems to increase the precision in modeling uncertain data. Therefore, there is a need for a new fuzzy set concept that models uncertain information and extract meaningful data from sequences regardless of sequence length. Trkarslan et. al. [55] have proposed the concept of consistency fuzzy set (CFS) which is expressed as an ordered pair whose components are the arithmetic mean and the consistency degree of the sequences of a FMS, respectively.

Recently, data science which has become increasingly important aims to collect, process and extract meaningful information from data and it unites statistic, mathematics and some computing science. A FMS contains a various data with sequences and it is possible to obtain meaningful data from these sequences via statistical tools such as arithmetic mean and standard deviation. On the other hand, the purpose of the use of a CFS is to do data science in fuzzy environment using some statistical methods. Ye et al. [65] studied on this concept in neutrosophic environment and Du and Ye [14] have merged this concept with cubic sets. A CFS reduces the information's reliance on the length of the sequence in an FMS and presents the information carried by the sequence in an FMS in a more compact form. Therefore, a CFS facilitates meaningful information extraction and reduces the effort of calculating information measures such as entropy and cross entropy used in decision making methods. We give Table 1 to compare CFSs with some existing FSs.

Table 1: Comparison of CFS with some existing FSs

Kind of fuzzy set	Multi-valued membership degree	Model hesitancy of DMs	Keep repetitive information	Contain statistical information	Represent compact information
FS [68]	×	×	×	×	×
HFS [54]	√	√	×	×	×
FMS [62]	√	√	√	×	×
CFS [55]	√	√	√	√	√

The concepts of entropy and cross entropy are important measurement methods used in the information theory and these concepts were proposed by Shannon [51]. Then the concept of cross entropy was improved by Kullback and Leibler [23] and it was modified by Lin [28]. Briefly, these concepts are related to quantity of the information and they measure the uncertainty. On the other hand, Zadeh [69] extended Shannon's entropy to the concept of fuzzy entropy. These entropy concepts look like related with each other, since they were constructed in order to measure uncertain information. However, there is a major difference between Shannon's entropy measures and fuzzy entropy measures. In fact, the later deals with vague and ambiguous uncertainties (non-probability), while the former tackles probabilistic uncertainties (randomness) [40]. The concept of fuzzy entropy is a fuzzy information measure which is defined by measuring the vagueness of a fuzzy event or a FS. Since Zadeh defined the concept of fuzzy entropy, various types of entropy have been defined in different FS settings in information theory. For example, De Luca and Termini [11] determined axioms of fuzzy entropy and proposed a fuzzy entropy that is based on Shannons function. Zhang et al. [72] have introduced some new entropy measures based on distance for interval-valued intuitionistic fuzzy sets. Cui and Ye [8] have proposed root entropy for simplified neutrosophic sets. Taruna et al. [53] have developed parametric generalized exponential entropy measure for intuitionistic vague sets. Morover, Ye et al. [66] introduced the concept of enthalpy by using the notion of entropy and applied it to multi-criteria decision making (MCDM). The concept of fuzzy cross entropy measure between FSs was introduced by Shang and Jiang [50]. This concept is a type of a fuzzy information measure and it can be considered as the degree of discrimination of two FSs. The concept has been widely used in fuzzy environment and it has been applied different fields. For example, Hu et al. [18] have studied on a novel object tracking algorithm by fusing color and depth for fuzzy cross entropy of single valued neutrosophic sets. Xu and Xia [61] have introduced the concepts of entropy and cross-entropy for hesitant fuzzy information and Wei [59] has proposed a new cross entropy measure for picture fuzzy sets and applied it to MCDM problems.

Many useful methods [22, 30, 44] have been developed by researchers to deal with MCDM problems at different fuzzy environment such as Complex Proportional Assessment (COPRAS) [48], Multi-Objective Optimization by a Ratio Analysis plus the Full Multiplicative Form (MULTIMOORA) [6, 42], Preference Ranking Organization Method for Enrichment of Evaluation (PROMETHEE) [19, 35], Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [5, 24], The Weighted Aggregates Sum Product Assessment (WASPAS) [32], Elimination Et Choice Translating Reality (ELECTRE) [2], VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR). In this paper we deal with the VIKOR method.

The VIKOR method [36] is a multi-criteria optimization and a MCDM method. The purpose of this method is to find a compromise solution which is a feasible solution closest to the ideal solution and rank the alternatives by

Table 2: Literature survey in different fuzzy settings for VIKOR approach

References	Approaches	Applications
Luo and Wang, 2017 [31]	intuitionistic fuzzy VIKOR	material handling selection problem
Wu et al., 2019 [60]	interval-valued intuitionistic fuzzy VIKOR	financing risk assessment of rural tourism projects
Liao and Xu, 2013 [27]	hesitant fuzzy VIKOR	the service quality among domestic airlines
Kutlu and Kahraman, 2019 [25]	spherical fuzzy VIKOR	warehouse site selection
Riaz and Tehrim, 2021 [45]	bipolar fuzzy VIKOR	laptop selection
Zhou and Chen, 2021 [74]	Pythagorean fuzzy VIKOR	the blockchain technology providers selection
Sarkar and Biswas, 2022[49]	Pythagorean fuzzy VIKOR	strategy evaluation
Salimian and Mousavi, 2022 [47]	Interval-valued intuitionistic fuzzy VIKOR	Digital Technology Strategies in Covid-19 Pandemic
Khan et.al., 2022 [21]	circular intuitionistic fuzzy VIKOR	health care waste disposal problems.
Ünver et al., 2022 [56]	intuitionistic fuzzy valued neutrosophic VIKOR	candidate selection problem
Yue et al., 2023 [67]	Interval-valued intuitionistic fuzzy VIKOR	software product quality assessment

taking into account the criteria. In the literature, there are many studies that compare the VIKOR method with some existing methods. For example, Opricovic and Tzeng [38] compared the VIKOR method and TOPSIS method from the perspective of aggregation function and found that although both methods calculate the distance of an alternative to the ideal solution the VIKOR method provides a compromise solution by mutual concessions, while the TOPSIS method obtains a solution with the farthest distance from the negative ideal solution and the shortest distance from the positive ideal solution without considering the relative importance degrees of these distances. Moreover, Opricovic and Tzeng [39] compared an extension of the VIKOR method with the TOPSIS, ELECTRE and PROMETHEE methods and revealed that the VIKOR method is superior in meeting conflicting and non-commensurable attributes. Later, a fuzzy VIKOR method [37] which relates to distance of an alternative to the ideal solution has been developed to solve MCDM problems in a fuzzy environment. Moreover, VIKOR method has been compared to other MCDM methods. For example, Zlaugotne et al. [76] have presented a study comparing VIKOR, TOPSIS, MULTIMOORA, PROMETHEE, COPRAS methods. As a result, they obtained that it is not really objective to compare the results obtained by different methods because results are similar but not the same. Moreover, Ceballos et al. [7] presented a comparative analysis for TOPSIS, VIKOR, MULTIMOORA and they reached that VIKORs ranking is very sensitive to the parameter γ which is the coefficient of the decision mechanism. Moreover, Dey at al. [13] and Lin et al. [29] presented a comparison of TOPSIS and VIKOR methods in various ranking problems.

According to Table 2, studies on the VIKOR method for different FSs are up-to-date and continuing. Moreover, the concept of entropy is often used in the VIKOR method to determine weights of criteria in MCDM process. For example, Fei et. al. [17] have proposed Demster Shafer VIKOR method based on Deng entropy. Mohsen and Fereshteh [34] have introduced an extended VIKOR method based on entropy measure for the failure modes risk assessment. Rani et. al. [43] have proposed a VIKOR approach based on entropy and divergence measures for Pythagorean fuzzy sets. Furthermore, the concept of fuzzy cross entropy is preferred by researchers in the VIKOR method. For example, Liang et. al. [26] have used a cross entropy measure for Pythagorean fuzzy sets to determine weights of criteria in VIKOR method. Zhao et. al. [73] have proposed an extended VIKOR method based on a fuzzy cross entropy measure between interval valued intuitionistic fuzzy sets. In this decision process, the proposed cross entropy measure is used to determine decision matrix. Rogulj et. al. [46] have proposed hybrid MCDM method based on VIKOR method and cross entropy measure for neutrosophic rough sets. In [46], the proposed cross entropy measure was used directly to rank alternatives. Wang et. al. [58] have presented a new VIKOR method based on entropy and cross-entropy measures for multi-valued neutrosophic sets. But there is no existing study in the literature that investigates the combination of the VIKOR method and fuzzy cross entropy measure for CFSs.

In this paper, we aim to present a novel VIKOR method based on an entropy and a cross entropy in the CFS environment. For this purpose, we present a fuzzy cross entropy measure and a fuzzy sine entropy measure for CFSs. Then, we extend the classical fuzzy VIKOR method to CF-VIKOR method with the help of a fuzzy cross entropy measure for CFSs. Later, we apply the CF-VIKOR method to a MCDM problem from the literature. Moreover, we use a fuzzy sine entropy measure for CFSs to determine unknown weights of criteria in the MCDM problem. Finally, to demonstrate the feasibility and effectiveness of the proposed method, we present a comparison analysis with the help of some rank correlation tests, and we conduct a validity test.

The rest of the paper is organized as follows. In Section 2, we recall the concepts of FMS, CFS and consistency fuzzy element (CFE) and introduce some set theoretic and algebraic operations for CFSs. In Section 3, we introduce a fuzzy cross entropy measure and a fuzzy sine entropy measure for CFSs and give some theoretical information. In Section 4, we propose the CF-VIKOR method with the help of the proposed cross entropy measure and consistency fuzzy sine entropy measure. We also give an application on MCDM to show the applicability of the improved method by considering a problem from the literature. Moreover, we give the sensitivity analysis of the proposed CF-VIKOR method. In Section 6, we compare the obtained results with some existing ones. We also solve the same problem by using some distance measures in the CF-VIKOR method and compare the results. In Section 7, we conclude the paper.

2 Preliminaries

In this section, the concepts of FMS and CFS are recalled. Then, the basic relations and operations for CFSs are proposed.

Definition 2.1. [33] *Let $X = \{x_1, ...x_m\}$ be a finite set. A FMS on X is given as*

$$\Omega = \left\{ \left\langle x_j, (\mu_\Omega^1(x_j), ..., \mu_\Omega^{n_j}(x_j)) \right\rangle : j = 1, ..., m \right\}, \tag{1}$$

where $(\mu_\Omega^1(x_j), ..., \mu_\Omega^{n_j}(x_j))$ is the membership sequence of the $j-th$ term and n_j is the length of the membership sequence of x_j.

Let

$$\Omega = \left\{ \left\langle x_j, (\mu_\Omega^1(x_j), ..., \mu_\Omega^{n_j}(x_j)) \right\rangle : j = 1, ..., m \right\},$$

and

$$\Phi = \left\{ \left\langle x_j, (\mu_\Phi^1(x_j), ..., \mu_\Phi^{n_j}(x_j)) \right\rangle : j = 1, ..., m \right\},$$

be two FMSs. Now we recall some basic relations and operations for FMSs [33]:

1. $\Omega \subseteq \Phi \Leftrightarrow \mu_\Omega^k(x_j) \leq \mu_\Phi^k(x_j)$, for any $k = 1, ..., n_j$, for any $j = 1, ..., m$,
2. $\Omega = \Phi \Leftrightarrow \mu_\Omega^k(x_j) = \mu_\Phi^k(x_j)$, for any $k = 1, ..., n_j$, for any $j = 1, ..., m$,
3. For any $k = 1, ..., n_j$, for any $j = 1, ..., m$, $\mu_{\Omega\cup\Phi}^k(x_j) = \mu_\Omega^k(x_j) \vee \mu_\Phi^k(x_j)$,
4. For any $k = 1, ..., n_j$, for any $j = 1, ..., m$, $\mu_{\Omega\cap\Phi}^k(x_j) = \mu_\Omega^k(x_j) \wedge \mu_\Phi^k(x_j)$,
5. $\Omega \oplus \Phi = \left\{ \left\langle x_j, (\mu_\Omega^1(x_j), ..., \mu_\Omega^{n_j}(x_j), \mu_\Phi^1(x_j), ..., \mu_\Phi^{n_j}(x_j)) \right\rangle : j = 1, ..., m \right\}$.

where the symbols " $\vee$ " and " $\wedge$ " are the maximum and minimum operations, respectively.
Now, we recall the concept of CFS.

Definition 2.2. [55] *Let $X = \{x_1, ...x_m\}$ be a finite set and let Ω be a FMS in X. The average value and the consistency degree of the membership sequence in Ω are defined as*

$$m_\Omega^{(j)} = \frac{1}{n_j} \sum_{k=1}^{n_j} \overset{k}{\mu}_\Omega(x_j), \tag{2}$$

and

$$c_\Omega^{(j)} = 1 - \sigma_\Omega^{(j)}, \tag{3}$$

for each $j = 1, ..., m$ where $\sigma_\Omega^{(j)} = \sqrt{\frac{1}{n_j - 1} \sum_{k=1}^{n_j} (m_\Omega^{(j)} - \overset{k}{\mu}_\Omega(x_j))^2}$ is the standard deviation of the jth membership sequence in Ω. A CFS C_Ω is defined as

$$C_\Omega = \left\{ \left\langle x_j, (m_\Omega^{(j)}, c_\Omega^{(j)}) \right\rangle : j = 1, ..., m \right\}. \tag{4}$$

Moreover, $\left\langle x_j, (m_\Omega^{(j)}, c_\Omega^{(j)}) \right\rangle$ for fixed $j = 1, ..., m$ is called as a CFE.

Now, we define some basic relations and operations for CFSs as follows:

Definition 2.3. *Let $X = \{x_1, ...x_m\}$ be a finite set, let*

$$\Omega = \left\{ \left\langle x_j, (\mu_\Omega^1(x_j), ..., \mu_\Omega^{n_j}(x_j)) \right\rangle : j = 1, ..., m \right\},$$

and

$$\Phi = \left\{ \left\langle x_j, (\mu_\Phi^1(x_j), ..., \mu_\Phi^{n_j}(x_j)) \right\rangle : j = 1, ..., m \right\},$$

be two FMSs and let

$$C_\Omega = \left\{ \left\langle x_j, (m_\Omega^{(j)}, c_\Omega^{(j)}) \right\rangle : j = 1, ..., m \right\},$$

and

$$C_\Phi = \left\{ \left\langle x_j, (m_\Phi^{(j)}, c_\Phi^{(j)}) \right\rangle : j = 1, ..., m \right\},$$

be CFSs corresponding to Ω and Φ, respectively.

1. $C_\Omega \subseteq C_\Phi \Leftrightarrow m_\Omega^{(j)}(x_j) \leq m_\Phi^{(j)}(x_j)$ *and* $c_\Omega^{(j)}(x_j) \leq c_\Phi^{(j)}(x_j)$, *for any* $j = 1, ..., m$,
2. $C_\Omega = C_\Phi \Leftrightarrow m_\Omega^{(j)}(x_j) = m_\Phi^{(j)}(x_j)$ *and* $c_\Omega^{(j)}(x_j) = c_\Phi^{(j)}(x_j)$, *for any* $j = 1, ..., m$,
3. *For any* $k = 1, ..., n_j$, *for any* $j = 1, ..., m$, $m_{C_\Omega \cup C_\Phi}^{(j)}(x_j) = m^{(j)}(\mu_\Omega^k(x_j) \vee \mu_\Phi^k(x_j))$ *and* $c_{C_\Omega \cup C_\Phi}^{(j)}(x_j) = c^{(j)}(\mu_\Omega^k(x_j) \vee \mu_\Phi^k(x_j))$,
4. *For any* $k = 1, ..., n_j$, *for any* $j = 1, ..., m$, $m_{C_\Omega \cap C_\Phi}^{(j)}(x_j) = m^{(j)}(\mu_\Omega^k(x_j) \wedge \mu_\Phi^k(x_j))$ *and* $c_{C_\Omega \cap C_\Phi}^{(j)}(x_j) = c^{(j)}(\mu_\Omega^k(x_j) \wedge \mu_\Phi^k(x_j))$, *for any* $k = 1, ..., n_j$, *for any* $j = 1, ..., m$,
5. $C_\Omega \oplus C_\Phi = \left\{ \left\langle x_j, \begin{matrix} (m^{(j)}(\mu_\Omega^1(x_j), ..., \mu_\Omega^{n_j}(x_j), \mu_\Phi^1(x_j), ..., \mu_\Phi^{n_j}(x_j)), \\ c^{(j)}(\mu_\Omega^1(x_j), ..., \mu_\Omega^{n_j}(x_j), \mu_\Phi^1(x_j), ..., \mu_\Phi^{n_j}(x_j))) \end{matrix} \right\rangle : j = 1, ..., m \right\}$,
6. *The complement C_Ω is given as*

$$C_\Omega^c := \left\{ \left\langle x_j, (1 - m_\Omega^{(j)}, c_\Omega^{(j)}) \right\rangle : j = 1, ..., m \right\}. \tag{5}$$

It is clear that $C_\Omega \cup C_\Phi$, $C_\Omega \cap C_\Phi$, $C_\Omega \oplus C_\Phi$ and C_Ω^c are CFSs.

3 Consistency fuzzy cross entropy measure and consistency fuzzy sine entropy measure

In this section, motivated by [64] we provide a consistency fuzzy cross entropy measure and we define its weighted version. Then, we propose a fuzzy sine entropy measure for CFSs.

Definition 3.1. *Let $X = \{x_1, ..., x_m\}$ be a finite set and let*

$$C_\Omega = \left\{ \left\langle x_j, (m_\Omega^{(j)}, c_\Omega^{(j)}) \right\rangle : j = 1, ..., m \right\},$$

and

$$C_\Phi = \left\{ \left\langle x_j, (m_\Phi^{(j)}, c_\Phi^{(j)}) \right\rangle : j = 1, ..., m \right\},$$

be two CFSs in X. Then a consistency cross entropy measure between Ω from Φ is given with

$$\begin{aligned} E(\Omega, \Phi) = &\frac{1}{m} \sum_{j=1}^{m} \left[m_\Omega^{(j)} \log_2 \frac{m_\Omega^{(j)}}{(m_\Omega^{(j)} + m_\Phi^{(j)})/2} + (1 - m_\Omega^{(j)}) \log_2 \frac{1 - m_\Omega^{(j)}}{1 - (m_\Omega^{(j)} + m_\Phi^{(j)})/2} \right] \\ &+ \frac{1}{m} \sum_{j=1}^{m} \left[c_\Omega^{(j)} \log_2 \frac{c_\Omega^{(j)}}{(c_\Omega^{(j)} + c_\Phi^{(j)})/2} + (1 - c_\Omega^{(j)}) \log_2 \frac{1 - c_\Omega^{(j)}}{1 - (c_\Omega^{(j)} + c_\Phi^{(j)})/2} \right]. \end{aligned}$$

Definition 3.2. *A weighted consistency cross entropy measure between* Ω *from* Φ *is given with*

$$\begin{aligned} WE(\Omega,\Phi) =& \sum_{j=1}^{m}\omega_j\left[m_\Omega^{(j)}\log_2\frac{m_\Omega^{(j)}}{(m_\Omega^{(j)}+m_\Phi^{(j)})/2}+(1-m_\Omega^{(j)})\log_2\frac{1-m_\Omega^{(j)}}{1-(m_\Omega^{(j)}+m_\Phi^{(j)})/2}\right]\\ &+\sum_{j=1}^{m}\omega_j\left[c_\Omega^{(j)}\log_2\frac{c_\Omega^{(j)}}{(c_\Omega^{(j)}+c_\Phi^{(j)})/2}+(1-c_\Omega^{(j)})\log_2\frac{1-c_\Omega^{(j)}}{1-(c_\Omega^{(j)}+c_\Phi^{(j)})/2}\right],\end{aligned}$$

where the importance of each element $x_j \in X$ *is* $\omega_j \in [0,1]$ *for* $j = 1, 2, ..., m$ *with* $\sum_{j=1}^{m} w_j = 1$.

Proposition 3.3. *The consistency cross entropy measure* E *satisfies the following properties:*
($\mathbf{P_1}$) $E(\Omega,\Phi) \geq 0$,
($\mathbf{P_2}$) $E(\Omega,\Phi) = E(\Omega^c,\Phi^c)$,
($\mathbf{P_3}$) $E(\Omega,\Phi) = 0$ *if and only if* $m_\Omega^{(j)} = m_\Phi^{(j)}$ *and* $c_\Omega^{(j)} = c_\Phi^{(j)}$, *for* $j = 1, ..., m$.

Proof. ($\mathbf{P_1}$) Based on the fuzzy cross entropy properties [50] (see, p. 429), we have the first sum as well as the second sum non-negative which yields that $E(\Omega,\Phi) \geq 0$.
($\mathbf{P_2}$) We have

$$\begin{aligned} E(\Omega,\Phi) =& \frac{1}{m}\sum_{j=1}^{m}\left[m_\Omega^{(j)}\log_2\frac{m_\Omega^{(j)}}{(m_\Omega^{(j)}+m_\Phi^{(j)})/2}+(1-m_\Omega^{(j)})\log_2\frac{1-m_\Omega^{(j)}}{1-(m_\Omega^{(j)}+m_\Phi^{(j)})/2}\right]\\ &+\frac{1}{m}\sum_{j=1}^{m}\left[c_\Omega^{(j)}\log_2\frac{c_\Omega^{(j)}}{(c_\Omega^{(j)}+c_\Phi^{(j)})/2}+(1-c_\Omega^{(j)})\log_2\frac{1-c_\Omega^{(j)}}{1-(c_\Omega^{(j)}+c_\Phi^{(j)})/2}\right]\\ =&\frac{1}{m}\sum_{j=1}^{m}\left[(1-m_\Omega^{(j)})\log_2\frac{1-m_\Omega^{(j)}}{1-(m_\Omega^{(j)}+m_\Phi^{(j)})/2}+m_\Omega^{(j)}\log_2\frac{m_\Omega^{(j)}}{(m_\Omega^{(j)}+m_\Phi^{(j)})/2}\right]\\ &+\frac{1}{m}\sum_{j=1}^{m}\left[c_\Omega^{(j)}\log_2\frac{c_\Omega^{(j)}}{(c_\Omega^{(j)}+c_\Phi^{(j)})/2}+(1-c_\Omega^{(j)})\log_2\frac{1-c_\Omega^{(j)}}{1-(c_\Omega^{(j)}+c_\Phi^{(j)})/2}\right]\\ =&E(\Omega^c,\Phi^c).\end{aligned}$$

($\mathbf{P_3}$) For $j = 1, ..., m$, if $m_\Omega^{(j)} = m_\Phi^{(j)}$ and $c_\Omega^{(j)} = c_\Phi^{(j)}$ then, $\log_2 1 = 0$. Therefore, we have $E(\Omega,\Phi) = 0$. Conversely, if $E(\Omega,\Phi) = 0$, then we yield that $m_\Omega^{(j)} = m_\Phi^{(j)}$ and $c_\Omega^{(j)} = c_\Phi^{(j)}$, for $j = 1, 2, ..., m$ (see, [50]). □

Obviously, the weighted consistency cross entropy measure WE satisfies $P_{1,2,3}$.

Now, we introduce a sine entropy measure for CFSs motivating from [9].

Definition 3.4. *Let* X *be a finite set and let* $C_\Omega = \{c_1, ..., c_m\}$ *be a CFS in* X, *where* $c_j = \left\langle (m_\Omega^{(j)}, c_\Omega^{(j)})\right\rangle$ $(j = 1, ..., m)$ *is the* $j-th$ *CFE in* C_Ω. *Then a fuzzy sine entropy measure of* C_Ω *is defined as follows:*

$$S(C_\Omega) = \frac{1}{2m}\sum_{j=1}^{m}\left[\sin\left(m_\Omega^{(j)}\pi\right)+\sin\left(c_\Omega^{(j)}\pi\right)\right]. \tag{6}$$

The fuzzy sine entropy measure S has the following properties:

Proposition 3.5. *The fuzzy sine entropy measure* S *satisfies the following properties:*
($\mathbf{P_1'}$) $S(C_\Omega) = 0$ *if* C_Ω *is a crisp set i.e.,* $c_j = \langle(1,1)\rangle$ *or* $c_j = \langle(0,1)\rangle$ *or* $c_j = \langle(1,0)\rangle$ *or* $c_j = \langle(0,0)\rangle$;
($\mathbf{P_2'}$) $S(C_\Omega) = 1$ *if and only if* C_Ω *is the fuzziest CFS, i.e.,* $c_j = \langle(0.5, 0.5)\rangle$ *(the fuzziest CFE) for any* $j = 1, ..., m$.
($\mathbf{P_3'}$) $S(C_\Omega) = S(C_\Omega^c)$ *where* C_Ω^c *is the complement of* C_Ω.

Proof. ($\mathbf{P_1'}$) The proof is trivial.
($\mathbf{P_2'}$) Consider the function f defined by $f(z) = \sin z\pi$ for $z \in [0,1]$. Clearly, f is a concave function with the global

maximum value $f(z) = 1$ at $z = 0.5$. Therefore, $m_\Omega^{(j)} = c_\Omega^{(j)} = 0.5$ for $j = 1, ..., m$. Conversely, let $c_j = \langle(0.5, 0.5)\rangle$ Therefore, $m_\Omega^{(j)} = c_\Omega^{(j)} = 0.5$ for $j = 1, ..., m$ implies that $S(C_\Omega) = 1$.
($\mathbf{P_3'}$) Since the complement of the CFE $c_j = \left\langle (m_\Omega^{(j)}, c_\Omega^{(j)}) \right\rangle$ in C_Ω is $c_j^c = \left\langle (1 - m_\Omega^{(j)}, c_\Omega^{(j)}) \right\rangle$ in C_Ω and $\sin((1-m_\Omega^{(j)})\pi) = \sin(m_\Omega^{(j)}\pi)$, we obtain that $S(C_\Omega) = S(C_\Omega^c)$. □

Remark 3.6. *In Proposition 3.5, we see that* $c_j = \langle(1, 1)\rangle$ *or* $c_j = \langle(0, 1)\rangle$ *or* $c_j = \langle(1, 0)\rangle$ *or* $c_j = \langle(0, 0)\rangle$*, then the CFS is the crisp set. However,* $c_j = \langle(1, 0)\rangle$ *or* $c_j = \langle(0, 0)\rangle$ *are not meaningful in the CFS setting. On the other hand, if* $m_\Omega^{(j)} = c_\Omega^{(j)} = 0.5$ *for any* $j = 1, ..., m$*, then the CFS is the fuzziest set and if* $\sin\left(m_\Omega^{(j)}\pi\right) + \sin\left(c_\Omega^{(j)}\pi\right) \geq \sin\left(m_\Phi^{(j)}\pi\right) + \sin\left(c_\Phi^{(j)}\pi\right)$ *for any* $j = 1, .., m$*, then* Ω *is fuzzier than* Φ*. Figure 3.6 visually explains* P_1' *and* P_2' *of Proposition 3.5. Although* $S(C_\Omega) = 0$ *is provided theoretically in case of* $c_j = \langle(1, 0)\rangle$ *or* $c_j = \langle(0, 0)\rangle$ *in* P_1' *of Proposition 3.5, this is not physically possible. In case* $c_j = \langle(1, 0)\rangle$*, for a sequence with an average of one to have a degree of consistency of zero, the sequence must be a sequence of zeros, which contradicts the situation where the average of the sequence is one. Similarly, for a sequence with an average of zero to have a degree of consistency of zero, the sequence must be a sequence of ones, which contradicts the average of the sequence for* $c_j = \langle(0, 0)\rangle$*. Moreover, the inequality*

$$\sin\left(m_\Omega^{(j)}\pi\right) + \sin\left(c_\Omega^{(j)}\pi\right) \geq \sin\left(m_\Phi^{(j)}\pi\right) + \sin\left(c_\Phi^{(j)}\pi\right),$$

determines the fuzziness degree of the fuzzy sets. This fact is shown with level curves in Figure 3.6. Note that, higher level curves indicate fuzzier CFEs.

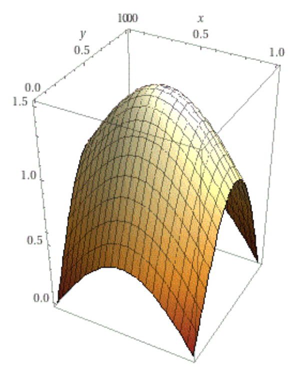

Figure 1: Illustration of the proposed fuzzy sine entropy measure

4 CF-VIKOR method for MCDM

In this section, we present a VIKOR method which is called the CF-VIKOR method that deals with the MCDM problems under consistency fuzzy environment. Then we apply it to a MCDM problem and we investigate the sensitivity analysis of the proposed method.

4.1 CF-VIKOR method

In this subsection, we present a stepwise algorithm for the CF-VIKOR method. Assume that the set of alternatives in an MCDM problem with unknown criteria weights is $\Psi = \{\Psi_1, ..., \Psi_m\}$ and the set of criteria is $A = \{a_1, ..., a_n\}$. The CF-VIKOR method can be given for MCDM problem with consistency fuzzy information as follows:

Step 1: Each alternative's appropriate evaluations $\Psi_i, (i = 1, ..., m)$ over criteria $a_j, (j = 1, ..., n)$ are represented by a FMS

$$\Psi_i = \left\{ \left\langle a_j, (\mu_{\Psi_i}^1(a_j), ..., \mu_{\Psi_i}^p(a_j)) \right\rangle : a_j \in A \right\},$$

and CFS C_{Ψ_i} are constructed by

$$C_{\Psi_i} = \left\{ \left\langle a_j, (m_{\Psi_i}^{(j)}, c_{\Psi_i}^{(j)}) \right\rangle : j = 1, ..., n \right\},$$

for each alternative Ψ_i $(i = 1, ..., m)$ from Definition 2.2.

Step 2: Construct the decision matrix $M = [b_{ij}]_{m \times n}$ for $i = 1, ..., m$ and $j = 1, ..., n$, where $b_{ij} = \left\langle a_j, \left(m_{C_{\Psi_i}}^{(j)}, c_{C_{\Psi_i}}^{(j)} \right) \right\rangle$ is a CFE for $j = 1, ..., n$. For each $j = 1, ..., n$ consider the criterion a_j as a CFS

$$a_j = \{ b_{ij} : i = 1, ..., m \}.$$

Then calculate $e_j := S(a_j)$.

Step 3: Calculate the unknown weights of each criterion using the fuzzy sine entropy weight model for CFSs shown below:

$$w_j = \frac{1 - e_j}{n - \sum_{k=1}^{n} e_k}.$$

Step 4: Compute the positive ideal solution a_j^+ and the negative ideal solution a_j^- for each criteria a_j as follows:

For benefit criteria;

$$a_j^+ = \left\langle a_j, \left(\max_i \left(m_{C_{\Psi_i}}^{(j)} \right), \max_i \left(c_{C_{\Psi_i}}^{(j)} \right) \right) \right\rangle,$$

and

$$a_j^- = \left\langle a_j, \left(\min_i \left(m_{C_{\Psi_i}}^{(j)} \right), \min_i \left(c_{C_{\Psi_i}}^{(j)} \right) \right) \right\rangle,$$

For cost criteria;

$$a_j^+ = \left\langle a_j, \left(\min_i \left(m_{C_{\Psi_i}}^{(j)} \right), \min_i \left(c_{C_{\Psi_i}}^{(j)} \right) \right) \right\rangle,$$

and

$$a_j^- = \left\langle a_j, \left(\max_i \left(m_{C_{\Psi_i}}^{(j)} \right), \max_i \left(c_{C_{\Psi_i}}^{(j)} \right) \right) \right\rangle.$$

Step 5: Compute the values S_i and R_i as follows:

$$S_i = \sum_{j=1}^{n} \omega_j \left(\frac{E(x_j^+, x_{ij})}{E(x_j^+, x_j^-)} \right), \qquad R_i = \max_j \omega_j \left(\frac{E(x_j^+, x_{ij})}{E(x_j^+, x_j^-)} \right),$$

for $i = 1, ..., m$, where E is the fuzzy cross entropy measure between CFSs defined in Definition 3.1. Then, compute the values of Q_i based on the results of S_i and R_i as follows:

$$Q_i = \gamma \left(\frac{S_i - S^*}{S^- - S^*} \right) + (1 - \gamma) \left(\frac{R_i - R^*}{R^- - R^*} \right),$$

where $S^- = \max_i S_i$, $S^* = \min_i S_i$, $R^- = \max_i R_i$ and $R^* = \min_i R_i$, for $i = 1, ..., m$.

γ is the coefficient of decision mechanism. The compromise solution can be elected by majority ($\gamma > 0.5$), consensus ($\gamma = 0.5$), or veto ($\gamma < 0.5$).

Step 6: Rank the values of S_i,R_i and Q_i with respect to descending order. An alternative with a lower score with respect to Q is a better alternative.

Step 7: If

$$Q(\Psi^{(2)}) - Q(\Psi^{(1)}) \geq \frac{1}{m-1},$$

and $\Psi^{(1)}$ is the best alternative with respect to S and R, then $\Psi^{(1)}$ is the compromise solution where $\left\{\Psi_{(i)}\right\}_{i=1}^{m}$ is the permutation of the alternatives such that $\Psi^{(1)}$ is the best ranked by Q (minimum).
If

$$Q(\Psi^{(2)}) - Q(\Psi^{(1)}) < \frac{1}{m-1},$$

then $\Psi^{(M)}$ is determined from

$$Q(\Psi^{(M)}) - Q(\Psi^{(1)}) < \frac{1}{m-1},$$

for maximum Ψ. So the alternatives $\Psi^{(1)}$,..., $\Psi^{(M)}$ are compromise solutions.
If $\Psi^{(1)}$ is not the best alternative with respect to S or R, then $\Psi^{(1)}$ and $\Psi^{(2)}$ are compromise solutions.

4.2 Numerical example

The following MCDM problem has been studied [65] under neutrosophic multi-valued set setting. Here this example is adapted by using only truth membership sequences of [65]. Throughout this section, we assume that $0 \log_2 0 := 0$ as in [1].

Example 4.1. *A customer wishes to purchase an appropriate car. There are four possible cars* $\Psi = \{\Psi_1, \Psi_2, \Psi_3, \Psi_4\}$ *and these cars are evaluated according to the criteria*

$$A = \{a_1\,(\textit{the fuel economy})\,, a_2\,(\textit{the price})\,, a_3\,(\textit{the amenity})\,, a_4\,(\textit{the safety})\}\,.$$

Step 1: *Let* Ψ_1, Ψ_2, Ψ_3 *and* Ψ_4 *be represented by following FMSs in criteria set* $A = \{a_1, a_2, a_3, a_4\}$*:*

$$\begin{aligned}
\Psi_1 &= \{\langle a_1, (0.5, 0.7)\rangle\,, \langle a_2, 0.4\rangle\,, \langle a_3, (0.7, 0.8)\rangle\,, \langle a_4, (0.1, 0.5)\rangle\}\,,\\
\Psi_2 &= \{\langle a_1, (0.7, 0.9)\rangle\,, \langle a_2, 0.7\rangle\,, \quad \langle a_3, 0.9\rangle\,, \quad \langle a_4, (0.5, 0.5)\rangle\}\,,\\
\Psi_3 &= \{\langle a_1, (0.3, 0.6)\rangle\,, \langle a_2, 0.2\rangle\,, \langle a_3, (0.6, 0.9)\rangle\,, \langle a_4, (0.4, 0.7)\rangle\}\,,\\
\Psi_4 &= \{\langle a_1, (0.8, 0.9)\rangle\,, \langle a_2, 0.3\rangle\,, \langle a_3, (0.1, 0.5)\rangle\,, \langle a_4, (0.4, 0.4)\rangle\}\,.
\end{aligned}$$

We obtain alternatives as CFSs as follows:

$$\begin{aligned}
C_{\Psi_1} &= \{\langle a_1, (0.6, 0.8586)\rangle\,, \langle a_2, (0.4, 1)\rangle\,, \langle a_3, (0.75, 0.9293)\rangle\,, \langle a_4, (0.3, 0.7172)\rangle\}\,,\\
C_{\Psi_2} &= \{\langle a_1, (0.8, 0.8586)\rangle\,, \langle a_2, (0.7, 1)\rangle\,, \langle a_3, (0.9, 1)\rangle\,, \langle a_4, (0.5, 1)\rangle\}\,,\\
C_{\Psi_3} &= \{\langle a_1, (0.45, 0.7879)\rangle\,, \langle a_2, (0.2, 1)\rangle\,, \langle a_3, (0.75, 0.7879)\rangle\,, \langle a_4, (0.55, 0.7879)\rangle\}\,,\\
C_{\Psi_4} &= \{\langle a_1, (0.85, 0.9293)\rangle\,, \langle a_2, (0.3, 1)\rangle\,, \langle a_3, (0.3, 0.7172)\rangle\,, \langle a_4, (0.4, 1)\rangle\}\,.
\end{aligned}$$

Step 2: *Construct the decision matrix:*

$$M = \begin{bmatrix}
\langle 0.6, 0.8586\rangle & \langle 0.4, 1\rangle & \langle 0.75, 0.9293\rangle & \langle 0.3, 0.7172\rangle\\
\langle 0.8, 0.8586\rangle & \langle 0.7, 1\rangle & \langle 0.9, 1\rangle & \langle 0.5, 1\rangle\\
\langle 0.45, 0.7879\rangle & \langle 0.2, 1\rangle & \langle 0.75, 0.7879\rangle & \langle 0.55, 0.7879\rangle\\
\langle 0.85, 0.9293\rangle & \langle 0.3, 1\rangle & \langle 0.3, 0.7172\rangle & \langle 0.4, 1\rangle
\end{bmatrix}.$$

Step 3: *The criteria weight vector is obtained as follows:*

$$W = (\omega_1, \omega_2, \omega_3, \omega_4) = (0.2233, 0.3256, 0.2590, 0.1921)\,.$$

Step 4: *First and second criteria are cost criteria and third and fourth ones are benefit criteria. Therefore, we have positive and negative ideal solution of each criteria as follows:*

$$
\begin{aligned}
a_1^+ &= \langle 0.45, 0.7879\rangle, & a_1^- &= \langle 0.85, 0.9293\rangle, \\
a_2^+ &= \langle 0.2, 1\rangle, & a_2^- &= \langle 0.7, 1\rangle, \\
a_3^+ &= \langle 0.9, 1\rangle, & a_3^- &= \langle 0.3, 0.7172\rangle, \\
a_4^+ &= \langle 0.55, 1\rangle, & a_4^- &= \langle 0.3, 0.7172\rangle.
\end{aligned}
$$

Step 5-6: *Compute the group utility value S, the individual regret value R and the compromise value Q for each alternative. (Let $\gamma = 0.5$.) Moreover, the ranking order of all the alternatives can be determined according to the decreasing order of S_i, R_i and Q_i values.*

Table 3: The values of S, R, and Q for all alternatives

	Ψ_1	Ψ_2	Ψ_3	Ψ_4	Ranking
S_i	0.3267	0.4742	0.2090	0.5106	$S_3 \succeq S_1 \succeq S_2 \succeq S_4$
R_i	0.1921	0.3256	0.1170	0.2590	$R_3 \succeq R_1 \succeq R_4 \succeq R_2$
$(Q_i)_{\gamma=0.5}$	0.3750	0.9396	0	0.8403	$Q_3 \succeq Q_1 \succeq Q_4 \succeq Q_2$

Step 7: *In Table 3, it can be seen that $Q_3 \succeq Q_1 \succeq Q_4 \succeq Q_2$, which means that Ψ_3 is the best ranked value in the context of Q. Moreover,*

$$Q(\Psi^{(2)}) - Q(\Psi^{(1)}) = 0.3750 \geq \frac{1}{(4-1)},$$

and so Ψ_3 is also the highest ranked value in terms of S_i and R_i, satisfying Conditions C_1 and C_2. This demonstrates that Ψ_3 is a one-of-a-kind compromise solution to the problem under consideration, which is both stable and advantageous. Table 4 and Figure 2 show the stability and uniqueness of the solution for various values of γ.

4.3 Sensitivity analysis

In this section, we examine how $\gamma \in [0, 1]$ affects the optimal solution. Sensitivity analysis demonstrates how the decision-making approach affects the outcome and is used to assess the long-term viability of the proposed method.

Table 4: Stability of the optimal solution for different values of γ

γ	Q_1	Q_2	Q_3	Q_4	Rankings	Optimal Solution
0.0	0.3598	1.0000	0.0000	0.6806	$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$	Ψ_3
0.1	0.3628	0.9879	0.0000	0.7125	$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$	Ψ_3
0.2	0.3658	0.9758	0.0000	0.7444	$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$	Ψ_3
0.3	0.3689	0.9638	0.0000	0.7764	$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$	Ψ_3
0.4	0.3719	0.9517	0.0000	0.8083	$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$	Ψ_3
0.5	0.3750	0.9396	0.0000	0.8403	$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$	Ψ_3
0.6	0.3780	0.9276	0.0000	0.8722	$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$	Ψ_3
0.7	0.3810	0.9155	0.0000	0.9041	$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$	Ψ_3
0.8	0.3841	0.9034	0.0000	0.9361	$\Psi_3 \succeq \Psi_1 \succeq \Psi_2 \succeq \Psi_4$	Ψ_3
0.9	0.3871	0.8914	0.0000	0.9680	$\Psi_3 \succeq \Psi_1 \succeq \Psi_2 \succeq \Psi_4$	Ψ_3
1.0	0.3902	0.8793	0.0000	1.0000	$\Psi_3 \succeq \Psi_1 \succeq \Psi_2 \succeq \Psi_4$	Ψ_3

Table 4 indicates the sensitivity analysis of Example 4.1. There are two situations in Table 4. If $0 \leq \gamma \leq 0.7$, then the order of the ranking is $\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$ and if $0.8 \leq \gamma \leq 1.0$ then the order of the ranking is $\Psi_3 \succeq \Psi_1 \succeq \Psi_2 \succeq \Psi_4$. It is seen that the change of parameters causes different rankings. This is to be expected. Ceballos et al. [7] proved that the results of VIKOR method are affected by the parameter. Though the ranking outcome is impressed by γ, Ψ_3 is always the best alternative and Ψ_1 is the second best alternative. In such cases, the consequences of $\gamma = 0.5$ are usually considered. Therefore, the sensitivity analysis shows that the conclusion reached by our method is stable and efficient. Figure 2 demonstrates the comparison values of four alternatives under the parameter γ.

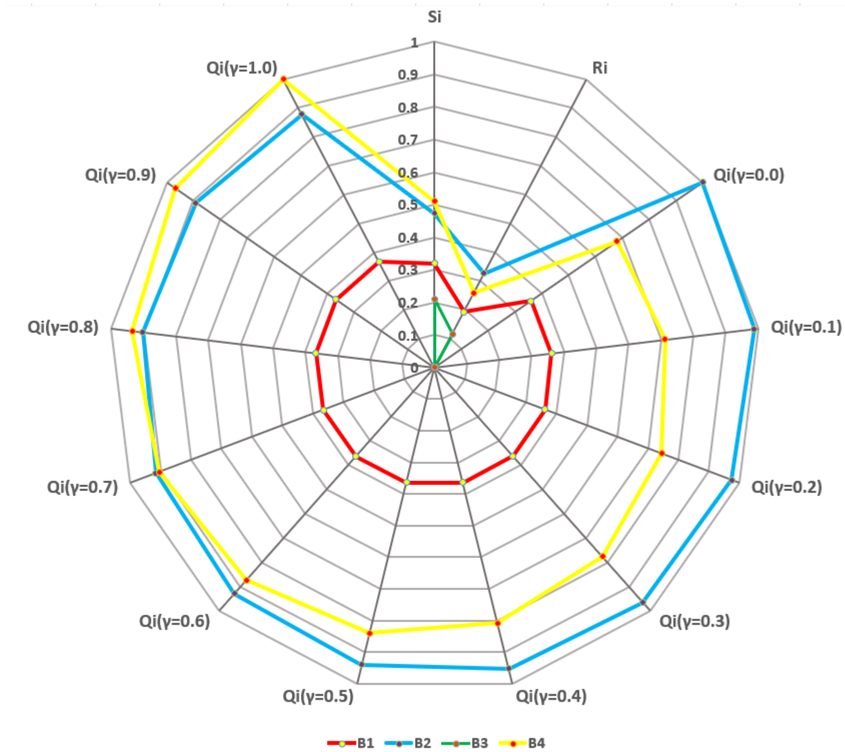

Figure 2: Sensitivity analysis for the alternatives under CFSs

5 Validity analysis of the proposed VIKOR method

The relationship between alternatives, the consistence of criteria, and impartial evaluations among decision makers all influence the MCDM method. Therefore, Wang and Triantaphyllou [57] proposed three test criteria for validating the presented method in MCDM. These three test conditions are applied to the proposed method in steps as follows in [57]:

First Condition: The best alternative does not change when a non-optimal alternative is replaced with a worse alternative without changing the importance of each criterion.

Second Condition: A useful MCDM method should have transitive property.

Third Condition: The order of the subproblems must match the order of the original problem when we divide the MCDM problem into smaller subproblems.

The proposed VIKOR method's validity is investigated stepwise as follows:

First Condition: We can select a non-optimal alternative Ψ_2 and then replace it with an arbitrary worse alternative $\hat{\Psi_2}$. While determining $\hat{\Psi_2}$, it is taken into account that the criteria are benefit and cost. Moreover, the degree of belonging of the criteria are determined to be arbitrary worse than Ψ_2. Assume

$$\hat{\Psi_2} = \{\langle a_1, (0.9, 0.9)\rangle, \langle a_2, 0.8\rangle, \langle a_3, 0.6\rangle, \langle a_4, (0.2, 0.2)\rangle\}.$$

Now, we examine whether the best alternative is changing or not for $\gamma = 0.5$. When the proposed VIKOR method is applied, we obtain new alternative order as $\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \hat{\Psi_2}$. Thus, test condition 1 is valid.

Second and Third Conditions: We divide the given MCDM into sub-problems:
$\{\Psi_1, \Psi_2, \Psi_3\}$, $\{\Psi_2, \Psi_3, \Psi_4\}$, $\{\Psi_3, \Psi_4, \Psi_1\}$ and $\{\Psi_4, \Psi_1, \Psi_2\}$. The rank results of each sub-problem get as $\Psi_3 \succeq \Psi_1 \succeq \Psi_2$, $\Psi_3 \succeq \Psi_4 \succeq \Psi_2$, $\Psi_3 \succeq \Psi_1 \succeq \Psi_4$ and $\Psi_1 \succeq \Psi_4 \succeq \Psi_2$, respectively, for $\gamma = 0.5$. Thus, the overall ranking can be seen

$\Psi_3 \succeq \Psi_1 \succeq \Psi_4 \succeq \Psi_2$. Therefore, transivity property is provided between these rankings. Test condition 2 and test condition 3 are also provided.

6 Comparison analysis

In this section, we compare this proposed method with other methods. In order to solve the MCDM problem given above, several different methods have been developed under different fuzzy environments in the literature. Firstly, Deli et al. [12] proposed weighted average and weighted geometric operator for bipolar neutrosophic sets. Then, Fan et al. [15] studied this problem under single valued neutrosophic multi-set setting and proposed a cosine similarity measure. Recently, Ye et al. [65] have introduced new decision making method and correlation coefficients. For the example above, the results obtained with different methods are given in the table as follow:

Table 5: The comparison of different methods

Method	Weight Vectors	the Obtained Results				Best Selections
		Ψ_1	Ψ_2	Ψ_3	Ψ_4	
Correlation coefficient S_{w_2} in [65]	$(0.5, 0.25, 0.125, 0.125)$ for $S_{w_{2,1}}$	**0.9586**	0.8485	0.9138	0.8881	Ψ_1
	$(0.35, 0.25, 0.25, 0.15)$ for $S_{w_{2,2}}$	**0.9412**	0.8509	0.9229	0.9179	Ψ_1
	$(0.25, 0.25, 0.25, 0.25)$ for $S_{w_{2,3}}$	**0.9349**	0.8505	0.9281	0.9148	Ψ_1
cosine measure in [15]	$(0.5, 0.25, 0.125, 0.125)$ for ρw	0.9535	0.9511	**0.9813**	0.9616	Ψ_3
distance measure derived from cosine and correlation measure in [55]	$(0.2233, 0.3256, 0.2590, 0.1921)$	0.0974	0.8582	**0**	0.7511	Ψ_3
Method	Weight Vector	the Obtained Results				Best Selection
		Ψ_1	Ψ_2	Ψ_3	Ψ_4	
average operator in [12]	$(0.5, 0.25, 0.125, 0.125)$ for $\tilde{s}$	0.50	0.52	**0.56**	0.54	Ψ_3
Method	Sine Entropy Weight Vector	the Obtained Results				Best Selection
		Ψ_1	Ψ_2	Ψ_3	Ψ_4	
the proposed method	$(0.2233, 0.3256, 0.2590, 0.1921)$	0.3750	0.9396	**0**	0.8403	Ψ_3

The results in Table 5 show that the best choice changes as the weights of the criteria and suggested methods change. While determining the best choice, the method we take the smallest evaluation result, while others accept the largest evaluation result. Thus, we bring a new evaluation method to the literature with the proposed method. All of the methods given in Table 5 are comparison methods. We compare the results of the multi-criteria decision making problem, which we solve using the method we present in the present paper, with the results of other methods used to solve the same problem, and create a comparison table in Table 5. Since the correlation coefficient and cosine similarity measures presented in [55] coincide for single set, we convert these concepts into distance measures by considering them as complements of similarity measures, and obtain the same result by using different formulas in the consistency fuzzy environment. While some results are consistent with our results, some are different. This is because of the effect of the decision making method on the results. As the decision method changes, the results in solving the problem may change. But our method is the first method in the literature for the consistency fuzzy environment and the developed CF-VIKOR approach not only improves the decision making reliability but also supplies a new influential way for DMs.

Correlation is the strength and direction of a relationship between two variables in statistics. A correlation coefficient's value can range from -1 to 1, with -1 implying a strongly negative relation, 0 implying no relation, and 1 indicating a significantly positive relation. Spearman's and Kendalls correlation coefficients measure the relationship between two columns of ranked data. Kendalls (non-parametric) and Spearmans (non-parametric) rank correlation coefficient assess statistical associations based on the ranks of the data.

6.1 Spearman's rank correlation coefficient (SRCC) for consistency analysis

We assess the ranking consistency of Example 4.1 by using [52]. The SRCC ρ is depicted below, and the test results are indicated in Figure 3:

$$\rho =: 1 - \frac{6}{n(n^2-1)} \sum_{i=1}^{n} d_i^2, \tag{7}$$

where n denotes the number of results and d_i denotes the difference in the ranks of the results.

	Sw2,1	Sw2,2	Sw2,3	ρw	$\tilde{s}$
The Proposed method $(Q_i)_{\gamma=0.5}$	0.8	0.8	0.8	0.8	0.4

Figure 3: The SRCCs for Example 4.1

The obtained values are considered as highly valid range because they are greater than 0.71 [52] except for the result of $\tilde{s}$. The reason for this difference may be due to the fact that no similarity or distance-based study has been carried out in [12], unlike other methods given in the 3. However, the result obtained for $\tilde{s}$ is not negative or too far from 0.71. Therefore, the values back up the strength of the results in terms of statistics. Figure 3 shows the SRCCs which show the consistency of our results.

6.2 Kendall's rank correlation coefficient (KRCC) for consistency analysis

Now, we evaluate the ranking consistency of Example 4.1 by using [20]. The KRCC τ is shown below

$$\tau =: \frac{C-D}{C+D}, \tag{8}$$

where C is the number of concordant pairs which is given with and

$$C = \{(i,j) : x_i < x_j \text{ and } y_i < y_j\},$$

and D is the number of discordant pairs which is given with

$$D = \{(i,j) : x_i < x_j \text{ and } y_i > y_j\},$$

and x_i, y_i for $i = 1, ..., j, ...n$ evaluation results of two different method. The test results are shown in Figure 4 :

	Sw2,1	Sw2,2	Sw2,3	ρw	$\tilde{s}$
The Proposed method $(Q_i)_{\gamma=0.5}$	0.6667	0.6667	0.6667	0.6667	0.3333

Figure 4: The KRCCs for Example 4.1

The numerical results presented in Figure 4 show that the result of proposed VIKOR method and other methods are consistent with each other %66 except for the result of $\tilde{s}$. This is a high percentage. It is thought that the reason for the difference in the results of $\tilde{s}$ is the same as that of SRCC. Figure 4 shows the KRCCs which show the consistency of our results.

7 Conclusions

The concept of CFS uses both the information from FMS and the information provided by the degree of consistency and average. Therefore, it contains more useful information than other FSs. In this paper, we proposed a novel VIKOR method (CF-VIKOR) based on the fuzzy cross entropy measure and sine entropy measure with consistency fuzzy information. In this method, we used a consistency fuzzy sine entropy measure to determine the weights of criteria. Moreover, we compare this method with existing methods to show the usefulness of CF-VIKOR method and we carry out a validity analysis and Spearman and Kendall rank correlation coefficients. The following are the main contributions of this paper:

• Based on average and standard deviation values, a consistency fuzzy set (CFS) can provide reasonable hybrid information about sequences in a fuzzy multi-set.(FMS).

• The concept of CFS reduces the dependence of information on the length of the sequence in FMS and presents the hybrid information in a more compact form.

• The concept of CFS contains statistical information with the average of the membership sequences as well as consistency degree which is constructed with the help of standard deviation of the membership sequences in a FMS.

• The primary principle behind the notion of CFS is that data science is tried to be done via of FSs in a fuzzy environment. For this aim, we propose a cross entropy measure and a sine entropy measure for this useful FS.

• The proposed cross entropy measure and a sine entropy measure provide a useful ranking method.

• The developed CF-VIKOR approach not only improves the decision making reliability but also supplies a new influential way for DMs.

In the future, the proposed method can be integrated with other MCDM methods which is given in introduction section and construct new entropy and cross entropy measures for different FSs.

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