International Journal Of
Industrial Electronics Control and Optimization
Print ISSN: 3517-2645
Online ISSN: 3568-2645
Homepage: https://ieco.usb.ac.ir/

Sliding Mode Control for Chaotic Systems with Unknown Uncertainties

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Article Info

Article type:
Research Article

Article history:
Received: 12-Dec-2023
Received in revised form: 14-Feb-2024
Accepted: 25-Feb-2024
Published online: 12-March-2024

Keywords:
Chaotic system,
Sliding mode control,
Time-dependent system,
Time-independent system,
Fixed-time stability.

ABSTRACT

Objective: In this article, time-varying chaotic systems with uncertainties, including external disturbances, are considered, and sliding mode control (SMC) is used to control such systems. To control these systems, an autonomous differential equation is first introduced. Then, based on this differential equation, a sliding surface is defined to control this chaotic system. This kind of controller is remarkable in that it removes the effects of disturbances, whether bounded or unbounded. Therefore, the system is known to be fixed-time stable. Where the trajectories of this chaotic system are not placed on the sliding surface, we have created creative controllers to place the trajectories on the sliding surface in finite time. Theoretical investigations show that such chaotic systems can be made fixed-time stable by applying the controls proposed in this study. Based on the findings of this study, the controllers are designed to eliminate all disturbances, whether bounded or unbounded. The results can be said to apply to chaotic, time-dependent, and time-independent systems. To further consolidate the results obtained in this article, two examples, namely the time-dependent system of the Gyro and the time-independent system of the Liu, are investigated, and the results were compared with previous works by other researchers.

I. Introduction

Chaos theory is an interdisciplinary scientific study and a branch of mathematics concerned with the underlying patterns and deterministic principles of dynamical systems that are very sensitive to the initial circumstances and were previously assumed to have fully random states of disorder and irregularity [1]. The butterfly effect is a term used in chaos theory to describe this occurrence. It is a simile used to describe how a butterfly may behave in Brazil while flapping its wings under specific conditions, which might then result in a storm in Texas [2]. As a result, it is not imaginable to forecast how these systems will behave in the long run. Although it can be assumed that random dynamics act chaotically, deterministic dynamic systems may also exhibit chaos, demonstrating that chaos need not be the result of an accidental element [3, 4]. It can be shown that chaotic behavior in continuous dynamical systems requires at least three variables. Chaotic movement is a frequent and acceptable nonlinear event that has lately gained increased attention due to its various uses. Some of its industrial applications include data processing, secure telecommunication systems, electrical converters, and chemical processes [5-8]. Since the early 1990s, ample research has been done on how to regulate chaos in the chaotic systems [9]. As a result, a number of researchers have now developed numerous strategies to combat the chaotic behavior in such dynamic systems. Here, we mention several papers. In [10], barrier function-based SMC approaches were used. In [11], stochastic delay methods were applied. In [12, 13], intelligent control methods based on neural networks were handled. In [14], the problem was challenged by using linear matrix inequalities. In [15, 16], to control such dynamic systems, sliding mode control (SMC) was introduced. In [17, 18], backstepping control was convincingly used. In [19, 20], adaptive control provided an effective solution. In [21, 22], the optimal control method has provided an efficient solution to the chaotic behavior of such systems. In [23, 24], the backstepping method, as a common method, was used for such chaotic systems. Among the control techniques mentioned above, sliding mode control has been successfully applied in a number of applications to enhance controller performance and address issues including immeasurable modes, input saturation, and others. It is a practical and effective method for dealing with the issue of uncertainty in nonlinear systems [25, 26]. However, simple adaptive control cannot be used to precisely regulate a nonlinear dynamical system with unknown model uncertainties. Additionally, the sliding mode control approach is a reliable and effective tool for engineering study of high-order nonlinear systems, both theoretically and practically. Among the existing control strategies, the SMC is a reliable and successful way for chaos control in chaotic systems [25]. However, the chattering problem, which occurs because the sign function of control input signal is discontinuous, affects the majority of typical SMC techniques. Recently in [26] , the undesirable chattering assumption was eliminated using an innovative chattering-free SMC technique. The design was developed with the hypothesis that the first derivatives and upper bounds of the uncertainty term are known. Unknown upper bounds for the uncertainty and its derivatives have not been studied in this case. SMC methods are frequently used in industrial settings. In [27], some new developments in SMC for networked control systems (NCSs) were investigated. First, a few innovative SMC methods to address NCSs with time delays, uncertainty, and disturbances are briefly described. Next, the issue of SMC for NCSs was covered. A SMC approach is also suggested in [28] for nonlinear systems with time delays and undetermined missing probability. The study by [29] examined nonlinear fractional systems with external disturbances and uncertainties. Using an appropriate sliding surface, they introduced a method for investigating and evaluating stability. They created a robust adaptive fractional sliding mode controller in response to the external disturbances and an unknown upper bound on uncertainties. The study of chaotic dynamics in environmental phenomena was addressed in article [30]. For this fractional-order system, they derived two distinct sliding-mode controllers to manage chaos. They created a new controlled system of equations both with and without uncertainties through this process. Additionally, the new systems' global stability is established. The study by [31] investigated chaos in the Bloch equation. In this work, the Bloch equation with and without delay was studied in relation to the Caputo fractional derivative. They explored the underlying chaos using a sliding-mode controller. The effectiveness of the controller was monitored in the presence of external disturbances and uncertainty.

In most of the studies reported in this paper, the researchers considered common sliding surfaces and then tried to put the trajectories of chaotic dynamic systems on this surface with complicated and challenging techniques. However, the present study presents an innovative sliding surface and controllers that disregard the boundedness of uncertainty and system disturbances in order to solve the upper bounds of unknown uncertainty.

The most significant contributions of the study are as follows:

- It develops a fixed-time controller for the stabilization of chaotic systems, utilizing an innovative sliding-mode surface.
- It offers a technique for establishing a limit on chaotic systems' fixed-time stability. without depending on the starting circumstances.
- It creates controllers without taking into account system disturbances and the uncertainty bound.

The rest of this article is divided into the following sections: A few explanations and definitions are provided in Section 2; the controller structure is both defined and formulated in Section 3; to demonstrate the effectiveness of the suggested control mechanism, simulation results are shown in Section 4; finally, some conclusions are reached in Section 5.

II. Preliminaries and system description

Nonlinear chaotic systems have a dynamic equation that is often expressed as follows:

$$\dot{x} = f(t,x) + d(t,x) + u(t). \qquad (1)$$

where $x = [x_1, x_2, \cdots, x_n]^T \in R$ represents the system state vector, $f(t,x): R^+ \times R^n \to R^n$ indicates a nonlinear function, $d(t,x): R^+ \times R^n \to R^n$ is the unknown uncertainty term that denotes model uncertainties mixed by unknown external disturbances and system unmodeled dynamics, and $u(t) \in R^n$ is the control signal. The number of control signals u and the number of state variables are assumed to be equal in the chaotic system equation (1).

Definition 1 [32]**:** An autonomous dynamic equation:

$$f: R^n \to R^n, \quad \dot{x}(t) = f\big(x(t)\big), \quad x(0) = x_0. \qquad (2)$$

is said to be fixed-time stable if

$$\exists\, t_* \;\; \forall\, t > t_*\,; \|x(t) = 0\| \;\wedge\; \lim_{t \to t_*} \|x(t)\| = 0. \qquad (3)$$

t_* is independent of the initial value of the autonomous differential equation.

Remark 1: In this definition, if we consider the dynamic system as $\dot{x}(t) = f\big(t\,, x(t)\big)$, the concept of fixed-time stability is described in a similar way.

The goal of this article is to develop a sliding surface and controllers for chaotic systems with model uncertainties combined with unidentified external perturbations in order to establish closed-loop systems that are fixed-time stable for any initial conditions. It indicates that the system's trajectory converges to the origin in a limited amount of time, irrespective of the initial conditions.

Remark 2: It should be noted that system (1) simply turns into this dynamical system:

$$\begin{cases} \dot{x}_1 = f_1(t,x) + d_1(t,x) + u_1(t), \\ \dot{x}_2 = f_2(t,x) + d_2(t,x) + u_2(t), \\ \vdots \\ \dot{x}_n = f_n(t,x) + d_n(t,x) + u_n(t). \end{cases} \quad (4)$$

Lemma 1: Suppose that $K > 0, M > 0$ and $x : R \to R$ is a continuous function. If there exists a function $F(x)$ such that:

$$\dot{x} = -K\,F(x). \quad (5)$$

And

$$\forall\, a,b \in R, \quad \left|\int_a^b \frac{dx}{F(x)}\right| < M. \quad (6)$$

then the autonomous differential equation (5) is fixed-time stable and $t_* < \frac{M}{K}$.

Proof: The result of equation (5) is:

$$\frac{dx}{F(x)} = -K\,dt. \quad (7)$$

upon careful evaluation of equation (7), it becomes apparent that:

$$\int_{x(0)}^{x(t)} \frac{dx}{F(x)} = -K\int_0^t d\tau. \quad (8)$$

therefore, it is obtained:

$$\int_{x(0)}^{x(t)} \frac{dx}{F(x)} = -Kt. \quad (9)$$

however, $\forall\, a,b \in R, \left|\int_a^b \frac{dx}{F(x)}\right| < M$, thus:

$$\left|\int_{x(0)}^{x(t)} \frac{dx}{F(x)}\right| = |-Kt| < M. \quad (10)$$

this gives the result $|t| < \frac{M}{K}$, hence the autonomous differential equation (5) is fixed-time stable and $t_* < \frac{M}{K}$. ■

Corollary 1 [33]: Suppose that $K > 0$:

$$A(x) = sign(x)(|x|+1)\sqrt{(|x|+1)^2 - 1}. \quad (11)$$

and $x: R \to R$ is a continuous differentiable function that satisfies the following conditions:

$$\dot{x} = -K\,A(x)\ , \quad x(0) = x_0. \quad (12)$$

then the system (12) is fixed-time stable and setting time is $t_* \le \frac{\pi}{K}$.

Fig. 1 shows that if we take $K = 2$ and $x(0) = 5$, the setting time for the solution curve of (12) is $t_* \le \frac{\pi}{2}$. Also, if we take $K = \frac{1}{2}$ and $x(0) = -7$, the setting time for the solution curve of (12) is $t_* \le 2\pi$.

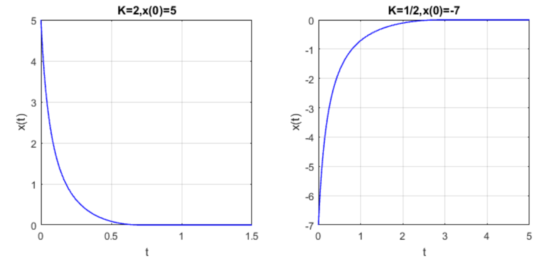

Fig. 1. solution curve for (12)

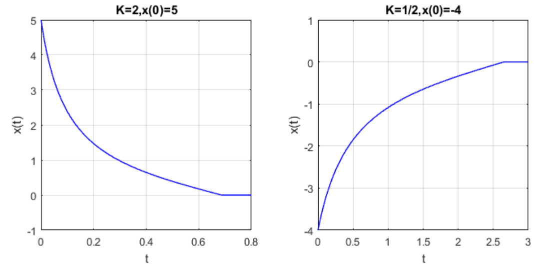

Fig. 2. Solution curves for (14)

Corollary 2: Suppose that:

$$K > 0\ ,\ B(x) = sign(x)\,(x^2+1). \quad (13)$$

and $x: R \to R$ is a continuous differentiable function that satisfies the following conditions:

$$\dot{x} = -K\,B(x)\ , \quad x(0) = x_0. \quad (14)$$

then the system (14) is fixed-time stable and setting time is $t \le \frac{\pi}{2K}$.

Proof: Assume that $F(x) = B(x)$ thus:

$$\int_{x(0)}^{x(t)} \frac{dx}{F(x)} = \int_{x(0)}^{x(t)} \frac{dx}{sign(x)(x^2+1)}. \quad (15)$$

therefore, it is obtained:

$$\int_{x(0)}^{x(t)} \frac{dx}{F(x)} = \left.\frac{tan^{-1}(x)}{sign(x)}\right|_{x(0)}^{x(t)} = -Kt. \quad (16)$$

however, for any θ , $|\,tan^{-1}(\theta)| < \frac{\pi}{2}$. Lemma 1 concluded that $t < \frac{\pi}{2K}$, hence the autonomous differential equation (14) is fixed-time stable and $t_* < \frac{\pi}{2K}$. ■

Fig. 2 shows that if we take $K = 2$ and $x(0) = 5$, the setting time for the solution curve of (14) is $t_* \le \frac{\pi}{4}$. Also, if we take $K = \frac{1}{2}$ and $x(0) = -4$, the setting time for the solution curve of (14) is $t_* \le \pi$.

III. Controller design by SMC

SMC is employed in the suggested control strategy to address the model uncertainties of the chaos system.

Remark 3: The system uncertainty terms $d_i(t,x)$ are assumed to be bounded in article [34], but this article does not make that same assumption.

A novel sliding mode controller is created for this part in order to provide fixed-time stable control over a nonlinear system. Two major steps are involved in the design of the proposed fixed-time controller:

- Constructing an appropriate sliding surface.
- Establishing an effective fixed-time control of the sliding motion within a specified setting time.

The nonlinear sliding mode is constructed in the following way to achieve system control (4) :

$$K > 0, \qquad s_i = x_i + \int_0^t K\, F(x_i). \tag{17}$$

It is clear that $s_i = 0$, $\dot{s}_i = 0$ if the system's trajectories are on the sliding surface.

Theorem 1: Assume that the sliding surface is dynamic (17). In the case where $s_i = 0$, the system is fixed-time stable, and the setting time T_1, is specified $T_1 \le \frac{\pi}{K}$ if $F(x) = A(x)$, or $T_1 \le \frac{\pi}{2K}$ if $F(x) = B(x)$. Its trajectories converge to the equilibrium $x_i(t) = 0$.

Proof: Assuming $s_i = 0$ and $\dot{s}_i = 0$, therefore $\dot{x}_i = -\,K\,F(x_i)$, Lemma 1 shows the theorem's validity. As a result, $x_i(t)$ converges to the origin, and the setting time T_1 is given $T_1 \le \frac{\pi}{K}$ according to corollary 1 or $T_1 \le \frac{\pi}{2K}$ according to corollary 2.

If $s_i \neq 0$, which indicates that the trajectories of system (4) are outside of the sliding surface, then we should construct a suitable controller to bring the trajectories into the sliding surface and maintain them there continuously. To reach this objective, the following theorem is offered:

Theorem 2: Assume that $K > 0$:

$$u_i(t) = \xi_i - K\,F(x_i) - f_i(t\,,x) - d_i(t\,,x). \tag{18}$$

With

$$\xi_i = -K\,F(s_i). \tag{19}$$

if the trajectories of system (4) are outside of the sliding surface, then the controller (18) brings the trajectories into the sliding surface. And reaching time T_2 is given $T_2 \le \frac{\pi}{K}$ according to corollary 1, $T_2 \le \frac{\pi}{2K}$ according to corollary 2.

Proof: It follows from calculation $\dot{s}_i$:

$$\begin{aligned} \dot{s}_i &= \dot{x}_i + K\,F(x_i), \\ &= f_i(t\,,x) + d_i(t\,,x) + u_i(t) + K\,F(x_i), \\ &= -K\,F(s_i). \end{aligned} \tag{20}$$

thus $\dot{s}_i = -\,K\,F(s_i)$, Lemma 1 proves the correctness of the theorem 2. As a result, s_i is fixed-time stable, and the setting time T_2 is determined by $T_2 \le \frac{\pi}{K}$ according to corollary 1 or $T_2 \le \frac{\pi}{2K}$ according to corollary 2.

Theorem 3: The state variables of the controlled system (4) can be stabilized into the origin fixed-timely if the controller $u_i(t)$ is defined as (18). The whole process time, T_3 is predicted by $T_3 \le T_1 + T_2$.

Proof: The conclusion of Theorem 3 is clear in light of Theorems 1–2.

Remark 4: The determination of fixed-time stability depends on the existence of a differential equation $\dot{x} = -K\,F(x)$ satisfying the conditions of Lemma 1. Furthermore, the initial conditions and the system characteristics had no effect on the control structure in this investigation. The method described here might be applicable to more kinds of chaotic dynamics.

IV. Numerical simulation

We executed two numerical simulations of the Gyro system and Liu's uncertain chaotic system in order to demonstrate how effectively the suggested technique performed in these two systems.

Example 1: The time-dependent Gyro system is investigated from the viewpoint put out in this article. Gyro system dynamics are described by:

$$f(t,x) = \begin{bmatrix} x_2 \\ b_1 x_2 + b_2 x_2^3 + g(x_1,t) \end{bmatrix}. \tag{21}$$

$$g(x_1,t) = b_3 \sin(x_1) - b_4^2 \frac{\left(1 - cos(x_1)\right)^2}{sin^3(x_1)} + Fsin(\omega t) sin(x_1).$$

$$d(t\,,x) = \begin{bmatrix} 1 + sin(x_2) cos(2x_1) \\ sin(x_1) \end{bmatrix}.$$

equation (21) can be considered as follows:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = b_1 x_2 + b_2 x_2^3 + g(x_1,t). \end{cases} \tag{22}$$

In [35], the dynamics of (22) were studied. Specifically, the system (22) with the parameters defined by the equations $b_1 = 0.5, b_2 = 0.05, b_3 = 1, b_4 = 10, F = 35, \omega = 2$ can behave chaotically. In Fig. 3 and Fig. 4, the chaos movement of the equation (22) with $x(0) = (-1,3)$ is depicted.

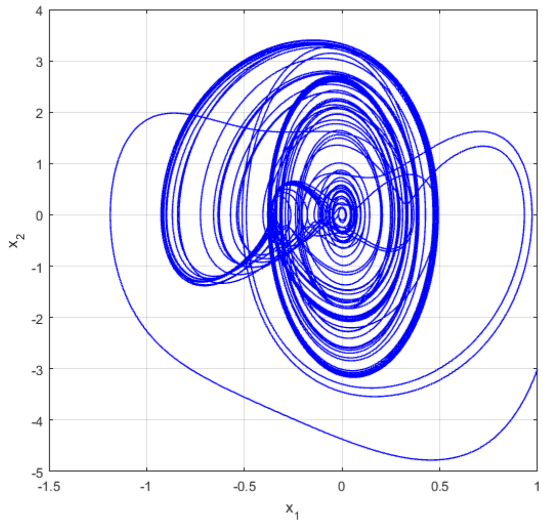

Fig. 3. The chaotic attractor of (22) with $x(0) = (-1{,}3)$

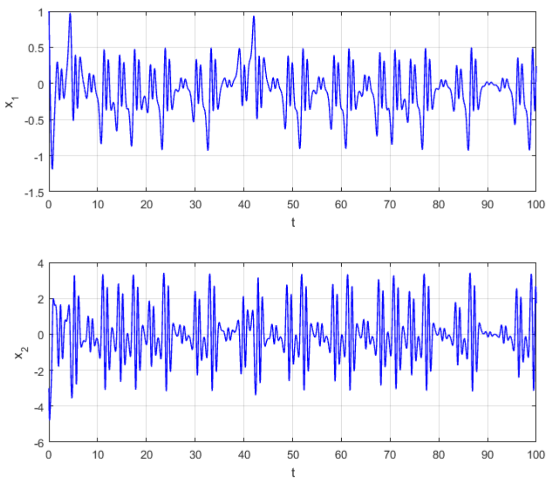

Fig. 4. $x_1(t)\,, x_2(t)$ of system (22)

Control over the chaotic system (22) is given by:

$$\begin{aligned} u_1(t) &= \xi_1 - KF(x_1) - x_2 - d_1(t\,,x), \\ u_2(t) &= \xi_2 - KF(x_2) - b_1x_2 - b_2x_2^3 - g(x_1,t) - d_2(t\,,x). \end{aligned} \quad (23)$$

That $\xi_1 = -KF(s_1), \xi_2 = -KF(s_2)$, $d_1(t\,,x) = 1 + \sin(x_2)cos(2x_1)$, $d_2(t\,,x) = sin(x_1)$ are the uncertainties. It can be observed that in (23), controllers $u_1(t)\,, u_2(t)$are designed to remove the effects of uncertainties $d_1(t\,,x)\,, d_2(t\,,x)$, whether they are bounded or unbounded. Based on the discussed results, we design efficient controllers such that (24) is fixed-time stable.

$$\begin{cases} \dot{x}_1 = x_2 + d_1(t\,,x) + u_1(t), \\ \dot{x}_2 = b_1x_2 + b_2x_2^3 + g(x_1,t) + d_2(t\,,x) + u_2(t), \end{cases} \quad (24)$$

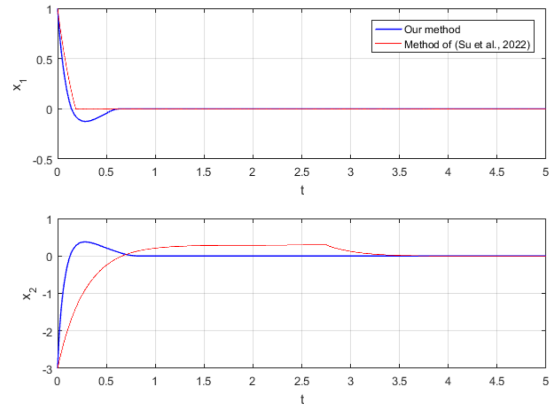

Fig. 5. With assumption $F(x) = A(x)$ and K= 1 , comparing our method with the method of article [32]

TABLE 1: EVALUTION OF RESULTS

Method	*t*	0	1	2	3	4	5
Our method	x_1	1	-2.8121e-19	-2.8121e-19	-2.8121e-19	-2.8121e-19	-2.8121e-19
	x_2	-3	-2.8121e-19	-2.8121e-19	-2.8121e-19	-2.8121e-19	-2.8121e-19
Method of Ref [32]	x_1	1	9.6686e-5	1.4082e-5	1.7498e-4	1.4867e-4	1.4312e-4
	x_2	-3	0.2085	0.2856	0.1211	6.2350e-4	1.2743e-5

Remark 5: The numerical simulations conducted in the Simulink environment demonstrate that the suggested method is more effective than the one suggested in [32]. Fig. 5 displays the controlled system's trajectories.

Table 1 demonstrates that x_1 and x_2 converge to zero using our strategy in less than a second. However, the convergence happens after three seconds in the technique suggested in [32]. Fig. 5 also reveals that x_1 and x_2 converge to zero in less than a second. However, the convergence occurs after three seconds in the method suggested in [32].

Example 2: In this part, the Simulink environment is used to assess the performance of the recommended method. Consider the Liu system [34], starting with the dynamic equation given below:

$$f(x) = \begin{bmatrix} -10x_1 + 10x_2 \\ -40x_1 + x_1x_3 \\ 4x_1^2 - 2.5x_3 \end{bmatrix}. \quad (25)$$

$$d(t\,,x) = \begin{bmatrix} sin(x_1) \\ sin(t) \\ sin(x_1) + sin(t) \end{bmatrix}. \quad (26)$$

The dynamical system can be considered as follows:

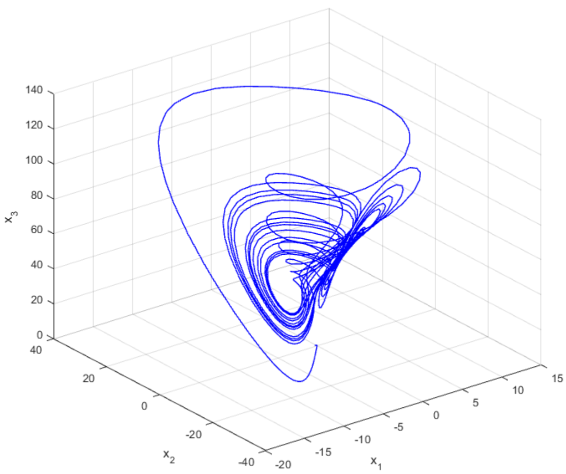

Fig. 6. The chaotic attractor of (27)

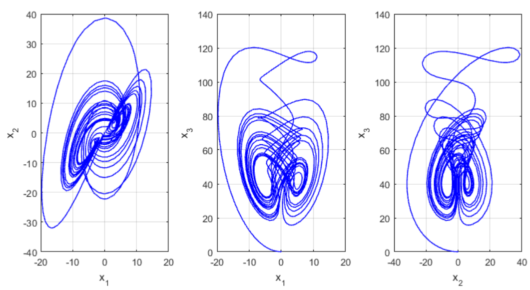

Fig. 7. The phase diagrams of states (27)

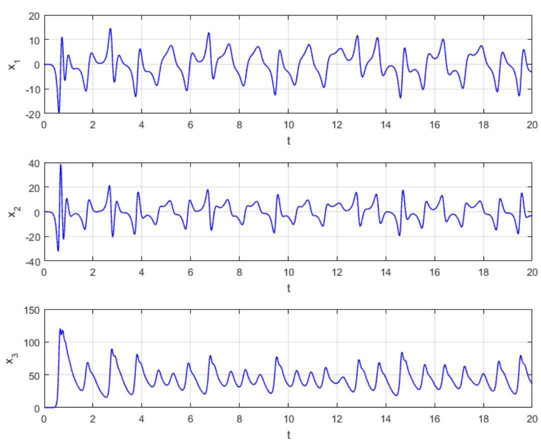

Fig. 8. $x_1(t), x_2(t), x_3(t)$ of (27) without controller

$$\begin{cases} \dot{x}_1 = -10x_1 + 10x_2 + sin(x_1), \\ \dot{x}_2 = -40x_1 + x_1x_3 + sin(t), \\ \dot{x}_3 = 4x_1^2 - 2.5x_3 + sin(x_1) + sin(t). \end{cases} \quad (27)$$

Due to the fact that Liu's chaotic system is a time-independent system, time is not a factor in the dynamics in this case. As a result, in (4), the function $f(x)$ has been utilized rather than $f(t,x)$. However, $d(t,x)$ is a combination of uncertainties and disturbances, and its values are indicated in (26). The chaotic movement of (27) with $x(0) = (-0.2, 0.3, 0.2)$ is shown in Fig. 6 to Fig. 8.

Although in our method, stabilization is possible for any time-dependent or time-independent system with any initial conditions without considering the boundary conditions for uncertainties and disturbances, but in example 2, we considered the initial conditions assumed in article [34]. We did this in order to be able to compare the results obtained in this article with those in [34]. Of course, it should be noted that in a non-linear system, initial conditions are important. A nonlinear system may become chaotic only under certain initial conditions. Therefore, the initial conditions cannot be chosen arbitrarily.
Considering the controllers, (27) can be written as:

$$\begin{cases} \dot{x}_1 = -10x_1 + 10x_2 + sin(x_1) + u_1(t), \\ \dot{x}_2 = -40x_1 + x_1x_3 + +sin(t) + u_2(t), \\ \dot{x}_3 = 4x_1^2 - 2.5x_3 + sin(x_1) + sin(t) + u_3(t). \end{cases} \quad (28)$$

Control over the chaotic system (28) is given by:

$$\begin{aligned} u_1(t) &= \xi_1 - KF(x_1) + 10x_1 - 10x_2 - d_1(t,x), \\ u_2(t) &= \xi_2 - KF(x_2) + 40x_1 - x_2x_3 - d_2(t,x), \\ u_3(t) &= \xi_3 - KF(x_3) - 4x_1^2 + 2.5x_3 - d_3(t,x). \end{aligned} \quad (29)$$

That $\xi_1 = -KF(s_1), \xi_2 = -KF(s_2), \xi_3 = -KF(s_3)$, $d_1(t,x) = \sin(x_1)$, $d_2(t,x) = sin(t)$, $d_3(t,x) = sin(x_1) + sin(t)$ are the uncertainties. It can be observed that in (29), controllers $u_1(t), u_2(t), u_3(t)$ are designed to remove the effects of uncertainties $d_1(t,x)$, $d_2(t,x), d_3(t,x)$ whether they are bounded or unbounded.
Based on the aforementioned results, we design robust controllers. The numerical simulations demonstrate that the suggested strategy is more effective than the one suggested in [34]. As a consequence, using the suggested method, Fig. 9 shows the simulation results.

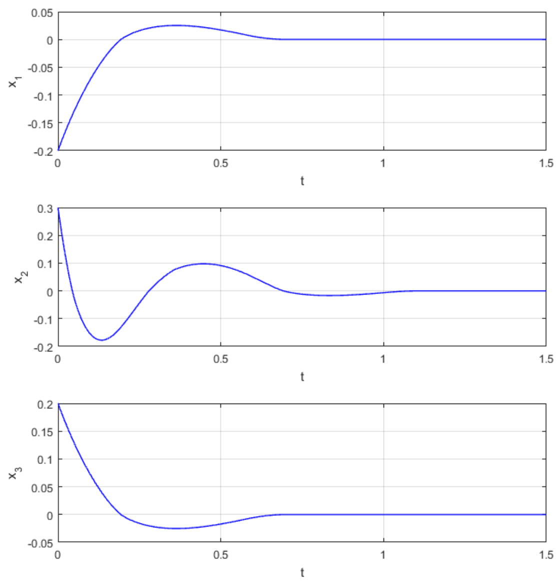

Fig. 9. $x_1(t), x_2(t), x_3(t)$ of (28) with $F(x) = A(x), K = 1$

Remark 6: In [34], a terminal sliding mode control strategy is designed and put into practice for a type of chaotic system with uncertainty. The dynamic uncertainties of the chaotic system were taken into account using sliding mode control (SMC), and the boundary problem of unknown model uncertainties was solved using a mix of SMC and an adaptive control strategy. In [34], by using an adaptive barrier function, chattering is fully removed. But in our method, by designing a suitable sliding surface, the control of Liu's system has been stabilized without any chattering in the control signals or the sliding surface curves of the system. In addition, in our method, the system becomes fixed-time stable. This example shows the effectiveness of our method.

V. Conclusions

In most of the articles mentioned in the references, researchers have considered a common sliding surface and then tried to stabilize a system with bounded uncertainty by using methods in adaptive control, optimal control, backstepping control, the barrier function, etc. Although these methods make the system stable, they often lead to complex calculations and the submission of special conditions to the system. But in this article, we sought to find an efficient sliding surface that is useful for most time-dependent and time-independent systems with uncertainties and disturbances that are not necessarily bounded. As it is clear in the proof of the theorems, using a suitable sliding surface does not require the use of Lyapunov's theorem. In this article, we have shown that in order to find a suitable sliding surface, one should first find the differential equation $\dot{x} = -F(x)$, which applies to condition $\left|\int_a^b \frac{dx}{F(x)}\right| < \infty$. Then, using this function $F(x)$, the sliding surface and controllers should be designed. Upcoming studies may focus on finding such functions. Furthermore, future research may focus on applying the results of this study to stabilize other kinds of dynamic systems with uncertainty and external disturbances.

References

[1] X. Zhang, X. Liu, and Q. Zhu, *Adaptive chatter free sliding mode control for a class of uncertain chaotic systems.* Applied Mathematics and Computation, 2014. 232: p. 431-435.

[2] E. Zeraoulia, and J.C. Sprott, *Robust chaos and its applications*. World Scientific series on nonlinear science. Series A, Monographs and treatises, v. 79. 2012, Singapore: World Scientific.

[3] C. Guanrong, *Chaos theory and applications: a new trend*. 2021, Akif AKGÜL. p. 1-2.

[4] M. Jun, *Chaos Theory and Applications：The Physical Evidence, Mechanism are Important in Chaotic Systems.* Chaos Theory and Applications, 2022. 4(1).

[5] S. Vaidyanathan, and C. Volos, *Advances and Applications in Chaotic Systems*. 1st 2016. ed. Studies in Computational Intelligence, 636. 2016, Cham: Springer International Publishing.

[6] A. Ouannas, et al., *Synchronization of Fractional Hyperchaotic Rabinovich Systems via Linear and Nonlinear Control with an Application to Secure Communications.* International Journal of Control, Automation and Systems, 2019. 17(9): p. 2211-2219.

[7] S. Mobayen, et al., *Barrier function-based adaptive nonsingular sliding mode control of disturbed nonlinear systems: A linear matrix inequality approach.* Chaos, Solitons & Fractals, 2022. 157: p. 111918.

[8] K. A. Alattas, et al., *Nonsingular Integral-Type Dynamic Finite-Time Synchronization for Hyper-Chaotic Systems.* Mathematics, 2022. 10(1): p. 115.

[9] C. K. Tse, *Controlling chaos and bifurcations in engineering systems, Guanrong Chen, Boca Raton, FL: CRC Press, 2000, ISBN 0-8493-0579-9.* International Journal of Robust and Nonlinear Control, 2001. 11(4): p. 393-397.

[10] Z. Liu, and H. Pan, *Barrier function-based adaptive sliding mode control for application to vehicle suspensions.* IEEE Transactions on Transportation Electrification, 2020. 7(3): p. 2023-2033.

[11] L. Zhang, C. Zhang, and D. Zhao, *Control of a class of chaotic systems by a stochastic delay method.* Kybernetika, 2010. 46(1): p. 38-49.

[12] P. Y. Xiong, et al., *Spectral entropy analysis and synchronization of a multi-stable fractional-order chaotic system using a novel neural network-based chattering-free sliding mode technique.* Chaos, Solitons & Fractals, 2021. 144: p. 110576.

[13] R. Wang, et al., *Fuzzy neural network-based chaos synchronization for a class of fractional-order chaotic systems: an adaptive sliding mode control approach.*

Nonlinear Dynamics, 2020. 100: p. 1275-1287.
[14] Z. Ma, et al., *Recursive Feasibility and Stability for Stochastic MPC based on Polynomial Chaos.* IFAC-PapersOnLine, 2023. 56(1): p. 204-209.
[15] A. Modiri, and S. Mobayen, *Adaptive terminal sliding mode control scheme for synchronization of fractional-order uncertain chaotic systems.* ISA transactions, 2020. 105: p. 33-50.
[16] M. H. Barhaghtalab, S. Mobayen, and F. Merrikh-Bavat. *Design of a global sliding mode controller using hyperbolic functions for nonlinear systems and application in chaotic systems*. in *2019 27th Iranian Conference on Electrical Engineering (ICEE)*. 2019. IEEE.
[17] S. Ha, et al., *Backstepping-based adaptive fuzzy synchronization control for a class of fractional-order chaotic systems with input saturation.* International Journal of Fuzzy Systems, 2019. 21: p. 1571-1584.
[18] S. Ha, H. Liu, and S. Li, *Adaptive fuzzy backstepping control of fractional-order chaotic systems with input saturation.* Journal of Intelligent & Fuzzy Systems, 2019. 37(5): p. 6513-6525.
[19] O. Mofid, S. Mobayen, and M.H. Khooban, *Sliding mode disturbance observer control based on adaptive synchronization in a class of fractional - order chaotic systems.* International Journal of Adaptive Control and Signal Processing, 2019. 33(3): p. 462-474.
[20] H. Jahanshahi, et al., *Entropy analysis and neural network-based adaptive control of a non-equilibrium four-dimensional chaotic system with hidden attractors.* Entropy, 2019. 21(2): p. 156.
[21] G. Rigatos, and M. Abbaszadeh, *Nonlinear optimal control and synchronization for chaotic electronic circuits.* Journal of Computational Electronics, 2021. 20: p. 1050-1063.
[22] C. Letellier, and J. P. Barbot, *Optimal flatness placement of sensors and actuators for controlling chaotic systems.* Chaos: An Interdisciplinary Journal of Nonlinear Science, 2021. 31(10).
[23] C. Liu, et al., *A new chaotic attractor.* Chaos, Solitons & Fractals, 2004. 22: p. 1031-1038.
[24] L. Crespo, and J.Q. Sun, *On the Feedback Linearization of the Lorenz System.* Journal of Vibration and Control - J VIB CONTROL, 2004. 10: p. 85-100.
[25] H. Su, et al., *Fixed time stability of a class of chaotic systems with disturbances by using sliding mode control.* ISA transactions, 2021. 118: p. 75-82.
[26] H. Li, et al., *Chaos control and synchronization via a novel chatter free sliding mode control strategy.* Neurocomputing, 2011. 74(17): p. 3212-3222.
[27] J. Hu, et al., *A survey on sliding mode control for networked control systems.* International Journal of Systems Science, 2021. 52(6): p. 1129-1147.
[28] J. Hu, et al., *Design of sliding-mode-based control for nonlinear systems with mixed-delays and packet losses under uncertain missing probability.* IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2019. 51(5): p. 3217-3228.
[29] F. Roshanravan, A. Heydari, *Sliding Mode Control Design for a Class of Nonlinear Fractional Systems with Application to Glucose-Insulin Systems.* International Journal of Industrial Electronics, Control and Optimization, 2002.
[30] M. K. Naik, et al., *Design of a fractional-order atmospheric model via a class of ACT-like chaotic system and its sliding mode chaos control.* Chaos: An Interdisciplinary Journal of Nonlinear Science, 2023. 33(2).
[31] C. Baishya, et al., *Chaos control of fractional order nonlinear Bloch equation by utilizing sliding mode controller.* Chaos, Solitons & Fractals, 2023. 174: p. 113773.
[32] H. Su, et al., *Robust fixed time control of a class of chaotic systems with bounded uncertainties and disturbances.* International Journal of Control, Automation and Systems, 2022. 20(3): p. 813-822.
[33] A. Rezaie, et al., *Design of a Fixed-Time Stabilizer for Uncertain Chaotic Systems Subject to External Disturbances.* Mathematics, 2023. 11(15): p. 3273.
[34] M. A. Sepestanaki, et al., *Chattering-Free Terminal Sliding Mode Control Based on Adaptive Barrier Function for Chaotic Systems With Unknown Uncertainties.* IEEE Access, 2022. 10: p. 103469-103484.
[35] Y. Chen, X. Li, and S. Liu, *Finite-time stability of ABC type fractional delay difference equations.* Chaos, Solitons & Fractals, 2021. 152: p. 111430.

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