Iranian Journal of Fuzzy Systems

Volume 20, Number 5, (2023), pp. 189-197

Original paper

Intuitionistic fuzzy type basic uncertain information

L. S. Jin[1], R. R. Yager[2], C. Ma[3], L. M. Lopez[4], R. M. Rodríguez[5], T. Senapati[6] and R. Mesiar[7]

[1]*School of Automobile and Traffic Engineering, Hubei University of Arts and Sciences, Xiangyang, 441053, China; School of Business, Nanjing Normal University, Nanjing, China*

[2]*Machine Intelligence Institute, Iona College, New Rochelle, NY*

[3]*Hubei Key Laboratory of Power System Design and Test for Electrical Vehicle, Hubei University of Arts and Science, Xiangyang, 441053, China; School of Automobile and Traffic Engineering, Hubei University of Arts and Sciences, Xiangyang, 441053, China*

[4,5]*Department of Computer Science, University of Jaen, 23071-Jaen, Spain*

[6]*Department of Mathematics, Padima Janakalyan Banipith, Kukrakhupi, Jhargram, 721517, India*

[7]*Faculty of Civil Engineering, Slovak University of Technology, Radlinskho 11, Sk-810 05 Bratislava, Slovakia; Institute for Research and Applications of Fuzzy Modeling, University of Ostrava, CE IT4Innovations, 30. dubna 22, 701 03 Ostrava, Czech Republic*

jls1980@163.com, Yager@panix.com, mmma1123@1s63.com, martin@ujaen.es, rmrodrig@ujaen.es
math.tapan@gmail.com, radko.mesiar@stuba.sk

Abstract

Recently, a new paradigm for uncertain information has been proposed that can effectively handle various types of uncertainty in decision-making problems. This approach utilizes a certainty degree, which is represented by a real number indicating the level of certainty associated with input values. However, just like intuitionistic fuzzy information can handle more problems that cannot be well modeled by fuzzy information, the certainty degree in basic uncertain information can also be intuitionistic fuzzy granule, which allows it to handle more uncertainty involved decision making situations. In this paper, we introduce the concept of intuitionistic fuzzy type basic uncertain information and explain its parameters. We also define a weighted arithmetic mean for aggregating this type of information and discuss different approaches for allocating induced weights based on trust preferred preference from four perspectives: (i) preference for higher certainty degrees; (ii) aversion to higher levels of uncertainty; (iii) preference for greater differences in certainty degrees; and (iv) preference for intuitionistic fuzzy certainties. Additionally, we explore trichotomic rules-based decision making using intuitionistic fuzzy type basic uncertain information. Finally, we present an objective-subjective evaluation numerical example utilizing these methods.

Keywords: Aggregation operator, basic uncertain information, information fusion, intuitionistic fuzzy type basic uncertain information, preference involved evaluation, rules-based decision making.

1 Introduction

Various preferences modeling techniques have been proposed to handle different types of uncertainty, including interval information, fuzzy information [36], vague information [12], probability information and possibility information [38]. Despite some mathematical similarities between certain types of uncertain data, such as interval numbers within unit intervals and intuitionistic fuzzy granules [1], they may still have distinct applications and meanings. Therefore, exploring their interrelations, transformations and standard expressions is crucial for advancing the theory and application of uncertain data.

In recent decades, the field of information fusion has placed great emphasis on dealing with uncertain information environments [2, 3, 17, 18, 20]. This has led to a surge in research focused on aggregation operators [13], which are

Corresponding Author: C. Ma

Received: April 2023; Revised: July 2023; Accepted: September 2023.

https://doi.org/10.22111/IJFS.2023.7840

highly valued for their consistent and objective mathematical frameworks. These operators have proven particularly useful in evaluation and decision-making scenarios where inputs may be uncertain or varied [8, 9, 10, 11, 15, 21, 32].

Basic uncertain information (BUI), a novel and intriguing normative uncertain paradigm, has recently been introduced [16, 26]. Subsequently, it has garnered attention for further exploration, investigation, and practical implementation [5, 6, 7, 22, 23, 25, 30, 31]. A BUI granule is expressed by a pair (a, c) in which $a \in [0, 1]$ is a concerned input value (or evaluation value) and $c \in [0, 1]$ is the certainty degree of a; dually, $1 - c \in [0, 1]$, the complement of certainty degree, is the uncertainty degree of a. When $c = 1$, $(a, 1)$ indicates the full certainty over input value a and thus in some situations it can be identified with the real input value a; when $c = 0$, $(a, 0)$ indicates the full uncertainty over input value a, and thus no effective information we can derive.

The level of certainty can have various interpretations, such as indicating the level of assurance a decision maker has in their estimated input value, the degree of belief in an input value, the reliability of the source providing an input value, the proportion of experts who endorse or accept a given input value in group decision-making scenarios, and so forth.

However, in some decision situations, the involved certainty can be still a type of uncertain information. We may take a new nested form $(a, \hat{c})$ where $\hat{c}$ is a given type of uncertain information. The level of uncertainty can be effectively represented by intuitionistic fuzzy granules, which are commonly encountered in various evaluation scenarios. While mathematically similar to interval numbers within a unit interval, intuitionistic fuzzy granules may possess distinct interpretations when applied to practical problems. For example, the form $(a, \hat{c})$ can model the group evaluation problem where $\hat{c}$ is an intuitionistic fuzzy granule (μ, ν) with $\mu + \nu \in [0, 1]$. Suppose $a \in [0, 1]$ represents a predicted value and a panel of experts can vote on whether they agree or disagree with the prediction. Then, the percentage of the experts who believe (respectively, do not believe) this predication can be denoted by $100\mu\%$ (respectively, $100\nu\%$); note that in intuitionistic fuzzy environment, it is possible that $\mu + \nu < 1$, which in this case means the reminder $100\,(1 - (\mu + \nu))\,\%$ of the experts have no opinions (or are indifferent) about whether to believe or not.

Given the current circumstances, it is necessary to introduce a novel approach for representing uncertain situations using intuitionistic fuzzy data. This study aims to formally establish the concept of basic uncertain information in an intuitionistic fuzzy framework, along with its relations to weighted arithmetic mean, allocation methods for induced weights, and rules-based decision-making.

The remaining part of this study is structured as follows. in Section 2, we establish specific terminologies. Section 3 formally defines intuitionistic fuzzy type basic uncertain information with some related parameters and aggregation. Section 4 explores various approaches to allocating induced weights based on preferences from four different perspectives. Section 5 delves into trichotomic rules-based decision making with intuitionistic fuzzy type basic uncertain information. 6, we illustrate the application of objectivity-subjectivity evaluation in water resource management through a numerical example. Finally, Section 7 provides concluding remarks for this study.

2 Review of some terminologies

A real vector of dimension n is denoted by $\mathbf{a} = (a_i)_{i=1}^n \in [0, 1]^n$. All of the closed intervals (or called interval values, interval numbers) $[a, b] \subseteq [0, 1]$ are denoted by $\mathcal{I}$. In addition, $[a, a]$ is sometimes identified with real number a if there is no confusion arising. For intervals, recall the lattice $(\mathcal{I}, \leq_{Int})$ with the standard partial order $\leq_{Int}$ such that $[a_1, b_1] \leq_{Int} [a_2, b_2]$ if and only if $a_1 \leq a_2$ and $b_1 \leq b_2$; denote $[a_1, b_1] <_{Int} [a_2, b_2]$ if and only if $[a_1, b_1] \leq_{Int} [a_2, b_2]$ and $[a_1, b_1] \neq [a_2, b_2]$.

Definition 2.1. [16, 26]

(i) A basic uncertain information (BUI) granule is expressed by a pair (a, c) in which $a \in [0, 1]$ is a concerned input value (or evaluation value) and $c \in [0, 1]$ is the certainty degree of a; $1 - c \in [0, 1]$ is the uncertainty degree of a.

(ii) The set of all BUI granules (a, c) is denoted by $\mathcal{B}$. A BUI vector is denoted by $(\mathbf{a}, \mathbf{c}) = (a_i, c_i)_{i=1}^n \in \mathcal{B}^n$ where $\mathbf{a} = (a_i)_{i=1}^n \in [0, 1]^n$ represents an input (or evaluation) vector while $\mathbf{c} = (c_i)_{i=1}^n \in [0, 1]^n$ is the certainty (degree) vector corresponding to a.

Definition 2.2. [1] *An intuitionistic fuzzy granule (also known as intuitionistic fuzzy number) is denoted by a pair $(\mu, \nu) \in [0, 1]^2$ such that $\mu + \nu \in [0, 1]$. $\mu \in [0, 1]$ is the membership degree and $\nu \in [0, 1]$ is called the non-membership degree. $\mu - \nu \in [-1, 1]$ is called the score of (μ, ν) and $\mu + \nu$ is called the accuracy degree of (μ, ν). The set of all intuitionistic fuzzy granules is denoted by $\mathcal{IF}$. A vector of n intuitionistic fuzzy granules is denoted by*

$$(\mu, \nu) = (\mu_i, \nu_i)_{i=1}^n \in (\mathcal{IF})^n.$$

Unless otherwise noted, the weight vector used in this work is normalized weight vector (of dimension n) $\mathbf{w} = (w_i)_{i=1}^n \in [0, 1]^n$ such that $\sum_{i=1}^n w_i = 1$.

As the mathematical structure of intuitionistic fuzzy granules is equivalent to that of intervals, it is possible to derive the following two orders directly from their interval counterparts. First, similar to the standard interval order $\leq_{Int}$, we naturally have the following counterpart of intuitionistic fuzzy order.

Definition 2.3. *The standard partial order for intuitionistic fuzzy granules*

The intuitionistic fuzzy order $\leq_{IF}$ and the corresponding poset $(\mathcal{IF}, \leq_{IF})$ is defined such that for two intuitionistic fuzzy granules $(\mu_1, \nu_1), (\mu_2, \nu_2) \in [0,1]^2$, $(\mu_1, \nu_1) \leq_{IF} (\mu_2, \nu_2)$ if and only if $\mu_1 \leq \mu_2$ and $\nu_1 \geq \nu_2$; denote $(\mu_1, \nu_1) <_{IF} (\mu_2, \nu_2)$ if and only if $(\mu_1, \nu_1) \leq_{IF} (\mu_2, \nu_2)$ and $(\mu_1, \nu_1) \neq (\mu_2, \nu_2)$.

Similar to $(\alpha, \beta)-$order [4] (defined for intervals), we can have the following counterpart of linear intuitionistic fuzzy order.

Definition 2.4. *The $(\alpha, \beta)-$order for intuitionistic fuzzy granules*

The linearly ordered set $(\mathcal{IF}, \leq_{(\alpha,\beta)})$ ($\alpha, \beta \in [0,1]$ and $\alpha \neq \beta$) for intuitionistic fuzzy granules is defined such that for two intuitionistic fuzzy granules $(\mu_1, \nu_1), (\mu_2, \nu_2) \in [0,1]^2$, $(\mu_1, \nu_1) \leq_{(\alpha,\beta)} (\mu_2, \nu_2)$ if and only if

$$(1-\alpha)\mu_1 + \alpha(1-\nu_1) < (1-\alpha)\mu_2 + \alpha(1-\nu_2) \text{ or } (1-\alpha)\mu_1 + \alpha(1-\nu_1) = (1-\alpha)\mu_2 + \alpha(1-\nu_2),$$

and $(1-\beta)\mu_1 + \beta(1-\nu_1) \leq (1-\beta)\mu_2 + \beta(1-\nu_2)$.

3 Intuitionistic fuzzy type basic uncertain information

In this segment, we provide the explanation for intuitionistic fuzzy type basic uncertain information.

Definition 3.1. *Intuitionistic fuzzy type basic uncertain information*

(i) A granule of intuitionistic fuzzy type basic uncertain information (IFBUI) is expressed in the form $(a, (\mu, \nu))$, in which $a \in [0,1]$ is the input value (or evaluation value) and $(\mu, \nu) \in \mathcal{IF}$ is the intuitionistic fuzzy certainty degree of a, measuring the degree of being trusted, convincing or believable, etc., of input value a. Specifically, we give $\mu \in [0,1]$ and $\nu \in [0,1]$ some different meanings in IFBUI; μ is called the certainty degree, ν is called the uncertainty degree, and $\mu - \nu \in [-1,1]$ is called the certainty difference (of certainty degree and uncertainty degree).

(ii) The set of all IFBUI granules is denoted by $\mathcal{IFB}$. A vector of IFBUI granules is denoted by $(\mathbf{a}, (\mu, \nu)) = (a_i, (\mu_i, \nu_i))_{i=1}^n$ in which a is called an evaluation vector and (μ, ν) is called an intuitionistic fuzzy vector formed by intuitionistic fuzzy certainty degrees.

Remark 3.2. *For an IFBUI granule $(a, (\mu, \nu))$, if $\mu + \nu = 1$, it actually degenerates into BUI granule (a, μ).*

Remark 3.3. *For two IFBUI granules $(a_1, (\mu_1, \nu_1))$ and $(a_1, (\mu_2, \nu_2))$, if $(\mu_1, \nu_1) <_{IF} (\mu_2, \nu_2)$, it means evaluation value a_1 is with lower certainty than a_2. When (μ_1, ν_1) and (μ_2, ν_2) are not comparable, we have no information about which evaluation value has a higher certainty. For any IFBUI granule, it attains maximum interval certainty degree $(1,0)$ and minimum interval certainty degree $(0,1)$.*

The interval weighted arithmetic mean and intuitionistic fuzzy weighted arithmetic mean are commonly known and reviewed as follows.

Definition 3.4. *The interval weighted arithmetic mean (with weight vector w) $ItWAM_{\mathbf{w}} : \mathcal{I}^n \to \mathcal{I}$ is defined such that*

$$ItWAM_{\mathbf{w}}([\mathbf{a}, \mathbf{b}]) = \left[\sum\nolimits_{i=1}^n w_i a_i, \sum\nolimits_{i=1}^n w_i b_i\right]. \tag{1}$$

Definition 3.5. *The intuitionistic fuzzy weighted arithmetic mean (with weight vector w) $IFWAM_{\mathbf{w}} : \mathcal{B}^n \to \mathcal{B}$ is defined such that*

$$IFWAM_{\mathbf{w}}(\mu, \nu) = \left(\sum\nolimits_{i=1}^n w_i \mu_i, \sum\nolimits_{i=1}^n w_i \nu_i\right). \tag{2}$$

The BUI weighted arithmetic mean is reviewed as follows.

Definition 3.6. [16] *The BUI weighted arithmetic mean (with weight vector w) $BWAM_{\mathbf{w}} : \mathcal{B}^n \to \mathcal{B}$ is defined such that*

$$BWAM_{\mathbf{w}}(\mathbf{a}, \mathbf{c}) = \left(\sum\nolimits_{i=1}^n w_i a_i, \sum\nolimits_{i=1}^n w_i c_i\right). \tag{3}$$

With the above analyses and preparations, we can directly define the weighted arithmetic means for IFBUI vectors.

Definition 3.7. *The intuitionistic fuzzy type basic uncertain information weighted arithmetic mean (with weight vector w)* $IFBWAM_{\mathbf{w}} : (\mathcal{IFB})^n \to \mathcal{IFB}$ *is defined such that*

$$IFBWAM_{\mathbf{w}}(\mathbf{a},(\mu,\nu)) = \left(\sum_{i=1}^{n} w_i a_i, (\sum_{i=1}^{n} w_i \mu_i, \sum_{i=1}^{n} w_i \nu_i)\right). \tag{4}$$

Remark 3.8. *We will not delve into the discussion of certain fundamental characteristics, such as continuity, monotonicity, and idempotency, that are evident in interval aggregation.*

4 Induced weights allocation with intuitionistic fuzzy type basic uncertain information

For a given vector of IFBUI granules $(\mathbf{a},(\mu,\nu))$, in the preceding section we have defined IFBUI weighted arithmetic mean, and there are different ways to determine the weight vector w for use. Here we consider the induced weights allocation [35]. The inducing information we adopt is in relation to intuitionistic fuzzy certainty degrees (μ,ν).

In most of the decision scenarios, decision makers may prefer those IFBUI granules $(a_i,(\mu_i,\nu_i))$ with large intuitionistic fuzzy certainty degrees. Observe that for two IFBUI granlues $(a_1,(\mu_1,\nu_1))$ and $(a_1,(\mu_2,\nu_2))$, if $(\mu_1,\nu_1)<_{IF}(\mu_2,\nu_2)$, then we necessarily have (i) $\mu_1 \le \mu_2$, (ii) $\nu_1 \ge \nu_2$, (iii) $\mu_1-\nu_1 < \mu_2-\nu_2$. But for two IFBUI granules $(a_1,(\mu_1,\nu_1))$ and $(a_1,(\mu_2,\nu_2))$, if $\mu_1 \le \mu_2$ (or $\nu_1 \ge \nu_2$), we do not necessarily have $\nu_1 \ge \nu_2$ (or $\mu_1 \le \mu_2$); if $\mu_1-\nu_1 < \mu_2-\nu_2$, we do not necessarily have $\mu_1 \le \mu_2$ or $\nu_1 \ge \nu_2$. In addition, neither the single condition $\mu_1 \le \mu_2$, nor $\nu_1 \ge \nu_2$, and nor $\mu_1-\nu_1 < \mu_2-\nu_2$ can guarantee $(\mu_1,\nu_1)<_{IF}(\mu_2,\nu_2)$. Hence, it is reasonable to model such preference from four different aspects: (i) certainty degree preferred preference, (ii) uncertainty degree averted preference, (iii) certainty difference preferred preference, and (iv) intuitionistic fuzzy certainty preferred preference.

Furthermore, after successfully modeling the aforementioned four preference types, we can employ a weight vector to calculate their weighted average. Various methods exist for determining the weight vector, including empirical approaches and mathematical tools. In this study, we will utilize the Laplace decision criterion and subsequently compute the average of these four preference types.

Indeed, one can also consider the poset $(\mathcal{IF},\le_{IF})$ and $(\alpha,\beta)-$order. We do not discuss such order in this work is partly because the determination of parameters α and β is not particularly clear in practice, not like the general acceptance of orness/andness determination methods in ordered weighted averaging (OWA) aggregation [33].

To realize and quantify each type of preference, a very powerful and convenient way is to take some desired basic unit monotonic (BUM) functions [34] (which serve as preference indicators), and then apply the recently proposed three-set method [19] (which can effectively avoid the difficulties of tied inducing values).

Recall that a BUM function $Q:[0,1]\to[0,1]$ is a non-decreasing function with $Q(0)=0$ and $Q(1)=1$. The orness of a BUM function is defined by $orness(Q)=\int_0^1 Q(t)dt$ and the andness of a BUM function is defined by $andness(Q)=1-orness(Q)$ [24]. When the selected BUM functions for use are convex (a commonly used system of functions is $Q(y)=y^n$ with $n\ge 1$) or concave (a commonly used system of functions is $Q(y)=1-(1-y)^n$ with $n\ge 1$), a larger BUM function can well model a stronger preference extent exhibited by decision maker, and vice versa.

Let $(S,\le_P)$ be a poset with partial order $\le_P$, and $\mathbf{x}=(x_i)_i^n \in S^n$ be a vector of poset values. Let Q be a BUM function. We can then use the three-set method [19] to determine a unique weight vector $\mathbf{w}=(w_i)_{i=1}^n$ for the poset valued vector $\mathbf{x}=(x_i)_i^n$ (i.e., w_i is assigned to index i).

The three-set method for generating weights [19].

Step 1: For each x_i, define three disjoint subsets of $\{1,...,n\}$: $A_i,B_i,E_i \subseteq \{1,...,n\}$ such that

$A_i=\{j\in\{1,...,n\}: x_i<_P x_j\}$, $B_i=\{j\in\{1,...,n\}: x_j<_P x_i\}$ and $E_i=\{1,...,n\}\backslash(A_i\cup B_i)$.

Step 2: Form an intermediate vector $\mathbf{s}=(s_i)_{i=1}^n \in [0,1]^n$ (which is not necessarily normalized) such that

$$s_i = \frac{Q(1-\frac{|B_i|}{n}) - Q(\frac{|A_i|}{n})}{|E_i|}. \tag{5}$$

where $|S|$ is the cardinality of finite set S.

Step 3: It can be shown that $\mathbf{s}\neq\mathbf{0}=(0,...,0)$ [19], and then after normalizing s, we obtain a normalized weight vector $\mathbf{w}=(w_i)_{i=1}^n$ by

$$w_i = \frac{s_i}{\sum_{k=1}^n s_k}. \tag{6}$$

Returning to the four types of preference previously listed, poset $(S,\le_P)$ and inducing vector $\mathbf{x}=(x_i)_i^n\in S^n$ can have the corresponding four detailed instances:

(i) $([0,1],\leq)$ and $\mu=(\mu_i)_i^n\in[0,1]^n$;
(ii) $([0,1],\geq)$ and $\nu=(\nu_i)_i^n\in[0,1]^n$;
(iii) $([-1,1],\leq)$ and $\mathbf{y}=(\mu_i-\nu_i)_i^n\in[-1,1]^n$;
(iv) $(\mathcal{IF},\leq_{IF})$ and $(\mu,\nu)=(\mu_i,\nu_i)_i^n\in\mathcal{IF}$.

Then, we can form four weight vectors $\mathbf{w}_1=(w_{1i})_{i=1}^n$, $\mathbf{w}_2=(w_{2i})_{i=1}^n$, $\mathbf{w}_3=(w_{3i})_{i=1}^n$ and $\mathbf{w}_4=(w_{4i})_{i=1}^n$ corresponding to μ, ν, $\mathbf{y}$ and (μ,ν), respectively, using three-set method four times. Lastly, we can take the mean of the four derived weight vectors and obtain a final weight vector $\mathbf{v}=\frac{1}{4}\sum_{k=1}^4\mathbf{w}_k$ as the preference which comprehensively embodies the four types of preferences. An illustrative example will be postponed until in Section 5.

5 Trichotomic rules-based decision making with intuitionistic fuzzy type basic uncertain information

Automated decision making scenarios greatly benefit from the application of rules-based decision making [27, 29, 37]. Moreover, the utilization of pre-established rules often facilitates the attainment of more rational and unbiased final decisions.

With a given IFBUI granule $(a,(\mu,\nu))$, we can design some complete automatic decision rules from elementary rule conditions, some of which are exemplified and roughly classified below:

(i) with $x\in[0,1]$, $\mu\geq x$, $\mu(1-\nu)\geq x$, $(1-\mu)\nu\leq x$,
(ii) with $x\in[-1,1]$, $\mu-\nu\geq x$,
(iii) with $(x,y)\in\mathcal{IF}$, $(x,y)\leq_{IF}(\mu,\nu)$.

The previous elementary rule conditions in (i) - (iii) are certainty preferred.

(iv) with $x\in(0,1]$, $\nu<x$, $\mu(1-\nu)<x$, $(1-\mu)\nu>x$,
(v) with $x\in(-1,1]$, $\nu-\mu<x$,
(vi) with $(x,y)\in\mathcal{IF}$, $(\mu,\nu)<_{IF}(x,y)$.

Meanwhile, the elementary rule conditions in (iv) - (vi) are uncertainty averted.

By logically combining multiple elementary rule conditions, decision makers can create various rules for making decisions based on their practical experiences. In what follows we only discuss some trichotomic rules because many categorized values are based on trichotomy. For example, intuitionistic fuzzy granules are actually based on trichotomy; in decision making, μ may represent the acceptance degree (or the percentage of who believe as mentioned in Introduction), ν may represent the refuse degree (or the percentage of who do not believe), and $1-(\mu+\nu)$ may represent the ignorance (or the percentage of who are indifferent). For another example, sometimes if a parameter of a product is larger than or equal to a threshold, we may then consider it as qualified; if it is smaller than another threshold, then it can be regarded as unqualified; else, some further analysis will be carried out to examine its quality.

The trichotomic decision rules discussed here can be directly derived from the corresponding thichotomic partitions which are derived by some well defined decision rules. Let $\{P_j\}_{j=1}^3$ be a partition with $\bigcup_{j=1}^3 P_j=\mathcal{IFB}$ and let { 1,2,3} be the decision space, and thus if $(a,(\mu,\nu))\in P_i$ $(i=1,2,3)$, then we will take decision i $(i\in\{1,2,3\})$. Below are cases of trichotomic partition in which we divide the domain $\mathcal{IFB}$ into three parts $\{P_i\}_{i=1}^3$ with $\bigcup_{i=1}^3 P_i=\mathcal{IFB}$.

Trichotomic partition (i):

$P_1=\{(z,(x,y))\in\mathcal{IFB}:z<d,x\geq q\}$, $P_2=\mathcal{IFB}\backslash(P_1\cup P_3)$, $P_3=\{(z,(x,y))\in\mathcal{IFB}:z\geq s,x\geq q\}$ $(0<d<s\leq 1,\ 0<q\leq 1)$.

Trichotomic partition (ii):

$P_1=\{(z,(x,y))\in\mathcal{IFB}:z<d,y\leq q\}$, $P_2=\mathcal{IFB}\backslash(P_1\cup P_3)$, $P_3=\{(z,(x,y))\in\mathcal{IFB}:z\geq s,y\leq q\}$ $(0<d<s\leq 1,\ 0<q\leq 1)$.

Trichotomic partition (iii):

$P_1=\{(z,(x,y))\in\mathcal{IFB}:z<d,x(1-y)\geq q\}$, $P_2=\mathcal{IFB}\backslash(P_1\cup P_3)$, $P_3=\{(z,(x,y))\in\mathcal{IFB}:z\geq s,x(1-y)\geq q\}$ $(0<d<s\leq 1,\ 0<q\leq 1)$.

Trichotomic partition (iv):

$P_1=\{(z,(x,y))\in\mathcal{IFB}:z<d,(1-x)y\geq q\}$, $P_2=\mathcal{IFB}\backslash(P_1\cup P_3)$, $P_3=\{(z,(x,y))\in\mathcal{IFB}:z\geq s,(1-x)y\geq q\}$ $(0<d<s\leq 1,\ 0<q\leq 1)$.

Trichotomic partition (v):

$P_1=\{(z,(x,y))\in\mathcal{IFB}:z<d,x-y\geq q\}$, $P_2=\mathcal{IFB}\backslash(P_1\cup P_3)$, $P_3=\{(z,(x,y))\in\mathcal{IFB}:z\geq s,x-y>q\}$ $(0<d<s\leq 1,\ -1<q\leq 1)$.

Trichotomic partition (vi):

$P_1=\{(z,(x,y))\in\mathcal{IFB}:z<d,(q,p)\leq_{IF}(x,y)\}$, $P_2=\mathcal{IFB}\backslash(P_1\cup P_3)$, $P_3=\{(z,(x,y))\in\mathcal{IFB}:z\geq s,(q,p)\leq_{IF}(x,y)\}$ $((0<d<s\leq 1,\ (0,1)\leq_{IF}(q,p))$.

The selection of parameters can be tailored to specific circumstances, allowing for the creation of a variety of trichotomic partitions in practical applications. An illustrative example using the above rules-based decision making is presented in Section 5.

6 Intuitionistic fuzzy type basic uncertain information in objectivity-subjectivity evaluation

In this segment, a numerical instance is provided that encompasses both objective and subjective assessments within an environment of basic uncertain information in the form of intuitionistic fuzzy type.

Assessing the feasibility of constructing a reservoir is a crucial aspect of water resource projects and evaluations [14]. There are four major criteria $\{C_i\}_{i=1}^4$ for evaluating the feasibility of building a reservoir as follows [28]:

C_1: Management, construction and hardware

C_2: Flood preventing and drought resisting ability

C_3: Environment production and water quality

C_4: Economic effect

Suppose an original weight vector $\mathbf{u} = (u_i)_{i=1}^4 = (0.2, 0.3, 0.4, 0.1)$, representing the relative importance between the four criteria.

Suppose the municipal authorities have deployed investigators to gather impartial assessments for each criterion, and they have also enlisted experts to assess the credibility of these evaluations.

Let $\mathbf{a} = (a_i)_{i=1}^4 \in [0,1]^4$ be a vector in which a_i is the objective evaluation value for criterion C_i $(i \in \{1,2,3,4\})$. Suppose for each individual objective evaluation a_i, there are $100\mu_i\%$ of the invited experts believe it, $100\nu_i\%$ of the invited experts do not believe it, and $100(1-\mu_i-\nu_i)\%$ of the invited experts have no judgments and opinions about whether or not to believe it. Hence, the above initial information can be conveniently modeled by a vector of intuitionistic fuzzy type basic uncertain information $(\mathbf{a}, (\mu, \nu)) = (a_i, (\mu_i, \nu_i))_{i=1}^4$.

To determine a subjective weight vector $\mathbf{v} = (v_i)_{i=1}^4$ for the four criteria $\{C_i\}_{i=1}^4$ which embody the preference of the government, suppose the government has a moderate trust-preferred preference that prefers to assign more weights to the criteria with higher certainty degrees. That is, such trust-preferred preference can be comprehensively embodied from the four aspects as discussed in Section 3: (i) certainty degree preferred preference, (ii) uncertainty degree averted preference, (iii) certainty difference preferred preference, and (iv) intuitionistic fuzzy certainty preferred preference. In addition, we adopt a moderate such preference, we may take the BUM function $Q(y) = 1-(1-y)^2$.

Assume $(\mathbf{a}, (\mu, \nu)) = (a_i, (\mu_i, \nu_i))_{i=1}^4$ satisfies

$$(a_1, (\mu_1, \nu_1)) = (0.5, (0.2, 0.5)),\ (a_2, (\mu_2, \nu_2)) = (0.4, (0.3, 0.4)),$$

$$(a_3, (\mu_3, \nu_3)) = (0.7, (0.4, 0.5)),\ (a_4, (\mu_4, \nu_4)) = (0.9, (0.3, 0.3)).$$

Then, we form four weight vectors $\mathbf{w}_1 = (w_{1i})_{i=1}^4$, $\mathbf{w}_2 = (w_{2i})_{i=1}^4$, $\mathbf{w}_3 = (w_{3i})_{i=1}^4$ and $\mathbf{w}_4 = (w_{4i})_{i=1}^4$ corresponding to $\mu = (\mu_i)_i^4 = (0.2, 0.3, 0.4, 0.3)$, $\nu = (\nu_i)_i^4 = (0.5, 0.4, 0.5, 0.3)$, $\mathbf{y} = (\mu_i - \nu_i)_i^4 = (-0.3, -0.1, -0.1, 0)$ and $(\mu, \nu) = (\mu_i, \nu_i)_i^4$, respectively, using three-set method four times with the same BUM function $Q(y) = 1-(1-y)^2$.

After calculation by (5) and (6), we have $\mathbf{w}_1 = (\frac{1}{16}, \frac{1}{4}, \frac{7}{16}, \frac{1}{4})$, $\mathbf{w}_2 = (\frac{1}{8}, \frac{5}{16}, \frac{1}{8}, \frac{7}{16})$, $\mathbf{w}_3 = (\frac{1}{16}, \frac{1}{4}, \frac{1}{4}, \frac{7}{16})$, and $\mathbf{w}_4 = (\frac{1}{16}, \frac{1}{4}, \frac{5}{16}, \frac{3}{8})$. Take the mean of the four derived weight vectors and obtain a final weight vector $\mathbf{v} = (v_i)_{i=1}^4 = \frac{1}{4}\sum_{k=1}^4 \mathbf{w}_k \doteq (0.078, 0.266, 0.281, 0.375)$ (i.e., $v_i = \frac{1}{4}\sum_{k=1}^4 w_{ki}$ for all $i \in \{1,2,3,4\}$). With equally considering the original objective weight vector $\mathbf{u} = (u_i)_{i=1}^4 = (0.2, 0.3, 0.4, 0.1)$ and the subjective preference derived weight vector $\mathbf{v} = (0.078, 0.266, 0.281, 0.375)$, we have a new weight vector $\mathbf{r} = (r_i)_{i=1}^4 = 0.5\mathbf{u} + 0.5\mathbf{v} = (0.139, 0.283, 0.3405, 0.2375)$ to embody an overall preference. Aggregating IFBUI granules by using intuitionistic fuzzy type basic uncertain information weighted arithmetic mean, we have

$$IFBWAM_{\mathbf{r}}(\mathbf{a}, (\mu, \nu)) = \left(\sum_{i=1}^4 r_i a_i, (\sum_{i=1}^4 r_i \mu_i, \sum_{i=1}^4 r_i \nu_i)\right) = (0.6348, (0.28015, 0.4242)).$$

At last, suppose we adopt the decision rules based on Trichotomic partition (vi) (discussed in Section 4), predefine a decision space: 1 infeasibility, 2 further analysis, 3 feasibility, and preset $d = 0.3$, $s = 0.6$, $(q, p) = (0.25, 0.25)$ with

$$P_1 = \{(z, (x, y)) \in \mathcal{IFB} : z < 0.3, (0.25, 0.25) \leq_{IF} (x, y)\},\ P_2 = \mathcal{IFB} \backslash (P_1 \cup P_3),$$

$$P_3 = \{(z, (x, y)) \in \mathcal{IFB} : z \geq 0.6, (0.25, 0.25) \leq_{IF} (x, y)\}.$$

Hence, since $(0.6348, (0.28015, 0.4242)) \in P_2$, then the current evaluation information available is not enough accurate and convincing. Therefore, given the current circumstances, it appears that additional evaluation is necessary for the city government to make a well-informed decision regarding the construction of the reservoir.

7 Conclusions

Conventional basic uncertain information is inadequate for situations where intuitionistic fuzzy modeling is used to express varying degrees of certainty. Hence, intuitionistic fuzzy type basic uncertain information can model the numerical certainty that consists of certainty degree and uncertainty degree which might not be complement to each other.

According to the inherent nature of intuitionistic fuzzy type basic uncertain information, we discussed the related induced weights allocation with preference from four different aspects: (i) certainty degree preferred preference, (ii) uncertainty degree averted preference, (iii) certainty difference preferred preference, and (iv) intuitionistic fuzzy certainty preferred preference. We then derived four corresponding weight vectors and averaged them to obtain a comprehensive weight vector that reflects the desired preference.

Trichotomic rules-based decision making is commonly used in evaluation practices and we discussed the related method under the environment of intuitionistic fuzzy type basic uncertain information. It is possible to derive a number of complete trichotomic automatic decision rules from certain elementary rule conditions with ease and effectiveness.

A practical demonstration is provided, showcasing the applicability of intuitionistic fuzzy type basic uncertain information in decision-making processes. This highlights its potential for further exploration and implementation across various domains.

Acknowledgement

We express our gratitude to the individuals who provided valuable feedback on our work, as their input was instrumental in enhancing its quality. This paper is partly supported by grant VEGA 1/0036/23. This paper is partly supported by grant VEGA 1/0036/23, Hubei Superior and Distinctive Discipline Group of New Energy Vehicle and Smart Transportation (Grant No. XKTD052323).

Conflict of interest

We have no conflicts of interest to disclose.

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87-96.

[2] M. Boczek, A. Hovana, O. Hutník, M. Kaluszka, *New monotone measure-based integrals inspired by scientific impact problem*, European Journal of Operational Research, **290** (2021), 346-357.

[3] M. Boczek, L. Jin, R. Mesiar, R. R. Yager, *On monotonicity of the interval sugeno integral*, IEEE Transactions on Fuzzy Systems, **29** (2021), 2825-2827.

[4] H. Bustince, J. Fernandez, A. Kolesárová, R. Mesiar, *Generation of linear orders for intervals by means of aggregation functions*, Fuzzy Sets and Systems, **220** (2013), 69-77.

[5] Z. S. Chen, L. Martínez, J. P. Chang, X. J. Wang, S. H. Xionge, K. S. Chin, *Sustainable building material selection: A QFD- and ELECTRE III-embedded hybrid MCGDM approach with consensus building*, Engineering Applications of Artificial Intelligence, **85** (2019), 783-807.

[6] Z. S. Chen, L. Martínez, K. S. Chin, K. L. Tsui, *Two-stage aggregation paradigm for HFLTS possibility distributions: A hierarchical clustering perspective*, Expert Systems with Applications, **104** (2018), 43-66.

[7] Z. S. Chen, X. J. Wang, K. S. Chin, K. L. Tsui, L. Martinez, *Individual semantics building for HFLTS possibility distribution with applications in domain-specific collaborative decision making*, IEEE Access, **6** (2018), 78803-78828.

[8] Z. S. Chen, C. Yu, K. S. Chin, L. Martínez, *An enhanced ordered weighted averaging operators generation algorithm with applications for multicriteria decision making*, Applied Mathematical Modelling, **71** (2019), 467-490.

[9] Z. S. Chen, X. Zhang, R. M. Rodríguez, W. Pedrycz, L. Martínez, *Expertise-based bid evaluation for construction-contractor selection with generalized comparative linguistic ELECTRE III*, Automation in Construction, **125** (2021), 103578.

[10] Z. S. Chen, X. Zhang, R. M. Rodríguez, W. Pedrycz, L. Martínez, M. J. Skibniewski, *Expertise-structure and risk-appetite-integrated two-tiered collective opinion generation framework for large-scale group decision making*, IEEE Transactions on Fuzzy Systems, **30** (2022), 5496-5510.

[11] Z. S. Chen, M. D. Zhou, K. S. Chin, A. Darko, X. J. Wang, W. Pedrycz, *Optimized decision support for BIM maturity assessment*, Automation in Construction, **149** (2023), 104808.

[12] W. L. Gau, D. J. Buehrer, *Vague sets*, IEEE Transactions on Systems, Man, and Cybernetics, **23** (1993), 610-614.

[13] M. Grabisch, J. L. Marichal, R. Mesiar, E. Pap, *Aggregation functions*, Cambridge University Press, In Proceedings of the 2008 6th International Symposium on Intelligent Systems and Informatics, Subotica, Serbia, (2008), 1-7.

[14] Q. Huang, J. Xu, *Rethinking environmental bureaucracies in river chiefs system (RCS) in China: A critical literature study*, Sustainability, **11** (2019), 1608.

[15] L. Jin, Z. S. Chen, J. Y. Zhang, et al., *Bi-polar preference based weights allocation with incomplete fuzzy relations*, Information Sciences, **621** (2023), 308-318.

[16] L. Jin, R. Mesiar, S. Borkotokey, M. Kalina, *Certainty aggregation and the certainty fuzzy measures*, International Journal of Intelligent Systems, **33** (2018), 759-770.

[17] L. Jin, R. Mesiar, A. Stupňanová, R. R. Yager, *Some generalized integrals applied in scientometrics and related evaluation*, IEEE Transactions on Emerging Topics in Computational Intelligence, **5** (2021), 846-853.

[18] L. Jin, R. Mesiar, A. Stupňanová, R. R. Yager, M. Kalina, *On some construction method and orness measure of bi-capacities*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, **29** (2021), 107-117.

[19] L. Jin, R. Mesiar, R. Yager, *Ordered weighted averaging aggregation on convex poset*, IEEE Transactions on Fuzzy Systems, **27** (2019), 612-617.

[20] L. Jin, R. Mesiar, R. R. Yager, *Some decision taking rules based on ordering determined partitions*, International Journal of General Systems, **50** (2021), 26-35.

[21] L. Jin, R. Mesiar, R. R. Yager, *On WA expressions of induced OWA operators and inducing function based orness with application in evaluation*, IEEE Transactions on Fuzzy Systems, **29** (2021), 1695-1700.

[22] L. S. Jin, Y. Q. Xu, Z. S. Chen, R. Mesiar, R. R. Yager, *Relative basic uncertain information in preference and uncertain involved information fusion*, International Journal of Computational Intelligence Systems, **15** (2022), 12.

[23] G. Li, R. R. Yager, X. Zhang, R. Mesiar, H. Bustince, L. Jin, *Comprehensive rules-based and preferences induced weights allocation in group decision-making with BUI*, International Journal of Computational Intelligence Systems, **15** (2022), 54.

[24] X. Liu, S. Han, *Orness and parameterized RIM quantifier aggregation with OWA operators: A summary*, International Journal of Approximate Reasoning, **48** (2008), 77-97.

[25] Z. Liu, F. Xiao, *An interval-valued exceedance method in MCDM with uncertain satisfactions*, International Journal of Intelligent Systems, **34** (2019), 2676-2691.

[26] R. Mesiar, S. Borkotokey, L. Jin, M. Kalina, *Aggregation under uncertainty*, IEEE Transactions on Fuzzy Systems, **26** (2018), 2475-2478.

[27] W. Pedrycz, *Fuzzy relational equations with generalized connectives and their applications*, Fuzzy Sets and Systems, **10** (1983), 185-201.

[28] G. Qian, E. Zhang, Z. Chen, R. Mesiar, R. R. Yager, L. Jin, *Consistent construction of evaluation threshold values and rules for heterogeneous linguistic input information*, International Journal of Computational Intelligence Systems, **14** (2021), 153.

[29] T. Takagi, M. Sugeno, *Fuzzy identification of systems and its applications to modeling and control*, IEEE Transactions on Systems, Man, and Cybernetics, **SMC-15** (1985), 116-132.

[30] Z. Tao, X. Liu, L. Zhou, H. Chen, *Rank aggregation based multi-attribute decision making with hybrid Z-information and its application*, Journal of Intelligent and Fuzzy Systems, **37** (2019), 4231-4239.

[31] Z. Tao, Z. Shao, J. Liu, L. Zhou, H. Chen, *Basic uncertain information soft set and its application to multi-criteria group decision making*, Engineering Applications of Artificial Intelligence, **95** (2020), 103871.

[32] P. Tiwari, *Generalized entropy and similarity measure for interval-valued intuitionistic fuzzy sets with application in decision making*, International Journal of Fuzzy System Applications, **10** (2021), 64-93.

[33] R. R. Yager, *On ordered weighted averaging aggregation operators in multicriteria decisionmaking*, IEEE Transactions on Systems, Man, and Cybernetics, **18** (1988), 183-190.

[34] R. R. Yager, *Quantifier guided aggregation using OWA operators*, International Journal of Intelligent Systems, **11** (1996), 49-73.

[35] R. R. Yager, D. P. Filev, *Induced ordered weighted averaging operators*, IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics), **29** (1999), 141-150.

[36] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (1965), 338-353.

[37] L. A. Zadeh, *Outline of a new approach to the analysis of complex systems and decision processes*, IEEE Transactions on Systems, Man, and Cybernetics, **SMC-3** (1973), 28-44.

[38] L. A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, **1** (1978), 3-28.