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Experimental Archaeology:
Investigating the Evolution of Hydrostatic Balances throughout History (An Experimental Approach)

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Abstract

More than two thousand years ago the first hydrostatic balance with a rudimentary structure was invented by the Greek sage “Archimedes.” Iranian scholars would later develop balances that were used not only for weighing but also for determining the density and titration of alloys.To appraise the path of evolution of these balances as well as their potentials, an experimental research program was undertaken by the Iranian Research Organisation for Science and Technology (IROST). Therefore, five different balances were selected, ranging from the oldest known in the history of science, the Archimedes balance, to the most sophisticated one, the balance of wisdom by Al-Khazini, which are separated by a lapse of time of about fourteen hundred years. Balances designed by Abu Rayhan Al-Biruni, Zakariya Al-Razi, and Omar Khayyam were the other three. In the course of the research, replicas of the selected balances were designed and fabricated according to the remaining design details in the related literature. The article covers the detailed design of each balance, their construction and functional test details as well as a comparison of their functions and their precision in densitometry and titration.

Keywords: Hydrostatic Balances, Archimedes, Al-Khazini, balance of Wisdom, Mizan al-Hikma, Razzi, Al-Biruni, Khayyam.

Article Type: Research Article

Introduction

Scale is one of the earliest tools that mankind has achieved in the history of trade and commerce. This study covers the most prominent designs of hydrostatic balances (scales) from the pre-Christian period, i.e. from roughly the 3rd century BCE, the time of the earliest design by Archimedes, to the Medieval Islamic period around the 12th century AD.

In his book De *Architectura*, Vitruvius provides detailed information about one of the earliest and pivotal scientific events which took place more than 2300 years ago. It marked a turning point in the field of hydraulics and fluid mechanics as well as densitometry and titration. When describing this event, Vitruvius states that Archimedes devised a method for measuring the volume of an object with an irregular shape. This would in turn provide the grounds for obtaining the density of that object which consequently determines whether it is made of a certain element. Therefore, Archimedes could measure the density of the material used in the construction of the crown (Vitruvius 1960). As a result, he proved that the crown was not made of pure gold, since he found a discrepancy between the density of the crown and that of the pure gold. This great scientific event can be considered the beginning of innovative thinking and visionary developments in the field of fluid mechanics. Following this, Archimedes constructed a balance for titration purposes by applying his principle (Al-Khazini 1967: 85–87).

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This in turn means that an object partially or fully submerged in a liquid will displace a certain amount of liquid. The weight of the displaced liquid will be equal to the value of the buoyant or upward force exerted on the object by the liquid. Later on, the subject would be resumed by many scholars and scientists of various nationalities such as Arabs and Iranians.

This article covers the results of the research work on re-construction, operation, precision, and calibration details of the balances made by Archimedes, the natural balance of Mohammad Zakariya Al-Razi, a balance designed by Al-Biruni, Khayyam's hydrostatic balance, and the balance of wisdom originally designed by Al-Khazini. However, the calibration process of the balances designed by Iranian scholars varied from the one by the Greeks, Archimedes, even though design-wise three of them proved to be very similar – namely those by Archimedes, Al-Razi, and Khayyam. Thus, only one of these schemes, *viz*. Al-Razi's, was reconstructed and tested as the representative of the three.

Symbols

d : Total number of divisions on the main beam of the balance.

d'_r : Number of divisions from the index of the low-density element on the scale's main beam to the right of the centre of the moving pan.

d'_l : Number of divisions from the left of the centre of the movable pan to the index of the denser element.

d_1: Number of divisions from the initial location of the precision moving balance weight to the middle of the scale.

d_2 : Number of divisions from the secondary location of the precision moving balance weight to the middle of the scale.

$\dot{D}$: Length of each "Shaier", in centimeters.

P : Actual density, in grams per cubic centimeter

ρ' : Density measured by the balances manufactured in this study, in grams per cubic centimeter

ρ'' : Density in grams per cubic centimeter, measured by original balances – recorded in existing and related documents of the 10th and 11th centuries AD

$\dot{\rho}$: Density of element with less density in grams per cubic centimeter

$\ddot{\rho}$: Density of element with higher density in grams per cubic centimeter

V : Actual volume, in cubic centimeters

v' : Volume measured by scales, in cubic centimeters

m: Real mass in grams

m_{WI} : Mass of the water outpoured from the Al-Biruni's balance for iron in grams

m_{WIP}: Mass of the outpoured water from Al-Biruni's balance for metal and plastic parts

M_{Au}: Mass of the gold element present in the alloy in grams

M_{Ag}: Mass of the silver element present in the alloy in grams

M_{alloy}: Mass of the alloy in grams

W : Measured mass of the body by the balance in grams

w' : Measured mass of the body by the balance after submerging in water in grams

E : Percentage error

E' :Average percentage error

k : Arm length (half length of the main beam): distance of both moving and stationary pans from the middle of the balance at the beginning in centimeters

l : Density of the denser element index (centre of element) to the middle of the main in centimeters

$k-l$: Length of grading interval of the desired alloy, in centimeters

The Project

In order to fill the gap in the current knowledge about precision weighing, titration, determination of density, and identification of the constituent elements of binary alloys, a project based on experimental archaeology was planned at the Mechanical Research Institute of the Iranian Research Organisation for Science & Technology (IROST). As part of this project, five more prominent scales in the history of weighing and titration were selected and with the available details and documentation in hand, prototyping was carried out before their performance was tested. The primary designers of these scales are as follows: Archimedes, Zakariya Al-Razi, Abu Rayhan Al-Biruni, Omar Khayyam, and Al-Khazini.

The information deriving from this research was impossible to obtain by examining the original artifacts, because what is available regarding these scales is simply limited to historical written documents with no actual manufactured examples being at disposal.

The Background

In the past, simple scales with two pans and a fulcrum were used to weigh various objects. More than two thousand years ago, an event in Greece sparked applying water scales or hydrostatic balances for precision measuring of weight, density, and purity of various objects and alloys (titration). According to Vitruvius, King Hiero of Syracuse ordered a votive crown which he wished to offer to a temple. Apparently, the king did not fully trust the goldsmith who was to make the crown. As a result, he asked Archimedes to determine whether some silver was used instead of pure gold in the structure of the crown by the goldsmith. Due to the fine and beautiful design of the crown, he asked Archimedes to apply a non-destructive approach to preclude any potential damages to the royal headgear. Therefore, Archimedes had to determine the volume of this irregularly shaped object without melting it, and through determine its density to compare it against the density of pure gold. With this in mind, one day he was taking a bath when he noticed that the increased level of water after the bowl slide into it. He therefore realized that the same effect could be used to measure the volume of the metals. Excited by the discovery, he cried out "Eureka", the Greek phrase for "I have found it!" The Vitruvius' tale continues to detail this measurement method (Vitruvius 1960; Sparavigna 2011), a subject that is out of the scope of the present paper. Archimedes eventually found out that the crown was not made of pure gold, and that the deceitful goldsmith had instead made it of a mixture of gold and cheaper and more colourable metals such as silver (Vitruvius 1960). Later, Archimedes developed one of the oldest designs for hydrostatic balances. This introduced a major improvement to the design of balances, which could determine the density and therefore the impurity of an object made of a certain element.

Today, the Archimedes' scale is also known as "the Absolute Scale." The Archimedes balance, with a relatively simple structure, was able to establish the original and basic concept of much more sophisticated balances known as hydrostatic balances. The most complete and comprehensive form of this type of scale appeared as the balance of wisdom many years later.

After Archimedes, Menelaus of Alexandria (living around 70–130 AD) and Sind ibn Ali (living in the 9th AD) studied and examined the design of the Archimedes scale closely (Al-Khazini 1967: A-D). The magnificent era of the Iranian hydrostatic scales began with Mohammad Zakariya Al-Razi. Abu Bakr Muhammad, born during the reign of the Samanid dynasty in 865 AD, is one of the greatest Iranian philosophers, physicians, and chemists (Mossaheb1988: 105; Halaby 1972: 115). He spent his early youth studying philosophy, medicine, mathematics, and astronomy. He has a great reputation in chemistry and medicine (Mossaheb 1988: 105; Halaby 1972: 115). His books, though partially lost in the sands of time, have been subjects of lectures in European universities until recently. Al-Razi categorized chemicals and investigated specific gravity (Mossaheb 1988: 105; Halaby 1972: 115). The "natural scale or Mizan al Tabie" is the name he gave to his balance, and he has a treatise on it in his *Al Mizan al Tabie val Amal* (Al-Khazini 1967: 88–95). The book is about "the kimia or the science of transforming metals into gold" on which he has also written four books (Al-Khazini 1967: 88–95; Mossaheb 1988: 105; Halaby 1972: 115).

After Al-Razi, Ibn Amid (912–970 AD) and Abu Ali Sina (981–1037 AD) also worked on the design of hydrostatic balances and were able to take steps to further complete their designs to achieve more precision in determining and detecting the impurities of composite objects. However, no written works and manuscripts about the details of the design of these balances are currently available (Al-Khazini 1967: A-D).

Years later, under the Ghaznavid rule, especially in the reign of Sultan Mahmud, another hydrostatic balance was advanced by Abu Rayhan Al-Biruni. He was born in 972 AD in Khwarazm, where he completed his studies. He has debated with Avicenna on scientific topics and is one of the most renowned Iranian scholars (Mossaheb 1988:105; Halaby 1972: 115). He was an eminent scholar, especially in philosophy, history, geography, astronomy and mathematics. Al-Biruni, for the first time, set up a specific gravity table for various elements (Ghazni 1982).His computational tables, the balance structure, and its functional directions are recorded in a treatise named "Maghala fel nasab allati bein al felezzat va al javaher fel hajm" which has also been presented by Al-Khazini in his *Mizan Al-Hikma* (*Balance of Wisdom*) (Al-Khazini 1967:48–49). Later, another balance with a structure similar and operating details very similar to Archimedes' was developed by Omar Khayyam. Apart from the structural design, it further closely resembles that of Archimedes in operating details (Khanikoff 1860: 87).

Another mechanical balance of the steelyard type is detailed in an article by Khayyam “Al Ghisstass al Mostaghim”, presented in Al-Khazini’s book *Mizan Al Hikma* (Al-Khazini 1967: 96–101; Abattouy 2005: 155–166; Khanikoff 1860: 1–128). As this balance was designed only for extremely precise weighing of gold and silver, it does not fall within the scope of this work.

At the same time as Khayyam, Abu Ḥatim al-Muẓaffar al-Isfizari, a great philosopher who authored many works in mathematics and devoted most of his career to studying Archimedes balance, was finally able to make significant improvements to the balance for the titration purposes (Hall 1973: 335–351; Zaimeche 2005; Al-Khazini 1967: A-D; Shahrzoori 2012: 394–398). He then presented the balance to the king Sultan Sanjar. As the Sultan’s treasurer realized that such a balance could reveal his fraud in the treasury, he planned for the destruction of the balance, an event that lead to al-Isfizari’s death due to the great grief induced by the loss of his balance (Shahrzoori 2012: 394–398; Al-Khazini 1967: A-D).

After al-Isfizari, Abd al-Rahman Al-Khazini, who studied mathematics and astronomy in the fifth and sixth centuries, followed the works of Archimedes and added further rules for the determination of the specific gravity of the bodies (Zaimeche 2005). He was also engaged in treasury service at the royal court and managed to repair, restore, and complete the broken balance of al-Isfizari (Moradi Ghiasabadi 2005). The new balance, which in some aspects superseded its earlier counterparts, was named by Al- Khazini the “balance of wisdom” (Al-Khazini 1967: 122–124). Also due to its superiority over its predecessors in many aspects, it was called “Mizan al-Jama” or universal balance (Al-Khazini 1967: 122–124). The book *Mizan al-Hikma* (*Balance of Wisdom*), one of the greatest works of the Islamic period in physics and mechanics, was written by Al-Khazini, where he describes the history of the invention of the “balance of wisdom” and the works of several other scholars such as Archimedes, al-Isfizari, Al-Biruni, Omar Khayyam and Al-Razi. Al-Khazini’s book also contains an anthology of the excerpts by various scholars (Ghorbani 1986: 82–120; Al-Khazini 1967: 122–124).

The following gives an outline of the five balances in chronological order. The designs, calibrations, operating details, and performance of the balances devised by Archimedes, Al-Razi, Al-Biruni, Omar Khayyam, and Al-Khazini will be described.

Archimedes’ Hydrostatic Balance

The balance is actually in the form of a “Bilancia a cavaliere,” an equal arms balance with a little mobile weight on one half of the balance’s beam. This scale was conceived and built by Archimedes over two thousand years ago and was basically used for titration, though also could be used for weighing purposes.

Details of the Balance

The Balance Structure

This balance not only is applied for the precise weighing of objects but also allows for the calibration of binary alloys. Its structure consists of a horizontal rod named “amud” (the main beam) which is held at the centre by a vertical base. Under equilibrium conditions, the “amud” stands parallel to the line of the horizon. One-half of this main beam (one of the arms) is divided into thirty equal parts. It is noted that the use of a “manghaleh” (precision moving balance weight) in this scale is mandatory. The weight is movable and is suspended on the scaled arm by a hook. It is used for obtaining the highest degree of precision when the balance is used in a hydrostatic mode. The pan suspended on the scaled side of the arm is used for placing silver and the other one for golden objects or alloys. Figure 1 gives a depiction of this balance.

Calibration

Al-Khazini’s detailed report on the calibration of the balance is presented here as a footnote[1] to avoid duplication, as it may be out of the scope of the article.

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[1] To calibrate the balance arm, the same weights of gold and silver are placed in their respective pans. The arm stands horizontally due to the state of equilibrium. In the next step, the pans are submerged in the water container. Therefore, the weights of both elements are reduced as much as the water weight whose volume is equal to the volumes of the elements. It can then be observed that the main beam loses its equilibrium towards the gold pan. To restore the equilibrium, the moving weight manghaleh, on the side of silver, is moved to a suitable position. After balancing the main beam, the space between the primary and secondary positions of the manghaleh is marked and the number of calibration reference lines from the centre of the main beam arm is recorded. Calibration of the position of the manghaleh is of paramount importance for the precision of the balance (Al-Khazini 1967: 122–124).

The Balance Function

Titration

To determine the gold content percentage of a binary alloy (consisting of gold and silver), one places a certain amount of the alloy in the fixed pan. Then an equal weight of gold is put on the movable pan. After the establishment of perfect balance, the pans are submerged in a water container. Therefore, the balance of the beam deteriorates and the re-establishment of equilibrium is achieved by replacing the “manghaleh” at the gold pan side. The position of “manghaleh” is marked (the primary position).

The same process is performed with an equal weight of silver and the new position of “manghaleh” is marked again (the secondary position) before the space between this position and the centre of the main beam is recorded (Al-Khazini 1967: 122–124). The ratio of the amount of gold in the alloy to the overall weight of the alloy is equal to the ratio of the number of reference calibration lines recorded for the secondary position of the manghaleh from the centre of the main beam to the number of calibration reference lines recorded for the primary position to the middle of the main beam (Equation 1) (Al-Khazini 1967: 122–124).

$$\frac{M_{Au}}{M_{alloy}} = \frac{d_2}{d_1} \qquad (1)$$

$$M_{Ag} = M_{alloy-}M_{Au} \qquad (2)$$

Al-Razi’s Balance

This scale was built by Zakariya Al-Razi in 926 AD and was limited in use to weight measurement and titration. Calibration of the “natural scale” could be performed by two different methods, each of which could affect the performance of the balance. The first of these roughly denotes the purity or impurity of elements and does not indicate a numerical value for it. However, in the second grading method, which applies “shaieraat”, the indices are divided into 12 parts which enable precise determination of the constituting elements of a binary sample (details of the process are presented in Section 2.2.1).

Details of the Balance

Description of the “Natural Scale”

Around the end of the 9th century AD, Al-Razi managed to devise a scale that enabled titration analysis of binary objects made of gold and silver (see Figure. 2). Generally, the scale can be calibrated in two ways (Al-Khazini 1967: 88–95, 20). This, in turn, would mean that there would be two methods for performing measurements. In the first, grading is done only by three indicators. In particular, the titration test will not specify the exact percentages of gold and silver contents, but will only determine which element has a greater share within the alloy. While the second method, categorized by using “shaieraat” (Ghorbani 1986: 82–120; Yassi 2009: 135–151), will yield a more accurate result which stands for the numerical measurement of the ratio of the elements within the alloy.

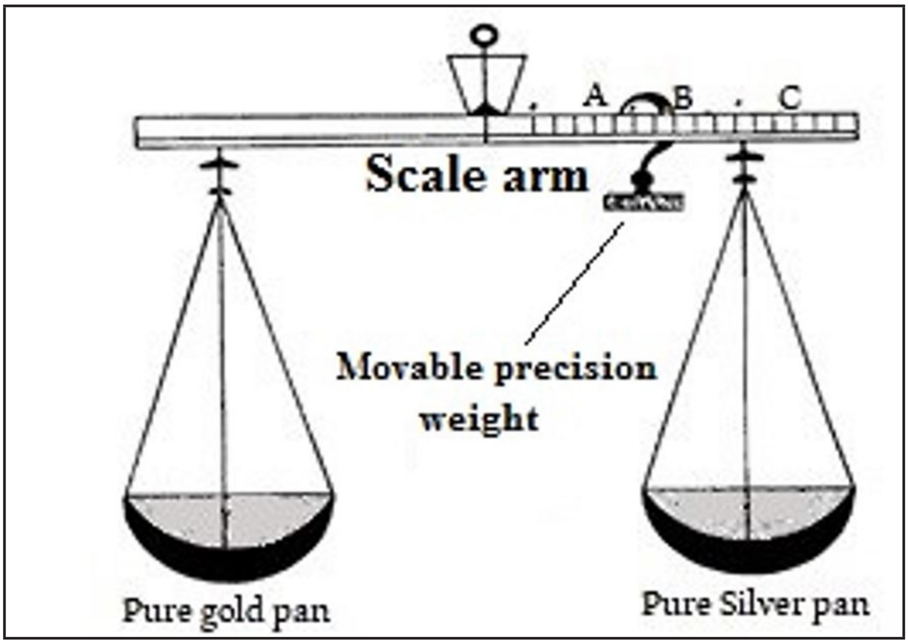

Figure. 1: *Archimedes’ balance (After: Al-Khazini 1967: 87)*

The Balance Structure

Both sides of the main beam has two suspending pans, one of them is fixed (the silver pan) and the other (the gold pan) is movable. The main beam is mounted on a base perpendicularly.

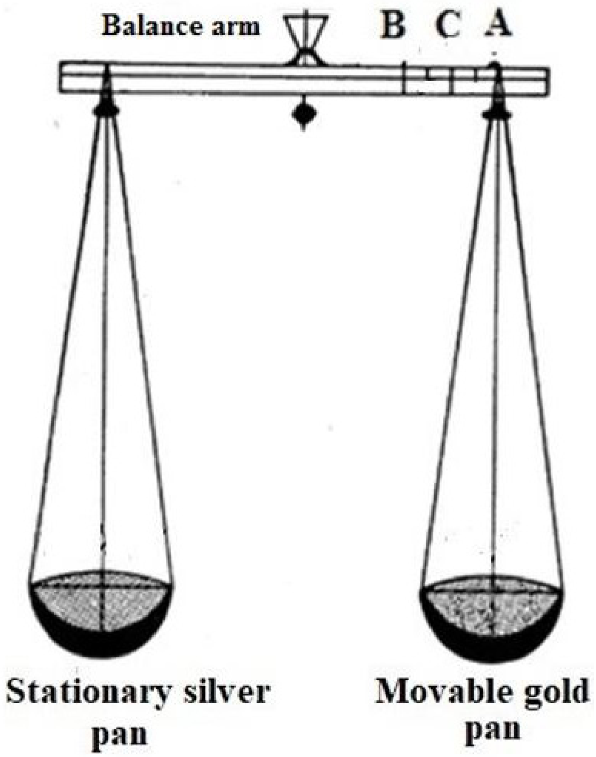

Figure. 2: *The Natural Balance by Al-Razi (After: Al-Khazini 1967: 90)*

Grading and Calibration

In order to operate the balance, it should be graded and calibrated. Calibration of the "natural scale" could be likewise performed through two different methods: "the ordinary or common" approach, and the "shaieraat" approach. [1]Each of these methods could affect the performance of the balance.

The first approach roughly indicates the purity or impurity of elements without giving a numerical value for them. However, in the second calibration method, which applies the "shaieraat", the indices are divided into 12 parts, enabling precise determination of the constituting elements of a binary sample. Figure 3 exhibits the view of the natural balance with "shaieraat" calibration on the right arm of the balance.

The Balance Function

Titration

Normal Approach

In order to measure the gold and silver content of an alloy, some pure silver is placed in the fixed pan and an equivalent amount of the alloy is placed in the movable pan (see Figure. 2). Then both pans are submerged into water and so the pans are filled with water. If the object to be measured consists of pure silver, the equilibrium of the balance is provided by placing the movable pan on the indicator "A." If the measured object consists of pure gold, the equilibrium of the balance is provided by placing the movable pan on the indicator "B." If the balance is under perfect equilibrium condition when the movable pan is situated at point "C," the object contains equal amounts of gold and silver. Otherwise, one of the following cases may be true:

I.If the movable pan, under equilibrium, stands between the indexes "A" and "C," it means that the percentage of silver in the alloy is more than the percentage of gold.

II.If the movable pan, again under equilibrium, stands between the "C" and "B" indices, it means that the percentage of the mass of gold is higher

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[1] A) Ordinary Calibration and grading

Step 1: The balance should be in perfect equilibrium at the beginning. The same weight amount of gold and silver are put into their respective pans. The position of the gold pan is marked on the main beam, by the letter "A".

Step 2: Then the pans are submerged into a water tank to fill them with water. Due to the lesser volume of the Gold in comparison to the Silver, for equal weights, the Gold pan allows more water in and therefore the overall equilibrium of the main beam is disrupted towards the gold pendant.

The movable pan is so moved that the equilibrium state is restored, the new position of the gold pendant is marked by the letter "B" and the midpoint between "A" and "B" is marked as "C." In this way, the scale is trimmed with the letters "A," "B" and "C (Figure. 2).

B) Calibration using "Shaieraat"

According to Figure 3, the movable pan is the gold or alloy pan and the fixed pan is the silver pan. Under equilibrium condition, the position of the movable pan is marked as "A," the position of the fixed pan is marked as "C," and the midpoint of the scale is marked with the index "B." Step 2 of the above is exactly repeated and the new position of the Gold pan is marked as point "D." In this way, the "A" index is called the Silver centre and the "D" index is called the Gold centre. For grading the main beam, according to Figure 3, the distance between the two indices "A" and "D" is divided into 12 equal parts, so-called "shaieraat" (Al-Khazini 1967: 88–93).

than the percentage of the mass of silver in the alloy (Al-Khazini 1967: 90–95).

Titration by “Shaieraat” Approach

As with the titration process mentioned in Section 2.3.1, to perform titration on an alloy of gold and silver, some pure silver is placed in the fixed pan, and the same amount of alloy is placed in the movable pan. Then both pans are submerged in water and when they are both filled with water, they are taken out. By moving the movable pan on the main beam until the state of balance is established and then by applying the “shaieraat” approach (see Figure. 3), as described in footnote 5, the process of titration is fulfilled[1].

Figure 4 presents a view of the natural balance scale reconstructed at the Iranian Research Organisation for Science and Technology (IROST).

Functional Issues

Due to the length of the components of the scale, such as the main beam, the base, and the threads by which the measuring pans are suspended, as well as the sensitivity of the mechanism to minor fluctuations, the balance has very high accuracy in titration measurements. It should be noted that before the calibration, it is necessary to know the constituent elements of the alloy. Therefore, the main beam is calibrated for that particular alloy, and therefore, it is impossible to use the same main beam for different alloys at the same time due to the possibility of interference with the calibrations.

In other words, before the calibration of any binary alloy, the calibration can be performed only by the prior knowledge of the constituent elements. Calibrating the scale for the measurement of various elements is a prerequisite for any measurement. Therefore, this may be regarded as a drawback for titration measurements by the natural balance.

Al-Biruni’s Balance

Al-Biruni constructed a balance that could be used for weight and specific weight measurements, elemental identification, and volume measurement. There are no calibrations on the arm of the balance and it operates through the Archimedes principle[2]. Al-Biruni calculated the specific gravity of metals, precious or semi-precious stones as well as the specific gravity of liquids with his scale to a great degree of precision, and presented his work in the book *Al Jamahir fil Javahir* (Rahimi 2015: 34–41; Al-Biruni 1995).

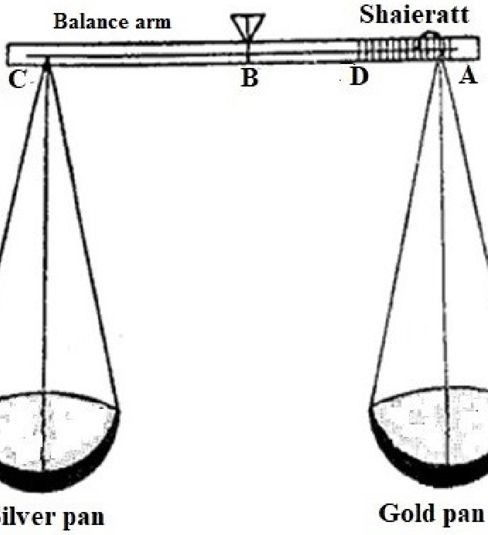

Figure. 3:*The Natural Balance calibrated by shaieraat (After:Al-Khazini 1967: 93)*

Details of the Balance

Balance Structure

As displayed in Figure 5, the body of this balance consists of two distinct parts, interconnected through a semi-pipe (halved through the cross-section along the length) called the “open pipe” in the original manuscript (Al-Khazini 1967: 48–49).

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[1] Process of Titration by “Shaieraat”

A. Referring to figure (3), if the number of “Shaiers” is taken from index A, the ratio of the number of the “shaieraat” of the position of the movable pan to the number 12, $^{d'_r}/_{d}$ represents the ratio of the presence of pure gold in the alloy (Equation 2). In this case, having the mass of the total alloy as well as the mass of the gold contained therein the mass of silver can then be obtained.

$$\frac{d'_r}{d} = \frac{M_{Au}}{M_{alloy}} \qquad (1)$$

$$M_{Ag} = M_{alloy} - M_{Au} \qquad (2)$$

B. In the case of counting the number of “shaieraats” from the index “d,” the ratio of the number of “shaieraats” of the position of the movable pan $^{d'_l}/_{d}$ (to the number 12, expresses the ratio of the amount of pure silver in the alloy (Equation 3). In this case, having the mass of the total alloy and the mass of the pure silver, the mass of pure gold could be obtained (Al-Khazini 1967, 124; Yassi et al. 2009: 88–93).

$$\frac{d'_l}{d} = \frac{M_{Ag}}{M_{alloy}} \qquad (3)$$

$$M_{Au} = M_{alloy} - M_{Ag} \qquad (4)$$

[2] Archimedes' principle states that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces.

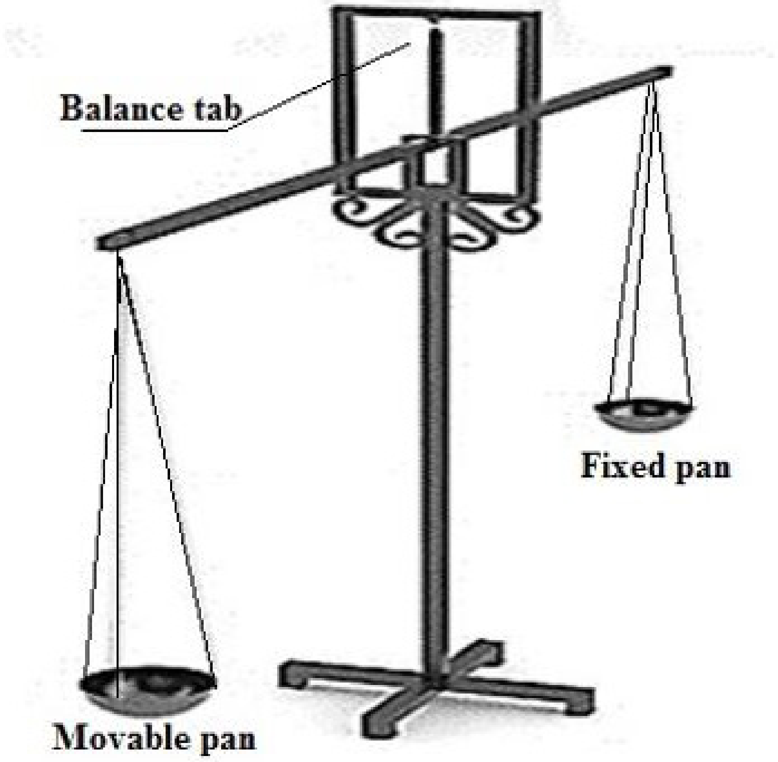

Figure. 4: The Natural Balance constructed during the research project at IROST (After :Yassi et al. 2009: 135–151)

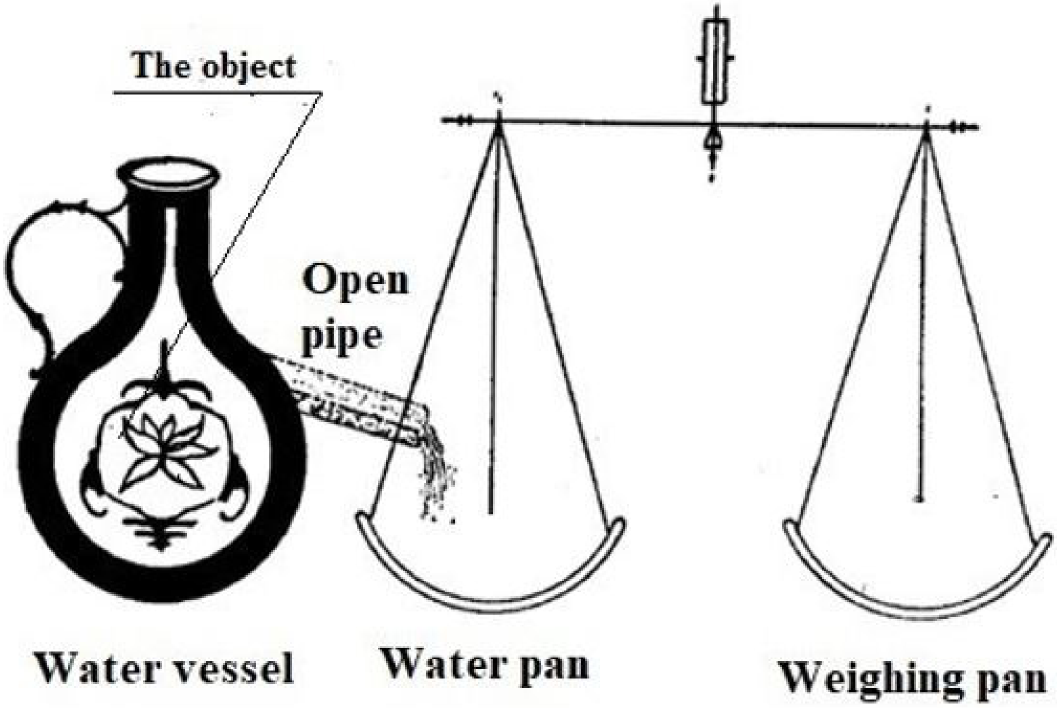

Figure. 5: Al-Biruni's balance (After: Al-Khazini 1967: 49)

One part consists of a simple scale that has a main beam, a falcon and a vertical base. Two weighing pans, namely the water pan and the weighing or stone pan, are suspended equally spaced from the centre of the main beam.

The other part consists of a laboratory flask-shaped vessel (a jug). The "open pipe" is connected to the neck of the container and directs the outflowing fluid from the container into the water pan. The relationship between these two parts is achieved through the outflow of water through the "open pipe" into the water pan (Al- Khazini 1967: 8–49).

Balance Function

Place the balance on a flat surface and the container alongside the balance so that the water drained out of the "open pipe" is poured into the water pan. Fill the container precisely up to the point of overflow into the "open pipe." To achieve a higher degree of precision, fill the container with water to a level higher than the overflow point and let it drain freely. The water level inside the container would then meet exactly the overflow point. An object of a known mass is inserted into the container, so the surface of the water inside increases and naturally overflows through the "open pipe" into the water pan. The mass of the outpoured water divided by the density of water (1 gr per cubic centimeter) represents the volume of the object (according to Archimedes' principle outlined in footnote 6). Thus, having the weight and volume of the object, its specific gravity can then be calculated.

Tables of Al-Biruni

By performing different tests on the balance, Al-Biruni was able to prepare tables which made possible the identification of metals and jewelry such as gold, azure, lead, silver, copper, etc., and the determination of the values of their densities. Examples of these tables are recorded in the book *Kitab al-Hikma* by Abd al-Rahman Al-Khazini (Rahimi 2015: 34–41).

Construction and Testing of the Replica

Al-Biruni's balance was reconstructed in light of the image and documentation in Mizan al-Hikma. No details of the dimensions of this scale are available in the remaining texts, therefore similar range of dimensions to other replicas was assumed for redesigning and reconstruction of the balance (see Figure. 6).

Khayyam's Hydrostatic Balance

Details of the Balance

Structure of the Balance

As with Razzi's and Archimedes', Khayyam's balance has two pans suspended on each side of the main beam, one of them is fixed (the silver pan) and the other one (the gold pan) is movable. The main beam is mounted on a perpendicular base.

Balance's Function

The balance function with its moving gold pan, calibrated main beam, and precision moving balance weight (*manghaleh*) is the same as those of Razzi and Archimedes. Its original design is mainly for titration purposes and densitometry is not a predefined function for Khayyam's balance.

Al-Khazini's Balance of Wisdom[1]

Details of the Balance

The balance of Wisdom by Al-Khazini is one of the most sophisticated and precise hydromechanical scales ever designed and built throughout Iranian engineering history (Al-Khazini 1967: 124; Yassi 2009: 135–151). Al-Khazini has described all the details of the design of this balance in his book *Al-Kitab Mizan Al-Hikma* (*The Balance of Wisdom*). Figure 7 gives the schematic representation of the balance in his book. This has many advantages and superiorities over other hydromechanical balances. Higher precision due to double suspension, triple complex action, and ability to detect the type and weight percentage of constituent elements of binary alloys (on the condition of knowing one of the elements) are among the prominent advantages of this device. The balance is also capable of measuring the weight and density of substances or alloys precisely. The balance is a hydrostatic balance of standard form with five scale pans, a rather complicated polyfill suspension, and a sensitive indicator tongue, using Archimedes' principle in its operations.

Structure of the Balance

The main body of the balance consists of a long vertical stand or the pilaster, a solid square frame fixed to the end of the pilaster, and a horizontal axis

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[1] Details of the balance of wisdom and all related tests, results, optimisations, etc. have been presented in an article titled "Al-Khazini's Balance of Wisdom. A Masterpiece of Medieval Engineering" (Nuncius 2019).

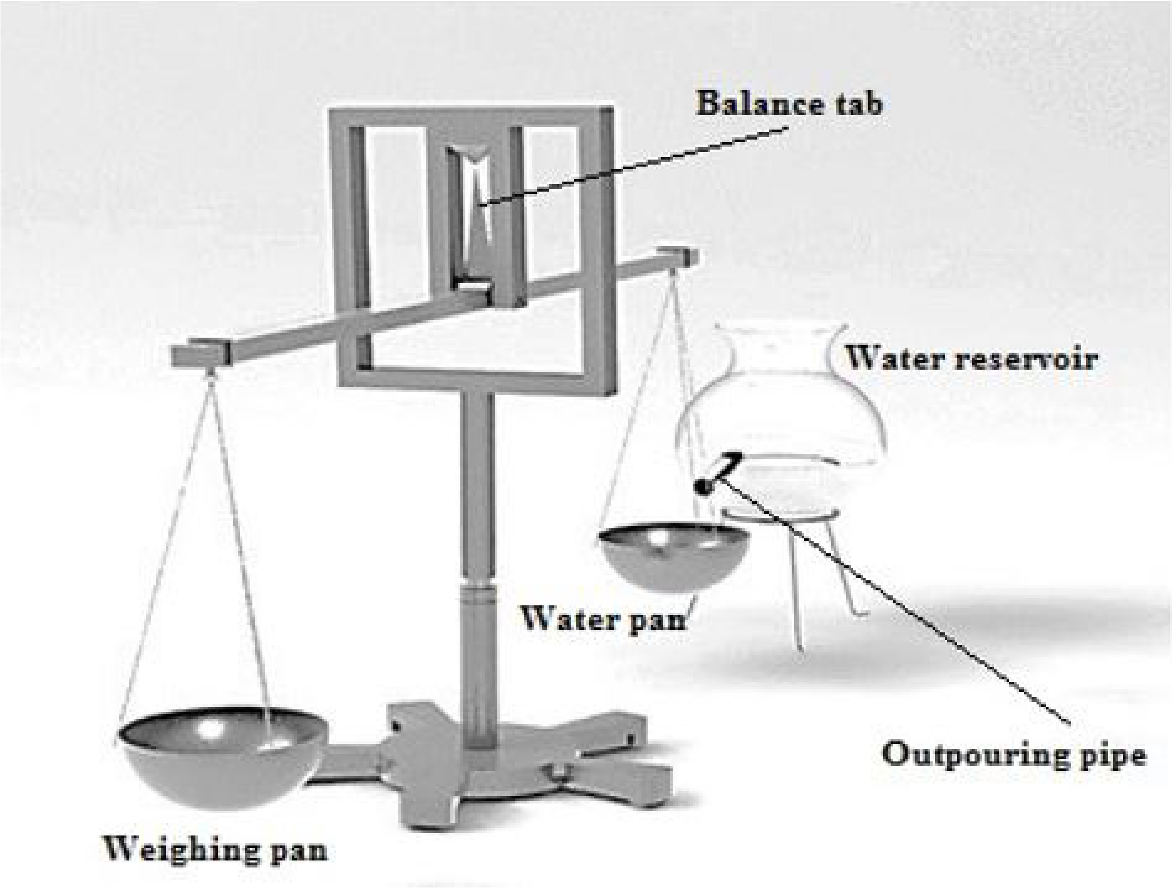

Figure. 6: *Al-Biruni's balance constructed during the research project at IROST (After: Yassi et al. 2009: 135–151)*

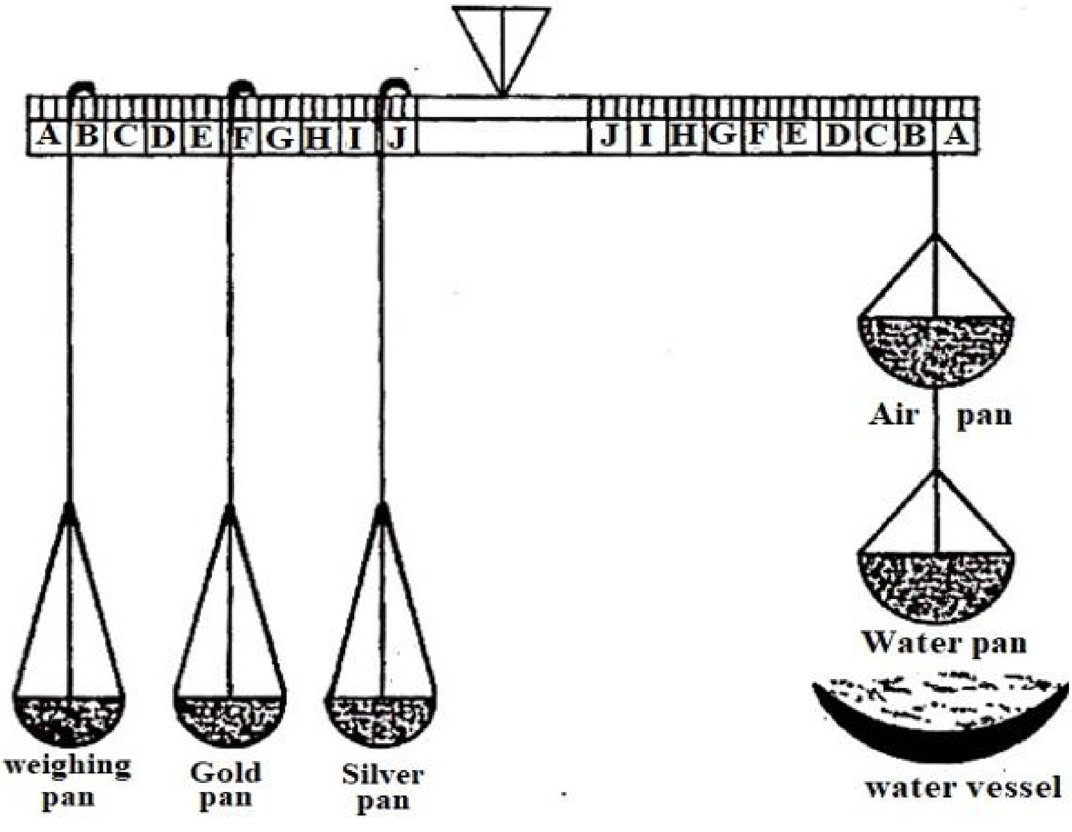

Figure. 7: *Balance of wisdom as represented by Al-Khazini (After: Al-Khazini 1967: 124)*

called "the main beam" that holds the suspending weighing pans.

The main beam is suspended from a trapezoidal pendant, "Alfiarine," via a transverse bar by several pieces of parallel vertical strings (the more the number of the strings, the higher the stability and precision of the balance). The trapezoidal pendant itself is also suspended from the square frame, so a dual suspension is achieved for higher precision. A small solid triangular equilibrium pointer, with its vertex pointing downwards, is fixed at the middle of the top horizontal side of the trapezoidal pendant. The perfect state of equilibrium is achieved when the falcon stands vertically and its vertex meets the vertex of the solid triangular pointer (Al-Khazini 1967: 124; Yassi *et al*. 2009: 135–151).

From one side of the main beam, the first air pan, "the criteria pan," is suspended. The air pan is otherwise called "the pan of the stone" in other sources. On the same arm are suspended two sub-pans which are used for titration. On the other side of the main beam, the water pan or "the governing pan" is suspended, which is always in a container of water and is suspended in double suspension under the second air pan. Figure 8 presents the reconstructed model of the balance of wisdom during the research work at IROST.

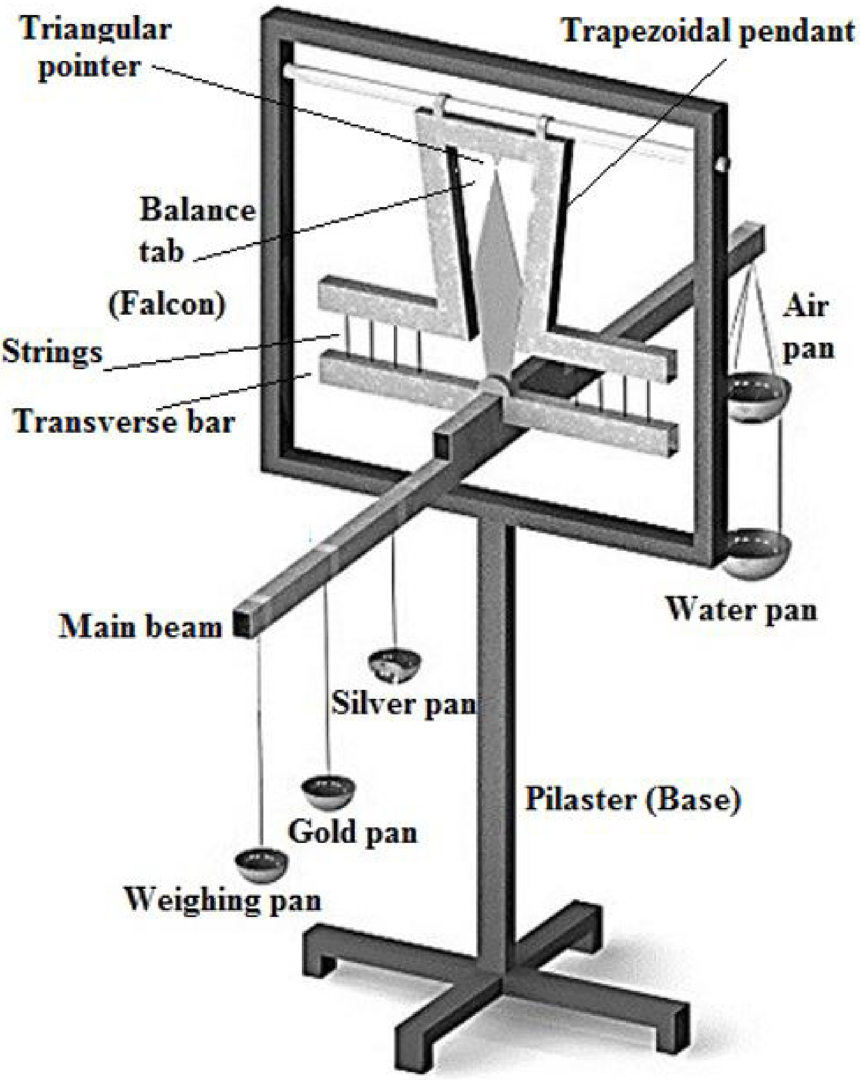

Figure. 8: *Balance of wisdom reconstructed at IROST (After: Yassi et al. 2009: 135–151)*

Grading and Calibration

Since precise calibration and grading of the balance is a prerequisite to obtaining precise results, the process should carefully be performed in advance. The following presents a step by step summary of calibration of the balance of wisdom:

The process for positioning the gold and the silver pan on the left arm of the balance:

1.Place all pans free of any masses in a position to obtain perfect equilibrium.

2.A certain amount of pure gold is added to the air pan and is counterbalanced by adding the appropriate stone weight to the weighing pan.

3.The gold is then removed and put in the water pan. This would in turn deteriorate the equilibrium condition of the balance.

4.The counterweight is put in the gold pan to restore the equilibrium. The gold pan is then moved to a new position on the left arm to achieve perfect equilibrium. The position of the gold pan is then marked on the left arm. The left arm could be calibrated for other elements applying the same process.

Titration of a mixture of gold and silver:

1.Put the mixture in the air pan and counterbalance it with appropriate weights in the weighing pan.

2.Now put the mixture in the water pan. The equilibrium deteriorates.

3.To restore equilibrium, the weights in the weighing pan should be distributed into the gold pan and the silver pan in such a manner that perfect equilibrium is again achieved.

4.The weights in these two pans represent the value of the constituent element in the alloy (mixture) (Yassi *et al*. 2009: 135–151; Al-Khazini 1967; Yassi and Yassi 2020).

Functions of the Balance of Wisdom

The functions of the balance include the following: weight measurement; density measurement; titration; identification of elements; determining the volume of objects of indefinite form; measuring time; and determining horizontality of surfaces (Al-Khazini 1967: 124; Yassi *et al*. 2009: 135–151).

Results and Discussions

During the research work, not only the design and technological aspects of five ancient balances were studied and compared but the evolution of hydrostatic balances using each one of the subject balances as a milestone throughout the path of the history was studied. Considering the latter, an evolutionary trajectory may be observed from Archimedes' design to the balance of wisdom from the titration point of view. But the point to note is that after Al-Razi, the scale of Al-Biruni lacks titration ability and its structure has not been designed for this purpose.

Of course, it is not strange because he identifies and classifies elements and minerals in nature in his field of studies by determining their specific gravity by his invented balance (Al-Khazini 1967: 48–49). After Al-Biruni, it comes down to a more sophisticated design, made by Abd al-Rahman Al-Khazini and known as "the balance of wisdom."

In terms of density measurement, it is important to note that Al-Biruni's scale is very precise because it has no limitations on the size of the objects, regardless of their physical form. Also, the volume of any solid object, even with a density lesser than that of water or any other liquid (on the conditions of insolubility in water and being denser than water), can be determined by this scheme.

Scales designed by Archimedes, Al-Razi, and Khayyam lack this feature and therefore densitometry may be regarded as an innovative step in the design of hydrostatic balances which can be observed in Al-Biruni's balance and the balance of wisdom.

Time measurement, horizontal leveling[1], and precise determination of the percentage of constituent elements of any binary alloys are further innovative features of the balance of wisdom.

The following presents the test results.

Comparison of Functions of the Scales in Vhronological Order

A brief analogy of the above-mentioned balances could bring about a clearer expression of the evolution of hydromechanical scales from Archimedes to Al-Khazini.

All the capabilities of the above scales, based on the available documentation and the test results performed on the balances, are summarized and presented in Table 1.

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[1] Al-Khazini has not made any notes on the leveling procedure for the balance, but a procedure was introduced by the author enabling the balance to be used as a leveling device (see section iii of the achievements).

Table .1: *A comparison of functions of the selected balances*

Functions **Balance**	**Leveling**	**Time**	**Element detection**	**Volume determination**	**Titration**	**Densitometry**	**weighing**
Archimedes	-	-	-	-	+	*	+
Razzi	-	-	-	-	+	*	+
Khayyam	-	-	-	-	+	*	+
Abu Rayhan	-	-	+	+	-	+	+
Balance of wisdom	+	+	+	+	+	+	+

*Not originally defined as the main function but possible to perform.

Precision Comparison of the Scales in Titration

The prime objective of hydrostatic balances is to measure the purity of binary alloys (or titration), and amongst the five selected scales only Al-Biruni's balance is incapable of titration. As mentioned earlier, out of the three almost similar balances, viz. Archimedes', Al-Razi's and Khayyam's, only Al-Razi's natural balance was reconstructed as the representative of the three given their similar design and performance.

According to the information presented in the book *Kitab Al-Mizan Al-Hikma* and concerning Section 2 above, "the natural scale," designed by Al-Razi, was reconstructed. To appraise the titration function of the scale, three samples of 250, 500, and 1000 grams of iron and aluminum compound were considered for different measurements. The balance was then calibrated for these two elements, and then respective measurements were performed. The results of the measurements and their consequent accuracies are presented in Table 2.

Also table 3 presents the titration results for the balance of wisdom of Al-Khazini, which was reconstructed according to the details presented by him in his book Balance of Wisdom.

Table 4 provides a comparative view of the titration precision of the balance of wisdom and Al-Razi's natural balance.

Testing and Comparison of Accuracy of Scales in Densitometry

Only two of the balances, namely that of Al-Biruni and the balance of wisdom, are capable of densitometry. Therefore, both balances were tested for densitometry and Table 5 represents the results (discrepancies less than 0.5% are taken as zero).

As seen from Table 5, the densitometry results of the test replicas are almost in perfect agreement with the original old versions. Also, Table 6 provides a densitometry precision comparison between the density values of four various elements determined by Al-Biruni's balance and the balance of wisdom against present-day values. Discrepancies are not more than 2%, and may thus be regarded as negligible in most cases.

Table. 2: *Titration test results against actual values for Al-Razi's balance*

		Aluminum content (gr)			**Iron content (gr)**		
		Actual value	**Titration result**	**Error (%)**	**Actual value**	**Titration result**	**Error (%)**
Overall mass of the sample (gr)	250	50	50	0	200	200	0
	250	330	330	0	170	170	0

Table. 3: *The titration results for the balance of wisdom*

Aluminum content (gr)		**Error**	**Iron content (gr)**		**Error**
Actual value	**Titration result**	**(%)**	**Actual value**	**Titration result**	**(%)**
199.1	193.6	2.8	125.6	131.1	4

Table .4: *Titration precision comparison of the test results of the balance of wisdom and Al- Razi's natural balance*

Balance	**Balance of Wisdom**	**Natural Balance**
Precision error E' (%)	3.400*	0*

* (Yassi *et al*. 2009: 135–151)

Table. 5: *Comparison of the densitometry results from the new replicas and the old versions*

P / **Element**	**(%)**	P' **Balance of wisdom (gr/Cm^3)**	P'' **Balance of wisdom* (gr/Cm^3)**	**(%)**	P' **Abu Rayhan gr/Cm^3**	P'' **Abu Rayhan* (gr/Cm^3)**
Brass	0	8.5	8.5	0.8	8.6	8.67
Copper	1.3	8.540	8.66	0	8.97	8.92
Mercury	0.2	13.5	13.56	0	13.7	13.74
Iron	0	7.70	7.7	0	7.8	7.69

*(Al-Khazini 1967: 124; Yassi *et al*. 2009: 135–151)

Table. 6: *Comparison of densitometry precision between old and today's values determined by the reconstructed samples.*

	Density (gr/cm3)			**Discrepancy (%)**	**Discrepancy (%)**
Element	**Balance of wisdom**	**Abu Rayhan's**	**Today's values**	**Balance of wisdom**	**Abu Rayhan's**
Brass	8.67	8.5	8.4	2	1.2
Copper	8.92	8.66	8.85	1	1.2
Mercury	13.74	13.56	13.56	1.3	0
Iron	7.69	7.7	7.8	1.4	1.2

Determination of Old Values of Densities

To determine the values of densities for various substances used throughout the research, the information about the weight reduction of substances in water in the book *Balance of Wisdom* was used. In one of the tables, Al-Khazini (1967: 57) presents the values of masses in air and in water for a certain amount of various substances. Strangely enough, he has used the value of a certain amount of clear water as the base value for determining the density values of other fluids (Al-Khazini 1967: 76–75). Therefore, using Archimedes' principle and having the two values for masses in the air and water of substances it is possible to determine the density value of a certain object.

For instance, according to the above-mentioned table, the following can generate the density value of a substance:

$$Density = \frac{Ma}{\left[\frac{(Ma-Mw)}{1(Density\ of\ the\ base\ fluid)}\right]} \qquad \text{(A)}$$

Where Ma is the mass of the substance in the air and Mw is the mass of the substance in water.

As an instance, the air mass value (Ma) for Mercury is given as 2400 Tassoo[1] . Its mass in water (Mw) is given as 2223 Tassoo. Integrating these values into equation (A), the value of the density for mercury will be equal to 15.56 Tassoo per unit volume or in today's terms, gr/cm^3.

Conflict of Interest

On behalf of all authors I, hereby, state that there is no conflict of interest whatsoever.

Data Availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author upon reasonable request, although all data generated or analyzed during this study are included in this published article.

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[1] Tassoo is an old mass unit equal to 1.24 mithqal.

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