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Original paper

A hybridized correlation coefficient technique and its application in classification process under intuitionistic fuzzy setting

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Abstract

Intuitionistic fuzzy set (IFS) is a reliable device for resolving uncertainty and haziness encountered in decision-making process. In most cases, the significance of IFSs are explored based on correlation measures in myriad of areas like in engineering, image segmentation, pattern recognition, diagnostic analysis, etc. Some methods for computing intuitionistic fuzzy correlation coefficient (IFCC) have been investigated, however with some inadequacies. In this present work, a new method of IFCC is developed to correct the drawbacks in some existing techniques in terms of mathematical presentation and the exclusion of the hesitation parameter to enhance reasonable output. A comparative analysis is presented to ascertain the edge of the new technique over some similar approaches. In addition, the new correlation coefficient technique is applied to discuss some pattern recognition problems. This new IFCC method could be investigated based on spherical fuzzy data, q-rung orthopair fuzzy data, and picture fuzzy data.

Keywords: Correlation measure, intuitionistic fuzzy sets, decision-making, pattern recognition.

1 Introduction

Pattern recognition has to do with the grouping of data based on an already gained knowledge for the purpose of inference. Pattern recognition is the art of categorizing patterns based on machine learning algorithm. Most often, pattern recognition process is enmeshed with uncertainties, which justifies the use of soft computing approach of IFSs [1]. IFS expands the sphere of fuzzy set [55] by including non-membership degree with the likelihood of hesitation margin to the membership degree of fuzzy set, and thereby enlarges the scope of fuzzy set to enhance its participation as a reliable soft computing tool in decision-making, pattern recognition, etc. Because of the practicality of IFS, the construct has been applied in medical diagnosis based on composite relation [9], distance measures [8], and similarity measures [33, 41, 45]. IFSs have been applied in numerous areas namely; career determination [15], decision-making [13, 37], etc. Some decision making approaches have been discussed based on intuitionistic fuzzy information [7, 26, 43, 47], and the concept of time series forecasting was discussed under intuitionistic fuzzy domain [39].

Many researchers have discussed the application of IFSs in pattern recognition using various soft computing tools. Some novel approaches for the calculation of similarity between IFSs were discussed and applied to pattern recognition [10, 36, 54]. In [32], a pattern recognition problem was discussed based on some new construction for similarity measures between IFSs, and Boran and Akay [3] presented a two-parametric similarity measure on IFSs and discussed its applications in the problems of pattern recognition. In [4], a new approach of calculation similarity between IFSs were discussed based on transformation techniques with pattern recognition application. Similarly, the idea of pattern recognition has been discussed based on distance measure using intuitionistic fuzzy information [27, 50]. Some measuring association tools between two fuzzy random variables with applications have discussed [42, 44].

In recent time, the idea of IFSs has been discussed in the education sphere [6, 34] and medical domain [16, 19, 35], respectively. Duan and Li [11] constructed intuitionistic similarities using implication operator and the corresponding

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logical metric spaces with application to solving a pattern recognition problem. In [21], some intuitionistic fuzzy distances were constructed and applied in decision making, and an application of IFS-TOPSIS on the level assessment of the surrounding socks was discussed [25]. In [14], an improved intuitionistic fuzzy similarity operator was constructed and used to discuss pattern recognition and management of emergency. Some applications of IFSs were discussed in [5, 20, 58, 56].

Correlation analysis is a statistical technique used to assess the strength of association between two, numerically continuous variables. This kind of analysis is deployed whenever a researcher wants to investigate whether there are possible relations between two variables. Similarly, the statistical measure that computes the strength of the association of two variables is called correlation coefficient. The construct of correlation analysis has been encapsulated with intuitionistic fuzzy information to enhance the applicability of IFSs in real life problems [24]. Intuitionistic fuzzy correlation analysis has been studied in probability spaces [28]. Hung [30] studied IFCC from statistical perspective, and the idea of IFCC based on centroid method has been studied [31].

Xu [51] introduced a new IFCC approach and applied the concept to disease diagnosis. The approach in [51] was modified by including the complete convention parameters of IFSs to boost accuracy [52]. Because of the drawbacks in the approaches in [51, 52], Huang and Guo [29] introduced a robust approach, however by considering only two parameters of IFSs. In [40, 46], the approach in [30] was independently modified by the inclusion of hesitation margin to avoid error of omission. Similarly, Zeng and Li [57] developed an intuitionistic fuzzy correlation coefficient approach which modified [24] by the inclusion of hesitation margin. Similar approaches of computing IFCC were studied in [53]. Some statistical approaches of computing IFCC based on variance and covariance have been studied and applied in cases of decision-making [12, 38, 48, 49]. In [17, 18], some new approaches of IFCC were computed based on JAVA computer programming. Certain correlation coefficient approaches based on connection number of set pair analysis and TOPSIS method with applications to decision-making problems have been discussed [22, 23]. The motivation of this paper is informed by the following:

- The IFCC methods in [24, 53, 57] lack the ability to compute the correlation coefficient between some IFSs like $\mathcal{A}_1 = \{\langle x_1, 1, 0\rangle, \langle x_2, 0, 0.3\rangle\}$ and $\mathcal{A}_2 = \{\langle x_1, 0, 0.3\rangle, \langle x_2, 1, 0\rangle\}$ in $X = \{x_1, x_2\}$.
- The IFCC methods in [51, 52] yield $0/0$, which is mathematically undefined whenever the IFSs are equal. Normally, the correlation coefficient of equal IFSs should be 1. Also, the approaches yield a perfect correlation coefficient even when the IFSs are not equal.
- The IFCC method in [29] does not include the definitive parameters of IFSs and so its result cannot be trusted.

In this work, we develop an efficient method to compute IFCC. This is obtained by the inclusion of the hesitation margin and the number of the parameters of IFSs to enhance reliability unlike the approach in [29]. This study seeks to hybridize the IFCC approaches in [29, 51, 52] to birth a new approach with reliable accuracy, reasonable interpretation, sound mathematical correctness, and in order to avoid error of omission, the approach includes the complete parameters of IFSs. The new approach is a hybridized method of the approaches in [29, 51, 52] because it crossbred the existing approaches with an enhanced performance and interpretation by

- extending the approach in [29] through the inclusion of hesitation margin and parametric number of IFSs, and
- employing parametric absolute difference, minimum parametric absolute difference, and maximum parametric absolute difference, respectively as seen in [29, 51, 52].

In this present study, we;

(i) reiterate and appraise the IFCC approaches in [29, 51, 52] to pinpoint their drawbacks.

(ii) develop a hybridized IFCC approach with reliable output, reasonable interpretation, mathematical correctness, and inclusive of the complete parameters of IFSs.

(iii) apply the hybridized IFCC approach in pattern recognition analysis of mineral fields and building materials.

(iv) present a comparative analysis between the hybridized IFCC approach and the obtainable techniques.

The organization of the paper is as follows: Section 2 discusses the preliminaries of IFSs and some existing IFCC approaches with the highlights of drawbacks of the existing IFCC approaches; Section 3 introduces the hybridized IFCC approach, characterizes some of its properties, presents its computational processes; Section 4 discusses pattern recognition in terms of the classifications of mineral fields and building materials; and Section 5 summarises the findings of the paper and gives recommendations for further research.

2 Intuitionistic fuzzy sets and their correlation measures

In this section, the fundamentals of IFSs are recalled for reference. Afterward, some existing IFCC approaches are enlisted and their limitations itemized to justify the development of a new IFCC method.

2.1 Preliminaries on IFSs

We take X as the universe of discourse in this work. Firstly, we reiterate the definition of fuzzy set as follows.

Definition 2.1. [55] *A fuzzy set represented by F in X is defined by*

$$F = \{\langle x, \alpha_F(x)\rangle \mid x \in X\},$$

where $\alpha_F(x)$ is a function $\alpha_F : X \to [0,1]$, which explains the degree of membership of $x \in X$.

Definition 2.2. [1] *An intuitionistic fuzzy set represented by L in X is of the form*

$$L = \{\langle x, \alpha_L(x), \beta_L(x)\rangle \mid x \in X\},$$

where $\alpha_L(x)$ and $\beta_L(x)$ are defined by the functions $\alpha_L : X \to [0,1]$ and $\beta_L : X \to [0,1]$, to describe the degrees of membership and non-membership of $x \in X$ with the property, $0 \leq \alpha_L(x) + \beta_L(x) \leq 1$.

The hesitation margin of an IFS L in X is defined by $\gamma_L(x) = 1 - \alpha_L(x) - \beta_L(x)$. The hesitation margin expresses the knowledge of the degree to whether $x \in X$ or $x \notin X$.

Definition 2.3. [1] *Given that L and M are IFSs in X, we define the following properties of IFSs:*

(i) *Complement;* $\overline{L} = \{\langle x, \beta_L(x), \alpha_L(x)\rangle | x \in X\}$, $\overline{M} = \{\langle x, \beta_M(x), \alpha_M(x)\rangle | x \in X\}$.

(ii) *Union;* $L \cup M = \{\langle x, \max\{\alpha_L(x), \alpha_M(x)\}, \min\{\beta_L(x), \beta_M(x)\}\rangle | x \in X\}$.

(iii) *Intersection;* $L \cap M = \{\langle x, \min\{\alpha_L(x), \alpha_M(x)\}, \max\{\beta_L(x), \beta_M(x)\}\rangle | x \in X\}$.

(iv) *Equality;* $L = M$ *iff* $\alpha_L(x) = \alpha_M(x)$ *and* $\beta_L(x) = \beta_M(x)$ *for all* $x \in X$.

(v) *Inclusion;* $L \subseteq M$ *iff* $\alpha_L(x) \leq \alpha_M(x)$ *and* $\beta_L(x) \geq \beta_M(x)$ *for all* $x \in X$.

Definition 2.4. [2] *Intuitionistic fuzzy values (IFVs) are ordered pairs of the form $\langle l, m\rangle$ with the property $l + m \leq 1$, where $l, m \in [0,1]$. In the IFVs $\langle l, m\rangle$, l represents the degree of membership, and m represents the degree of nonmembership, respectively.*

2.2 Some correlation coefficients of IFSs

Some existing approaches of finding correlation coefficient for IFSs are reiterated before the introduction of the new approach.

Definition 2.5. [24] *If L and M are IFSs in X, then the coefficient of correlation between L and M denoted by $\rho(L, M)$ is a function ρ: $IFS \times IFS \to [0,1]$ such that the following conditions hold:*

(i) $0 \leq \rho(L, M) \leq 1$,

(ii) $\rho(L, M) = 1$ *iff* $L = M$,

(iii) $\rho(L, M) = \rho(L, M)$.

When $\rho(L, M)$ reaches 1, it shows that L and M have strong correlation. Again, if $\rho(L, M)$ reaches 0 then L and M have weak correlation. But if $\rho(L, M) = 0$ then L and M have no correlation. Hence, greater correlation coefficient shows better performance rating.

Now, for IFSs L and M in $X = \{x_1, \cdots, x_n\}$ where $n < \infty$, the following approaches are recalled.

2.2.1 Gerstenkon and Manko [24]

The notion of correlation coefficient was first discussed by Gerstenkon and Manko [24], and it is given as follows:

$$\rho_1(L,M)=\frac{\mathcal{C}(L,M)}{\sqrt{\mathcal{T}(L)}\sqrt{\mathcal{T}(M)}}, \tag{1}$$

where

$$\left.\begin{aligned}\mathcal{C}(L,M)&=\sum_{i=1}^{N}\Big(\alpha_L(x_i)\alpha_M(x_i)+\beta_L(x_i)\beta_M(x_i)\Big)\\ \mathcal{T}(L)&=\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big)\\ \mathcal{T}(M)&=\sum_{i=1}^{N}\Big(\alpha_M^2(x_i)+\beta_M^2(x_i)\Big)\end{aligned}\right\}. \tag{2}$$

The obvious limitation of (1) is the omission of hesitation margin from the computation and the inability to measure the correlation of some IFSs, and thus the output from this approach cannot be trusted.

Example 2.6. *Suppose* $\mathcal{A}_1$ *and* $\mathcal{A}_2$ *are IFSs given by* $\mathcal{A}_1=\{\langle x_1,1,0\rangle,\langle x_2,0,0.3\rangle\}$ *and* $\mathcal{A}_2=\{\langle x_1,0,0.3\rangle,\langle x_2,1,0\rangle\}$ *in* $X=\{x_1,x_2\}$.

Applying (1) we get $\mathcal{C}(\mathcal{A}_1,\mathcal{A}_2)=0$, $\mathcal{T}(\mathcal{A}_1)=\mathcal{T}(\mathcal{A}_2)=1.09$, and so $\rho_1(\mathcal{A}_1,\mathcal{A}_2)=\dfrac{0}{\sqrt{1.09\times 1.09}}=0$. Clearly, this output is a misinformation of the correlation between $\mathcal{A}_1$ and $\mathcal{A}_2$.

2.2.2 Zeng and Li [57]

By considering the limitation in the approach of Gerstenkon and Manko [24], a new approach was introduced by Zeng and Li [57] taking into account hesitation margin as seen in (3).

$$\rho_2(L,M)=\frac{\mathcal{C}(L,M)}{\sqrt{\mathcal{T}(L)}\sqrt{\mathcal{T}(M)}}, \tag{3}$$

where

$$\left.\begin{aligned}\mathcal{C}(L,M)&=\frac{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)\alpha_M(x_i)+\beta_L(x_i)\beta_M(x_i)+\gamma_L(x_i)\gamma_M(x_i)\Big)}{N}\\ \mathcal{T}(L)&=\frac{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)+\gamma_L^2(x_i)\Big)}{N}\\ \mathcal{T}(M)&=\frac{\sum_{i=1}^{N}\Big(\alpha_M^2(x_i)+\beta_M^2(x_i)+\gamma_M^2(x_i)\Big)}{N}\end{aligned}\right\}. \tag{4}$$

The limitation of (3) is the inability to measure the correlation of some IFSs. Applying (3) to Example 2.6, we get $\mathcal{C}(\mathcal{A}_1,\mathcal{A}_2)=\dfrac{0}{2}=0$, $\mathcal{T}(\mathcal{A}_1)=\mathcal{T}(\mathcal{A}_2)=\dfrac{1.58}{2}=0.79$, and so $\rho_2(\mathcal{A}_1,\mathcal{A}_2)=\dfrac{0}{\sqrt{0.79\times 0.79}}=0$. Similarly, this output is a misinformation of the correlation between $\mathcal{A}_1$ and $\mathcal{A}_2$.

2.2.3 Xu et al. [53]

Three methods of computing correlation coefficient between IFSs were discussed in [53]. The first approach modified the approach in [24], and it is given by

$$\rho_3(L,M)=\frac{\mathcal{C}(L,M)}{\max\Big\{\sqrt{\mathcal{T}(L)},\sqrt{\mathcal{T}(M)}\Big\}}, \tag{5}$$

where

$$\left.\begin{aligned} \mathcal{C}(L,M) &= \sum_{i=1}^{N}\Big(\alpha_L(x_i)\alpha_M(x_i)+\beta_L(x_i)\beta_M(x_i)\Big) \\ \mathcal{T}(L) &= \sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big) \\ \mathcal{T}(M) &= \sum_{i=1}^{N}\Big(\alpha_M^2(x_i)+\beta_M^2(x_i)\Big) \end{aligned}\right\}. \tag{6}$$

Similarly, (5) discards hesitation margin from the computation. Applying (5) to Example 2.6, we get $\mathcal{C}(\mathcal{A}_1,\mathcal{A}_2)=0$, $\mathcal{T}(\mathcal{A}_1)=\mathcal{T}(\mathcal{A}_2)=1.09$, and so $\rho_3(\mathcal{A}_1,\mathcal{A}_2)=\dfrac{0}{\max\{\sqrt{1.09},\sqrt{1.09}\}}=0$. Similarly, this output is a misinformation of the correlation between $\mathcal{A}_1$ and $\mathcal{A}_2$.

The other two approaches in [53] were based on the approach in [24], namely:

$$\rho_4(L,M)=\frac{\mathcal{C}(L,M)}{\max\Big(\sqrt{\mathcal{T}(L)},\sqrt{\mathcal{T}(M)}\Big)}, \tag{7}$$

$$\rho_5(L,M)=\frac{\mathcal{C}(L,M)}{\sqrt{\mathcal{T}(L)}\sqrt{\mathcal{T}(M)}}, \tag{8}$$

where

$$\left.\begin{aligned} \mathcal{C}(L,M) &= \sum_{i=1}^{N}\Big(\alpha_L(x_i)\alpha_M(x_i)+\beta_L(x_i)\beta_M(x_i)+\gamma_L(x_i)\gamma_M(x_i)\Big) \\ \mathcal{T}(L) &= \sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)+\gamma_L^2(x_i)\Big) \\ \mathcal{T}(M) &= \sum_{i=1}^{N}\Big(\alpha_M^2(x_i)+\beta_M^2(x_i)+\gamma_M^2(x_i)\Big) \end{aligned}\right\}. \tag{9}$$

By simplification, it is observed that (3) and (8) are equivalent. It is worthy to note that (5) and (7) are not reliable correlation measures because they do not yield perfect correlation coefficient whenever the IFSs are equal. To see this, let us recall the following:

$$\rho_3(L,M)=\frac{\sum_{i=1}^{N}\Big(\alpha_L(x_i)\alpha_M(x_i)+\beta_L(x_i)\beta_M(x_i)\Big)}{\max\Big(\sqrt{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big)},\sqrt{\sum_{i=1}^{N}\Big(\alpha_M^2(x_i)+\beta_M^2(x_i)\Big)}\Big)},$$

If $L=M$, then

$$\begin{aligned} \rho_4(L,M) &= \frac{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big)}{\max\Big(\sqrt{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big)},\sqrt{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big)}\Big)} \\ &= \frac{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big)}{\sqrt{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big)}} \\ &= \sqrt{\sum_{i=1}^{N}\Big(\alpha_L^2(x_i)+\beta_L^2(x_i)\Big)} \\ &\neq 1. \end{aligned}$$

Similarly, $\rho_4(L,M)\neq 1$ if $L=M$.

Applying (7) and (8) to Example 2.6, we get $\mathcal{C}(\mathcal{A}_1, \mathcal{A}_2) = 0$, $\mathcal{T}(\mathcal{A}_1) = \mathcal{T}(\mathcal{A}_2) = 1.58$, and so $\rho_4(\mathcal{A}_1, \mathcal{A}_2) = \dfrac{0}{\max\{\sqrt{1.58}, \sqrt{1.58}\}} = 0$ and $\rho_5(\mathcal{A}_1, \mathcal{A}_2) = \dfrac{0}{\sqrt{1.58 \times 1.58}} = 0$. Again, these outputs are misinformation of the correlation between $\mathcal{A}_1$ and $\mathcal{A}_2$.

2.2.4 Xu [51]

In [51], an approach for estimating correlation coefficient between IFSs were developed from different perspective. The approach is as follows:

$$\rho_6(L, M) = \frac{1}{2N} \sum_{i=1}^{N} \Big(\frac{\Delta\alpha_{\min} + \Delta\alpha_{\max}}{\Delta\alpha_i + \Delta\alpha_{\max}} + \frac{\Delta\beta_{\min} + \Delta\beta_{\max}}{\Delta\beta_i + \Delta\beta_{\max}} \Big), \tag{10}$$

where

$$\left.\begin{aligned} \Delta\alpha_i = |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_i = |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\alpha_{\min} = \min_i |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_{\min} = \min_i |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\alpha_{\max} = \max_i |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_{\max} = \max_i |\beta_L(x_i) - \beta_M(x_i)| \end{aligned}\right\}. \tag{11}$$

The approach lacks reliability due to information loss occasioned by the omission of hesitation margin and its inability to measure the correlation of some IFSs.

Example 2.7. *Suppose* $\mathcal{B}_1$ *and* $\mathcal{B}_2$ *are IFSs given by* $\mathcal{B}_1 = \{\langle x_1, 0.4, 0.3\rangle, \langle x_2, 0.3, 0.2\rangle\}$ *and* $\mathcal{B}_2 = \{\langle x_1, 0.3, 0.2\rangle, \langle x_2, 0.2, 0.1\rangle\}$ *in* $X = \{x_1, x_2\}$.

Applying (10), we get the correlation coefficient using the information in Table 1.

Table 1: Computational Process

X	$\Delta\alpha_i$	$\Delta\beta_i$
x_1	0.1	0.1
x_2	0.1	0.1

We see that

$$\Delta\alpha_{\min} = \Delta\alpha_{\max} = 0.1, \Delta\beta_{\min} = \Delta\beta_{\max} = 0.1.$$

Hence

$$\begin{aligned} \rho_6(\mathcal{B}_1, \mathcal{B}_2) &= \frac{1}{4}\left[\frac{(0.1+0.1)}{(0.1+0.1)} + \frac{(0.1+0.1)}{(0.1+0.1)} + \frac{(0.1+0.1)}{(0.1+0.1)} + \frac{(0.1+0.1)}{(0.1+0.1)}\right] \\ &= 1. \end{aligned}$$

This result does not corroborate with Definition 2.5, and so it is not reliable.

2.2.5 Xu and Cai [52]

Due to the limitation of the approach in [51], an enhanced correlation measure was envisaged to mitigate the setback and improve reliability. The approach for measuring correlation coefficient in [52] is:

$$\rho_7(L, M) = \frac{1}{3N} \sum_{i=1}^{N} \Big(\frac{\Delta\alpha_{\min} + \Delta\alpha_{\max}}{\Delta\alpha_i + \Delta\alpha_{\max}} + \frac{\Delta\beta_{\min} + \Delta\beta_{\max}}{\Delta\beta_i + \Delta\beta_{\max}} + \frac{\Delta\gamma_{\min} + \Delta\gamma_{\max}}{\Delta\gamma_i + \Delta\gamma_{\max}} \Big), \tag{12}$$

where

$$\left.\begin{aligned} \Delta\alpha_i = |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_i = |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\gamma_i = |\gamma_L(x_i) - \gamma_M(x_i)| \\ \Delta\alpha_{\min} = \min_i |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_{\min} = \min_i |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\gamma_{\min} = \min_i |\gamma_L(x_i) - \gamma_M(x_i)| \\ \Delta\alpha_{\max} = \max_i |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_{\max} = \max_i |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\gamma_{\max} = \max_i |\gamma_L(x_i) - \gamma_M(x_i)| \end{aligned}\right\}. \tag{13}$$

The approach lacks reliability due to its inability to measure the correlation of some IFSs. Applying (12) to Example 2.7, we get the correlation coefficient using the information in Table 2.

Table 2: Computational Process

X	$\Delta\alpha_i$	$\Delta\beta_i$	$\Delta\gamma_i$
x_1	0.1	0.1	0.2
x_2	0.1	0.1	0.2

It follows that

$$\Delta\alpha_{\min} = 0.1,\ \Delta\beta_{\min} = 0.1,\ \Delta\gamma_{\min} = 0.2$$

$$\Delta\alpha_{\max} = 0.1,\ \Delta\beta_{\max} = 0.1,\ \Delta\gamma_{\max} = 0.2.$$

Hence

$$\begin{aligned}\rho_7(\mathcal{B}_1, \mathcal{B}_2) &= \frac{1}{6}\left[\frac{(0.1+0.1)}{(0.1+0.1)} + \frac{(0.1+0.1)}{(0.1+0.1)} + \frac{(0.1+0.1)}{(0.1+0.1)} + \frac{(0.1+0.1)}{(0.1+0.1)} + \frac{(0.2+0.2)}{(0.2+0.2)} + \frac{(0.2+0.2)}{(0.2+0.2)}\right] \\ &= 1.\end{aligned}$$

This result does not corroborate with Definition 2.5 since $\mathcal{B}_1 \neq \mathcal{B}_2$, and so it is not reliable.

2.2.6 Huang and Guo [29]

The approaches in [51, 52] were observed to have some limitations. First, the approaches yield 0/0, which is mathematically undefined whenever the IFSs are equal. Normally, the correlation coefficient of equal IFSs should be 1. Secondly, the approaches yield a perfect correlation coefficient 1 even when the IFSs are not equal. Due to these setbacks, Huang and Guo [29] introduced a novel approach as follows:

$$\rho_8(L, M) = \frac{1}{2N}\sum_{i=1}^{N}\Big(\mu_i(1-\Delta\alpha_i) + \nu_i(1-\Delta\beta_i)\Big), \tag{14}$$

where

$$\left.\begin{aligned}\mu_i &= \frac{c-\Delta\alpha_i-\Delta\alpha_{\max}}{c-\Delta\alpha_{\min}-\Delta\alpha_{\max}} \\ \nu_i &= \frac{c-\Delta\beta_i-\Delta\beta_{\max}}{c-\Delta\beta_{\min}-\Delta\beta_{\max}}\end{aligned}\right\}, \tag{15}$$

for $c > 2$, and

$$\left.\begin{aligned}\Delta\alpha_i = |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_i = |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\alpha_{\min} = \min_i |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_{\min} = \min_i |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\alpha_{\max} = \max_i |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_{\max} = \max_i |\beta_L(x_i) - \beta_M(x_i)|\end{aligned}\right\}. \tag{16}$$

One cannot rely on the results from this method because it omits the hesitation margin component in the computation.

Example 2.8. *Suppose* $\mathcal{C}_1$ *and* $\mathcal{C}_2$ *are IFSs given by* $\mathcal{C}_1 = \{\langle x_1, 0.9, 0.1\rangle, \langle x_2, 0.7, 0.2\rangle\}$ *and* $\mathcal{C}_2 = \{\langle x_1, 0.1, 0.8\rangle, \langle x_2, 0.6, 0.4\rangle\}$ *in* $X = \{x_1, x_2\}$.

Applying (14), we get the correlation coefficient using the information in Table 3.

Table 3: Computational Process

X	$\Delta\alpha_i$	$\Delta\beta_i$
x_1	0.8	0.7
x_2	0.1	0.2

We see that

$$\Delta\alpha_{\min} = 0.1, \Delta\alpha_{\max} = 0.8,$$
$$\Delta\beta_{\min} = 0.2, \Delta\beta_{\max} = 0.7.$$

Thus,

$$\mu_1 = \frac{3 - 0.8 - 0.8}{3 - 0.1 - 0.8} = 0.6667,\ \nu_1 = \frac{3 - 0.7 - 0.7}{3 - 0.2 - 0.7} = 0.7619,$$
$$\mu_2 = \frac{3 - 0.1 - 0.8}{3 - 0.1 - 0.8} = 1,\ \nu_2 = \frac{3 - 0.2 - 0.7}{3 - 0.2 - 0.7} = 1.$$

Hence

$$\begin{aligned}\rho_8(\mathcal{C}_1, \mathcal{C}_2) &= \frac{1}{4}\Big[0.6667(1 - 0.8) + (1 - 0.1) + 0.7619(1 - 0.7) + (1 - 0.2)\Big] \\ &= 0.5155.\end{aligned}$$

This result shows that a minimum correlation exists between the IFSs. That is, the correlation coefficient has a low performance index.

3 Hybridized intuitionistic fuzzy correlation coefficient

In this work, we introduce and discuss an efficient method to compute the correlation coefficient for IFSs. This is obtained by the inclusion of the hesitation margin and the number of the parameters of IFSs to enhance reliability unlike the approach in [29]. In fact, this approach hybridizes the intuitionistic fuzzy correlation coefficient approaches in [29, 51, 52]. The new approach is a hybridized method of the approaches in [29, 51, 52] because it crossbred the existing approaches with an enhanced performance and interpretation by

- extending the approach in [29] through the inclusion of hesitation margin and parametric number of IFSs, and
- employing parametric absolute difference, minimum parametric absolute difference, and maximum parametric absolute difference, respectively as seen in [29, 51, 52].

By combining the attributes of the approaches in [29, 51, 52], a new method is developed which resolves the limitations of the approaches in [29, 51, 52].

Assume there are two arbitrary IFSs L and M in $X = \{x_1, \cdots, x_n\}$ where $n < \infty$, then the correlation coefficient for the IFSs can be measured by:

$$\tilde{\rho}(L, M) = \frac{1}{3N}\sum_{i=1}^{N}\Big(\mu_i(1 - \Delta\alpha_i) + \nu_i(1 - \Delta\beta_i) + \pi_i(1 - \Delta\gamma_i)\Big), \tag{17}$$

where

$$\left.\begin{aligned}\mu_i &= \frac{c - \Delta\alpha_i - \Delta\alpha_{\max}}{c - \Delta\alpha_{\min} - \Delta\alpha_{\max}} \\ \nu_i &= \frac{c - \Delta\beta_i - \Delta\beta_{\max}}{c - \Delta\beta_{\min} - \Delta\beta_{\max}} \\ \pi_i &= \frac{c - \Delta\gamma_i - \Delta\gamma_{\max}}{c - \Delta\gamma_{\min} - \Delta\gamma_{\max}}\end{aligned}\right\}, \tag{18}$$

for $c > 2$, and

$$\left.\begin{aligned}\Delta\alpha_i = |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_i = |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\gamma_i = |\gamma_L(x_i) - \gamma_M(x_i)| \\ \Delta\alpha_{\min} = \min_i |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_{\min} = \min_i |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\gamma_{\min} = \min_i |\gamma_L(x_i) - \gamma_M(x_i)| \\ \Delta\alpha_{\max} = \max_i |\alpha_L(x_i) - \alpha_M(x_i)|,\ \Delta\beta_{\max} = \max_i |\beta_L(x_i) - \beta_M(x_i)| \\ \Delta\gamma_{\max} = \max_i |\gamma_L(x_i) - \gamma_M(x_i)|\end{aligned}\right\}. \tag{19}$$

In some cases, it is of necessity to consider the weights of elements of X while computing the correlation coefficient. For instance, in multi-attribute decision-making cases, every feature does has dissimilar significant and so needs to be apportioned a dissimilar weight. By considering the weights of the elements of X, (17) becomes

$$\tilde{\rho}_\omega(L,M)=\frac{1}{3}\sum_{i=1}^{N}\omega_i\Big(\mu_i(1-\Delta\alpha_i)+\nu_i(1-\Delta\beta_i)+\pi_i(1-\Delta\gamma_i)\Big), \tag{20}$$

where the parameters are the same as in (18) and (19), and $\omega_i \geq 0$ for $\sum_{i=1}^{N}\omega_i=1$. If $\omega=\left(\frac{1}{N},\frac{1}{N},\cdots,\frac{1}{N}\right)^T$, then (17) and (20) are the same.

3.1 Numerical illustrations of the new IFCC approach

Some examples of IFSs are considered to illustrate the steps involve in the computation of correlation coefficient based on the new approach.

Example 3.1. *Suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are IFSs in $X=\{x_1,x_2,x_3\}$ defined by*

$$\mathcal{L}_1=\{\langle x_1,0.1,0.2,0.7\rangle,\langle x_2,0.2,0.1,0.7\rangle,\langle x_3,0.1,0.6,0.3\rangle\},$$

$$\mathcal{L}_2=\{\langle x_1,0.3,0.0,0.7\rangle,\langle x_2,0.2,0.2,0.6\rangle,\langle x_3,0.3,0.0,0.7\rangle\}.$$

By mere observation, $\mathcal{L}_1$ and $\mathcal{L}_2$ are related since $\mathcal{L}_1\subseteq\mathcal{L}_2$. We calculate the correlation coefficient concerning the IFSs via the new approach using the information in Table 4.

Table 4: Computational Process

X	$\Delta\alpha_i$	$\Delta\beta_i$	$\Delta\gamma_i$
x_1	0.2	0.2	0.0
x_2	0.0	0.1	0.1
x_3	0.2	0.6	0.4

where

$$\Delta\alpha_{\min}=0.0,\ \Delta\beta_{\min}=0.1,\ \Delta\gamma_{\min}=0.0,$$

$$\Delta\alpha_{\max}=0.2,\ \Delta\beta_{\max}=0.6,\ \Delta\gamma_{\max}=0.4.$$

Thus

$$\mu_1=\frac{3-0.2-0.2}{3-0.0-0.2}=0.9286,\ \nu_1=\frac{3-0.2-0.6}{3-0.1-0.6}=0.9565,\ \pi_1=\frac{3-0.0-0.4}{3-0.0-0.4}=1,$$

$$\mu_2=\frac{3-0.0-0.2}{3-0.0-0.2}=1,\ \nu_2=\frac{3-0.1-0.6}{3-0.1-0.6}=1,\ \pi_2=\frac{3-0.1-0.4}{3-0.0-0.4}=0.9615,$$

$$\mu_3=\frac{3-0.2-0.2}{3-0.0-0.2}=0.9286,\ \nu_3=\frac{3-0.6-0.6}{3-0.1-0.6}=0.7826,\ \pi_3=\frac{3-0.4-0.4}{3-0.0-0.4}=0.8462.$$

Hence

$$\begin{aligned}\tilde{\rho}(\mathcal{L}_1,\mathcal{L}_2) &= \frac{1}{9}\Big((0.9286\times 0.8)+(0.9565\times 0.8)+(1\times 1)+(1\times 1)+(1\times 0.9)+(0.9615\times 0.9)\\ &+ (0.9286\times 0.8)+(0.7826\times 0.4)+(0.8462\times 0.6)\Big)\\ &= 0.7597.\end{aligned}$$

This result corroborates the relationship between $\mathcal{L}_1$ and $\mathcal{L}_2$.

Example 3.2. *Suppose $\mathcal{M}_1$ and $\mathcal{M}_2$ are IFSs in $X=\{x_1,x_2,x_3,x_4\}$ defined by*

$$\mathcal{M}_1=\{\langle x_1,0.7,0.2,0.1\rangle,\langle x_2,0.6,0.1,0.3\rangle,\langle x_4,0.5,0.4,0.1\rangle\},$$

$$\mathcal{M}_2=\{\langle x_1,0.8,0.1,0.1\rangle,\langle x_2,0.7,0.1,0.2\rangle,\langle x_3,0.3,0.4,0.3\rangle\}.$$

Similarly, we compute the correlation coefficient between the IFSs through the new approach using the information in Table 5.

Table 5: Computational Process

X	$\Delta\alpha_i$	$\Delta\beta_i$	$\Delta\gamma_i$
x_1	0.1	0.1	0.0
x_2	0.1	0.0	0.1
x_3	0.2	0.6	0.3
x_4	0.5	0.6	0.1

where

$$\Delta\alpha_{\min} = 0.1,\ \Delta\beta_{\min} = 0.0,\ \Delta\gamma_{\min} = 0.0,$$

$$\Delta\alpha_{\max} = 0.5,\ \Delta\beta_{\max} = 0.6,\ \Delta\gamma_{\max} = 0.3.$$

So,

$$\mu_1 = \frac{3-0.1-0.5}{3-0.1-0.5} = 1,\ \nu_1 = \frac{3-0.1-0.6}{3-0.0-0.6} = 0.9583,\ \pi_1 = \frac{3-0.0-0.3}{3-0.0-0.3} = 1,$$

$$\mu_2 = \frac{3-0.1-0.5}{3-0.1-0.5} = 1,\ \nu_2 = \frac{3-0.0-0.6}{3-0.0-0.6} = 1,\ \pi_2 = \frac{3-0.1-0.3}{3-0.0-0.3} = 0.963,$$

$$\mu_3 = \frac{3-0.2-0.5}{3-0.1-0.5} = 0.9583,\ \nu_3 = \frac{3-0.6-0.6}{3-0.0-0.6} = 0.75,\ \pi_3 = \frac{3-0.3-0.3}{3-0.0-0.3} = 0.8889,$$

$$\mu_4 = \frac{3-0.5-0.5}{3-0.1-0.5} = 0.8333,\ \nu_4 = \frac{3-0.6-0.6}{3-0.0-0.6} = 0.75,\ \pi_4 = \frac{3-0.1-0.3}{3-0.0-0.3} = 0.963.$$

Hence

$$\begin{aligned}\tilde{\rho}(\mathcal{M}_1, \mathcal{M}_2) &= \frac{1}{12}\Big((1 \times 0.9) + (0.9583 \times 0.9) + (1 \times 1) + (1 \times 0.9) + (1 \times 1) + (0.963 \times 0.9) \\ &+ (0.9583 \times 0.8) + (0.75 \times 0.4) + (0.8889 \times 0.7) + (0.8333 \times 0.5) + (0.75 \times 0.4) + (0.963 \times 0.9)\Big) \\ &= 0.7334,\end{aligned}$$

which interprets the correlation between $\mathcal{M}_1$ and $\mathcal{M}_2$.

3.2 Comparative analysis

The superiority of the new IFCC method over the existing IFCC methods is unveiled by presenting a comparative analysis as follows. By applying the new IFCC method to Example 2.6, we have $\tilde{\rho}(\mathcal{A}_1, \mathcal{A}_2) = 0.3333$, while the IFCC methods in [24, 53, 57] give $\rho_1(\mathcal{A}_1, \mathcal{A}_2) = 0.0$, $\rho_2(\mathcal{A}_1, \mathcal{A}_2) = 0.0$, $\rho_3(\mathcal{A}_1, \mathcal{A}_2) = 0.0$, $\rho_4(\mathcal{A}_1, \mathcal{A}_2) = 0.0$, and $\rho_5(\mathcal{A}_1, \mathcal{A}_2) = 0.0$.

Though the correlation between $\mathcal{A}_1$ and $\mathcal{A}_2$ is weak by mere observation, the IFCC methods in [24, 53, 57] give a misleading interpretation that the correlation does not exist at all. On the contrary, the new IFCC method gives a correlation value that tallies with the mere observation. This proves the advantage of the new IFCC methods over the methods in [24, 53, 57].

By applying the new IFCC method to Example 2.7, we have a correlation coefficient $\tilde{\rho}(\mathcal{B}_1, \mathcal{B}_2) = 0.8667$, while the IFCC methods in [51, 52] give correlation coefficients $\rho_6(\mathcal{B}_1, \mathcal{B}_2) = 1$ and $\rho_7(\mathcal{B}_1, \mathcal{B}_2) = 1$. Of course, a strong correlation exists between $\mathcal{B}_1$ and $\mathcal{B}_2$ but certainly not perfect. Correlation coefficient can only be perfect if $\mathcal{B}_1 = \mathcal{B}_2$. This speaks to the limitation of the IFCC methods in [51, 52]. Again, this proves the advantage of the new IFCC methods over the methods in [51, 52].

Finally, we apply the new IFCC method to Example 2.8, and get a correlation coefficient $\tilde{\rho}(\mathcal{C}_1, \mathcal{C}_2) = 0.6437$. By applying the IFCC method in [29], we have $\rho_8(\mathcal{C}_1, \mathcal{C}_2) = 0.5155$. The new IFCC method is more reliable compare to the IFCC method [29] because it take account of all the parametric definition of the concerned IFSs. It is observed that as the hesitation margin becomes smaller, the new IFCC method yields a result with high performance index compare to the IFCC method in [29], which underscores the limitation of the IFCC method [29] and proves the advantage of the new IFCC methods.

The new approach is an improved version of the method in [29] with high performance rating and reliability because it does not provide any leeway for information loss as seen in [24, 29, 51].

3.3 Some properties of the new IFCC approach

To substantiate the validity of the new approach, we present some of its properties.

Theorem 3.3. *Suppose L and M are two IFSs in X, then the new IFCC $\tilde{\rho}(L, M)$ satisfies the commutative property:*

(*i*) $\tilde{\rho}(L, M) = \tilde{\rho}(M, L)$,

(*ii*) $\tilde{\rho}_\omega(L, M) = \tilde{\rho}_\omega(M, L)$.

Proof. To prove (i), we recall that

$$\tilde{\rho}(L, M) = \frac{1}{3N} \sum_{i=1}^{N} \Big(\mu_i(1 - \Delta\alpha_i) + \nu_i(1 - \Delta\beta_i) + \pi_i(1 - \Delta\gamma_i) \Big),$$

and then

$$\begin{aligned} \tilde{\rho}(L, M) &= \frac{1}{3N} \sum_{i=1}^{N} \Bigg(\mu_i \Big(1 - |\alpha_L(x_i) - \alpha_M(x_i)| \Big) + \nu_i \Big(1 - |\beta_L(x_i) - \beta_M(x_i)| \Big) + \pi_i \Big(1 - |\gamma_L(x_i) - \gamma_M(x_i)| \Big) \Bigg) \\ &= \frac{1}{3N} \sum_{i=1}^{N} \Bigg(\mu_i \Big(1 - |\alpha_M(x_i) - \alpha_L(x_i)| \Big) + \nu_i \Big(1 - |\beta_M(x_i) - \beta_L(x_i)| \Big) + \pi_i \Big(1 - |\gamma_M(x_i) - \gamma_L(x_i)| \Big) \Bigg) \\ &= \tilde{\rho}(M, L), \end{aligned}$$

which proofe (i). The prove of (ii) is similar to (i).

□

Theorem 3.4. *If L and M are two IFSs in X, then the new correlation coefficient $\tilde{\rho}(L, M)$ satisfies*

(*i*) $\tilde{\rho}(L, M) = 1$ *iff* $L = M$,

(*ii*) $\tilde{\rho}_\omega(L, M) = 1$ *iff* $L = M$.

Proof. First, we establish (i). Suppose $L = M$. Then $|\alpha_L(x_i) - \alpha_M(x_i)| = 0$, $|\beta_L(x_i) - \beta_M(x_i)| = 0$, and $|\gamma_L(x_i) - \gamma_M(x_i)| = 0$. Deductively,

$$\Delta\alpha_i = \Delta\beta_i = \Delta\gamma_i = 0,$$

$$\Delta\alpha_{\min} = \Delta\beta_{\min} = \Delta\gamma_{\min} = 0, \text{ and}$$

$$\Delta\alpha_{\max} = \Delta\beta_{\max} = \Delta\gamma_{\max} = 0.$$

Thus $\mu_i = \nu_i = \pi_i = 1$, and thus $\tilde{\rho}(L, M) = \frac{1}{3N} \sum_{i=1}^{N} 3N = 1$.

Conversely, if $\tilde{\rho}(L, M) = 1$ then L and M have perfect relation, and so $L = M$. Hence, (i) holds. The prove of (ii) is similar to (i).

□

Theorem 3.5. *Suppose $\tilde{\rho}(L, M)$ and $\tilde{\rho}_\omega(L, M)$ are correlation coefficients between IFSs L and M in X, then $\tilde{\rho}(L, M) \in [0, 1]$ and $\tilde{\rho}_\omega(L, M) \in [0, 1]$.*

Proof. We need to prove that $0 \le \tilde{\rho}(L, M) \le 1$, i.e. $\tilde{\rho}(L, M) \ge 0$ and $\tilde{\rho}(L, M) \le 1$. Certainly, $\tilde{\rho}(L, M) \ge 0$. Now, we show that $\tilde{\rho}(L, M) \le 1$.

To see this, let us assume that

$$\sum_{i=1}^{N} \Big(\mu_i(1 - \Delta\alpha_i) \Big) = \xi, \; \sum_{i=1}^{N} \Big(\nu_i(1 - \Delta\beta_i) \Big) = \eta,$$

$$\sum_{i=1}^{N} \Big(\pi_i(1 - \Delta\gamma_i) \Big) = \kappa.$$

By Cauchy-Schwarz inequality's principle, we get

$$\begin{aligned}\tilde{\rho}(L,M) &= \frac{1}{3N}\sum_{i=1}^{N}\Big(\mu_i(1-\Delta\alpha_i)+\nu_i(1-\Delta\beta_i)+\pi_i(1-\Delta\gamma_i)\Big)\\ &\leq \frac{\sum_{i=1}^{N}\Big(\mu_i(1-\Delta\alpha_i)\Big)+\sum_{i=1}^{N}\Big(\nu_i(1-\Delta\beta_i)\Big)+\sum_{i=1}^{N}\Big(\pi_i(1-\Delta\gamma_i)\Big)}{3N}\\ &= \frac{\xi+\eta+\kappa}{3N}.\end{aligned}$$

Thus,

$$\begin{aligned}\tilde{\rho}(L,M)-1 &= \frac{\xi+\eta+\kappa}{3N}-1=\frac{\xi+\eta+\kappa-3N}{3N}=-\frac{(3N-\xi-\eta-\kappa)}{3N}\\ &\leq 0,\end{aligned}$$

which implies that $\tilde{\rho}(L,M)\leq 1$. Hence, $\tilde{\rho}(L,M)\in[0,1]$. Similarly, the proof of $\tilde{\rho}_\omega(L,M)\in[0,1]$ follows. □

4 Application examples

We demonstrate the hand-on relevance of the new correlation coefficient approach and the similar approaches [29, 51, 52] in cases of pattern recognition to project the viability of the new approach. To start with, pattern recognition is the art of categorizing patterns based on machine learning algorithm. Pattern recognition has to do with the grouping of data based on an already gained knowledge to aid inference. In most cases, the art of pattern recognition is enmeshed with uncertainties, which justifies the use of soft computing approach of IFCC technique.

To achieve this, we suppose there are known patterns within a sample space and an unknown pattern within the same space that needed to be grouped into any of the similar known pattern using IFCC technique. The correlation concerning the known pattern and the unknown pattern which yields the greatest correlation coefficient value determine the grouping or classification. The intuitionistic fuzzy data presented in [50] is employed for the application discussions.

4.1 Pattern recognition of mineral fields

Table 6: Mineral Fields as IFVs

	Feature Space					
Patterns	s_1	s_2	s_3	s_4	s_5	s_6
$\alpha_{\hat{C}_1}$	0.739	0.033	0.188	0.492	0.020	0.739
$\beta_{\hat{C}_1}$	0.125	0.818	0.626	0.358	0.628	0.125
$\gamma_{\hat{C}_1}$	0.136	0.149	0.186	0.150	0.352	0.136
$\alpha_{\hat{C}_2}$	0.124	0.030	0.048	0.136	0.019	0.300
$\beta_{\hat{C}_2}$	0.665	0.825	0.800	0.648	0.823	0.653
$\gamma_{\hat{C}_2}$	0.211	0.145	0.152	0.216	0.158	0.047
$\alpha_{\hat{C}_3}$	0.449	0.662	1.000	1.000	1.000	1.000
$\beta_{\hat{C}_3}$	0.387	0.298	0.000	0.000	0.000	0.000
$\gamma_{\hat{C}_3}$	0.164	0.040	0.000	0.000	0.000	0.000
$\alpha_{\hat{C}_4}$	0.280	0.521	0.470	0.295	0.188	0.735
$\beta_{\hat{C}_4}$	0.715	0.368	0.423	0.658	0.806	0.118
$\gamma_{\hat{C}_4}$	0.005	0.111	0.107	0.047	0.006	0.147
$\alpha_{\hat{C}_5}$	0.326	1.000	0.182	0.156	0.049	0.675
$\beta_{\hat{C}_5}$	0.452	0.000	0.725	0.765	0.896	0.263
$\gamma_{\hat{C}_5}$	0.222	0.000	0.093	0.079	0.055	0.062
$\alpha_{\hat{M}}$	0.629	0.524	0.210	0.218	0.069	0.658
$\beta_{\hat{M}}$	0.303	0.356	0.689	0.753	0.876	0.256
$\gamma_{\hat{M}}$	0.068	0.120	0.101	0.029	0.055	0.086

Firstly, we think through a case of pattern recognition of certain mineral fields. Given there are five categories of mineral fields which are featured in the content of six minerals, and there is a category of typical hybrid mineral.

We represent the five categories of the typical hybrid mineral by IFSs $\hat{C}_1$, $\hat{C}_2$, $\hat{C}_3$, $\hat{C}_4$, and $\hat{C}_5$ in the feature space $S = \{s_1, \cdots, s_6\}$. Assuming there is another unclassified category of hybrid mineral $\hat{M}$, then we determine which field this unclassified category of hybrid mineral $\hat{M}$ can be classified with. The IFVs of the mineral fields are given in Table 6.

In order to classify the unknown hybrid mineral $\hat{M}$, we compute its correlation coefficients with each $\hat{C}_i$, for $i = 1, 2, 3, 4, 5$ using the new correlation coefficient approach and similar approaches [29, 51, 52] to obtain the following results:

Our new approach yields;

$$\tilde{\rho}(\hat{C}_1, \hat{M}) = 0.7880,\ \tilde{\rho}(\hat{C}_2, \hat{M}) = 0.7541,\ \tilde{\rho}(\hat{C}_3, \hat{M}) = 0.6121,$$

$$\tilde{\rho}(\hat{C}_4, \hat{M}) = 0.8532,\ \tilde{\rho}(\hat{C}_5, \hat{M}) = 0.8723,$$

which shows that the unknown hybrid mineral $\hat{M}$ can be classified with $\hat{C}_5$ since $\tilde{\rho}(\hat{C}_5, \hat{M})$ is the greatest.
The approach of Xu [51] yields;

$$\rho_6(\hat{C}_1, \hat{M}) = 0.7934,\ \rho_6(\hat{C}_2, \hat{M}) = 0.7602,\ \rho_6(\hat{C}_3, \hat{M}) = 0.7602,$$

$$\rho_6(\hat{C}_4, \hat{M}) = 0.7595,\ \rho_6(\hat{C}_5, \hat{M}) = 0.8455.$$

The approach of Xu and Cai [52] yields;

$$\rho_7(\hat{C}_1, \hat{M}) = 0.8074,\ \rho_7(\hat{C}_2, \hat{M}) = 0.7718,\ \rho_7(\hat{C}_3, \hat{M}) = 0.7596,$$

$$\rho_7(\hat{C}_4, \hat{M}) = 0.7580,\ \rho_7(\hat{C}_5, \hat{M}) = 0.8210.$$

The approach of Huang and Guo [29] yields;

$$\rho_8(\hat{C}_1, \hat{M}) = 0.7480,\ \rho_8(\hat{C}_2, \hat{M}) = 0.6871,\ \rho_8(\hat{C}_3, \hat{M}) = 0.4626,$$

$$\rho_8(\hat{C}_4, \hat{M}) = 0.8016,\ \rho_8(\hat{C}_5, \hat{M}) = 0.8473.$$

Table 7 presents the results of the IFCC values.

Table 7: Results for Mineral Fields Classification

IFCC Methods	$(\hat{C}_1, \hat{M})$	$(\hat{C}_2, \hat{M})$	$(\hat{C}_3, \hat{M})$	$(\hat{C}_4, \hat{M})$	$(\hat{C}_5, \hat{M})$	Classifications
New IFCC	0.7880	0.7541	0.6121	0.8532	0.8723	$\hat{M}$ belongs to $\hat{C}_5$
Xu [51]	0.7934	0.7602	0.7602	0.7595	0.8455	$\hat{M}$ belongs to $\hat{C}_5$
Xu and Cai [52]	0.8074	0.7718	0.7596	0.7580	0.8210	$\hat{M}$ belongs to $\hat{C}_5$
Huang and Guo [29]	0.7480	0.6871	0.4626	0.8016	0.8473	$\hat{M}$ belongs to $\hat{C}_5$

Similarly, the existing IFCC approaches yield the same pattern recognition, but the new approach shows that a better correlation exists between the unknown hybrid mineral $\hat{M}$ and the mineral field $\hat{C}_5$.

4.2 Pattern recognition of building materials

In this second case, a pattern recognition problem regarding the classification/grouping of some building materials is considered. Assuming there are four given classes of building material, which are represented by IFSs $\hat{M}_1$, $\hat{M}_2$, $\hat{M}_3$, and $\hat{M}_4$ in the feature space $S = \{s_1, s_2, \cdots, s_{12}\}$.
Given another kind of unknown building material $\hat{N}$, we seek to associate the unknown pattern $\hat{N}$ with any of the appropriate known patterns based on the intuitionistic fuzzy correlation measures. The intuitionistic fuzzy information of the patterns are presented in Table 8.

Table 8: Building Materials as IFVs

	Feature Space											
Patterns	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}
$\alpha_{\hat{M}_1}$	0.173	0.102	0.530	0.965	0.420	0.008	0.331	1.000	0.215	0.432	0.750	0.432
$\beta_{\hat{M}_1}$	0.524	0.818	0.326	0.008	0.351	0.956	0.512	0.000	0.625	0.534	0.126	0.432
$\gamma_{\hat{M}_1}$	0.303	0.080	0.144	0.027	0.229	0.036	0.157	0.000	0.160	0.034	0.124	0.136
$\alpha_{\hat{M}_2}$	0.510	0.627	1.000	0.125	0.026	0.732	0.556	0.650	1.000	0.145	0.047	0.760
$\beta_{\hat{M}_2}$	0.365	0.125	0.000	0.648	0.823	0.153	0.303	0.267	0.000	0.762	0.923	0.231
$\gamma_{\hat{M}_2}$	0.125	0.248	0.000	0.227	0.151	0.115	0.141	0.083	0.000	0.093	0.030	0.009
$\alpha_{\hat{M}_3}$	0.495	0.603	0.987	0.073	0.037	0.690	0.147	0.213	0.501	1.000	0.324	0.045
$\beta_{\hat{M}_3}$	0.387	0.298	0.006	0.849	0.923	0.268	0.812	0.653	0.284	0.000	0.483	0.912
$\gamma_{\hat{M}_3}$	0.118	0.099	0.007	0.078	0.040	0.042	0.041	0.134	0.215	0.000	0.193	0.043
$\alpha_{\hat{M}_4}$	1.000	1.000	0.857	0.734	0.021	0.076	0.152	0.113	0.489	1.000	0.386	0.028
$\beta_{\hat{M}_4}$	0.000	0.000	0.123	0.158	0.896	0.912	0.712	0.756	0.389	0.000	0.485	0.912
$\gamma_{\hat{M}_4}$	0.000	0.000	0.020	0.108	0.083	0.012	0.136	0.131	0.122	0.000	0.129	0.060
$\alpha_{\hat{N}}$	0.978	0.980	0.798	0.693	0.051	0.123	0.152	0.113	0.494	0.987	0.376	0.012
$\beta_{\hat{N}}$	0.003	0.012	0.132	0.213	0.876	0.756	0.721	0.732	0.368	0.000	0.423	0.897
$\gamma_{\hat{N}}$	0.019	0.008	0.070	0.094	0.073	0.121	0.127	0.155	0.138	0.013	0.201	0.091

To obtain the grouping of the unknown building material $\hat{N}$ with $\hat{M}_i$, for $i = 1, 2, 3, 4$, we calculate its correlation coefficients with each $\hat{M}_i$ using the new approach and similar approaches [29, 51, 52] to get the following results:
New approach yields;

$$\tilde{\rho}(\hat{M}_1, \hat{N}) = 0.6414,\ \tilde{\rho}(\hat{M}_2, \hat{N}) = 0.6118,$$

$$\tilde{\rho}(\hat{M}_3, \hat{N}) = 0.8143,\ \tilde{\rho}(\hat{M}_4, \hat{N}) = 0.9632,$$

which shows that the unknown building material $\hat{N}$ can be associated with building material $\hat{M}_4$ because the correlation coefficient between $(\hat{M}_4, \hat{N})$ is the greatest.
The approach of Xu [51] yields;

$$\rho_6(\hat{M}_1, \hat{N}) = 0.8098,\ \rho_6(\hat{M}_2, \hat{N}) = 0.7030,$$

$$\rho_6(\hat{M}_3, \hat{N}) = 0.8086,\ \rho_6(\hat{M}_4, \hat{N}) = 0.8113.$$

The approach of Xu and Cai [52] yields;

$$\rho_7(\hat{M}_1, \hat{N}) = 0.8195,\ \rho_7(\hat{M}_2, \hat{N}) = 0.7184,$$

$$\rho_7(\hat{M}_3, \hat{N}) = 0.7854,\ \rho_7(\hat{M}_4, \hat{N}) = 0.8289.$$

The approach of Huang and Guo [29] yields;

$$\rho_8(\hat{M}_1, \hat{N}) = 0.5182,\ \rho_8(\hat{M}_2, \hat{N}) = 0.4818,$$

$$\rho_8(\hat{M}_3, \hat{N}) = 0.7550,\ \rho_8(\hat{M}_4, \hat{N}) = 0.9642.$$

Table 9 presents the results of the IFCC values.

Table 9: Results for Classification of Building Materials

IFCC Methods	$(\hat{M}_1, \hat{N})$	$(\hat{M}_2, \hat{N})$	$(\hat{M}_3, \hat{N})$	$(\hat{M}_4, \hat{N})$	Classifications
New IFCC	0.6414	0.6118	0.8143	0.9632	$\hat{N}$ belongs to $\hat{M}_4$
Xu [51]	0.8098	0.7030	0.8086	0.8113	$\hat{N}$ belongs to $\hat{M}_4$
Xu and Cai [52]	0.8195	0.7184	0.7854	0.8289	$\hat{N}$ belongs to $\hat{M}_4$
Huang and Guo [29]	0.5182	0.4818	0.7550	0.9642	$\hat{N}$ belongs to $\hat{M}_4$

From the existing IFCC approaches, it follows that the unknown building material $\hat{N}$ can be associated with building material $\hat{M}_4$, akin to the interpretation from the new approach. In the whole, our approach is better than the approach in [29, 51, 52].

5 Conclusions

In this work, we have developed a new IFCC approach which measures correlation reliably better than the existing approaches [29, 51, 52]. The new approach is an improved version of the method in [29] with a better reliability rating because it does not provide any leeway for information loss unlike the approaches in [24, 29, 51]. The properties of the new approach were discussed, and easy to follow illustrative examples of the approach were provided. By comparative analysis, it has been shown that where the existing approaches fail, the new IFCC approach gives a better measure of correlation. Finally, the new approach was applied to tackle problems of pattern recognition because of its flexibility in decision-making. The following are some of advantages of the new approach; (i) it incorporates the complete parameters of IFSs to avoid error of omission, (ii) it hybridizes the IFCC approaches in [29, 51, 52] with reliable output, reasonable interpretation, and mathematical correctness, (iii) it can correctly measure the correlation between two similar IFSs, and also two equal IFSs unlike [51, 52], (iv) it possesses better performance rating which enhances reliable interpretation than the other tri-parametric approaches in [52, 53, 57]. In future studies, the new approach could be investigated in TOPSIS method, multiple criteria decision-making, and multiple group attributes decision-making based on spherical fuzzy data, q-rung orthopair fuzzy data, and picture fuzzy data.

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