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Original paper

Type-3 fuzzy system for dynamic system control

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Abstract

Many dynamic processes are characterized by parametric or structural uncertainties due to internal and external disturbances. Existing deterministic models could not handle the uncertainties inherent in these processes. A valuable alternative to control these processes is the use of a type-3 fuzzy system. Since type-3 fuzzy systems use three-dimensional membership functions, they have more capacity to model uncertainties. This paper introduces the design of a type-3 fuzzy logic system (FLS) for the control of dynamic plants. Utilizing type-3 fuzzy logic, the architecture of the type-3 fuzzy control system (T3FCS) is proposed. The knowledge base of the controller is constructed and its design stages are presented. The inference mechanism of type-3 FLS is developed using α slices and interval type-3 membership functions. The proposed type-3 FLS is utilized for controlling nonlinear dynamic plants. The modeling of the proposed T3FCS is performed and transient response characteristic is derived using different stepwise excitation signals. A comparison of the designed system with the type-1 FLS-based system is provided. The obtained simulation result demonstrates the efficiency of using the proposed type-3 FLS in the control of dynamic systems characterized by uncertainties.

Keywords: Fuzzy controller, type-3 membership function, type-3 fuzzy logic, control system.

1 Introduction

In industry, many dynamic processes are characterised by uncertainties arising from the presence of internal and external disturbances that affect the operating mode of these processes. These conditions require the use of advanced theories for accurate control of dynamic systems. The researchers have conducted a set of research studies in controlling dynamic processes characterized by uncertainties. The fuzzy Logic theory invented by L.Zadeh is one of the important scientific approaches used in solving such kinds of problems [27]. Fuzzy logic provides treatment of uncertainties and has shown great improvement in control system development for these problems [27]. Zadeh also proposed type-2 and type-n fuzzy logics to deal with high uncertainty degrees. In the future, the mathematical background of the type-2 fuzzy logic system was developed by Mendel [25] in order to design fuzzy systems and improve their performance by handling uncertainties. The membership function (MF) of type-2 FLS is two-dimensional which leads to increasing complexity of type-2 fuzzy logic systems (T2FLS). Mendel proposed interval T2FLS to simplify operations and reduce the computational cost, i.e. complexity of T2FLS. T2FLS is widely used for solving many problems in engineering and science. Those are time-series prediction [5, 6], control and identification [14, 27], classification [16] problems.

Various control structures have been designed using fuzzy logic to solve engineering problems. The reference [17] introduced a hybrid controller using active disturbance rejection control and sliding mode control to improve the control performance. The paper [11] presented indirect adaptive control of linear and nonlinear systems. High uncertainties and fuzziness can be handled by high-order fuzzy relations of T2FLS. As indicated in [9], more fuzziness in the system design increases the ability to handle uncertainty. Recently few research studies related to type-3 FLS were developed [18, 26]

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to model high levels of uncertainty. In [21] type-3 FLS was designed for power control and battery charging planning for photovoltaic battery hybrid systems. The system has shown superior performance under different conditions in contrast to other types of systems. In [7] interval type-3 FLS was used for autonomous mobile robot control. The Bee colony optimisation search is employed to find the parameters of the controller. The paper [3] provided an approach to the design of interval type-3 FLS using α planes. The developed system is employed to solve some benchmarking problems. In [13] integration of interval T3FLS and neural networks was used for multiple time series prediction. The system is tested using the Dow Jones Index and COVID-19 data sets. In [20] the authors used interval T3FLS in sound speaker control. In [2], a robust type-3 fuzzy model predictive control is employed for regulating the output voltage of the dc/dc interface system. T2FL is integrated with the neural networks to mitigate the destructive effect of constant power load. The paper [22] presents a flow-meter fault detection approach using non-singleton T3FLS. The parameters of the membership functions were trained using an unscented Kalman filter and the parameters of the rules were adjusted using a correntropy Kalman filter. In [24], T3FLS and unscented Kalman filter algorithm are employed for solving singular multi-pantograph differential equations. Experimental results are provided to show the suitability of the developed system. Using the Lyapunov approach the the system stability is analyzed. In [12], the basic differences between type-1, type-2 and type-3 fuzzy sets were analysed. Interval type-3 FLS is designed and implemented in Matlab for robotic control. The paper [10] presented interval type-3 FLS with uncertain type-2 non-singleton input numbers. The structure of the system is designed using α levels. A gradient method is applied for the training of the system. The system is used for predicting the surface temperature of a hot strip mill. In [8], interval type-3 FLS has been designed for the prediction of the air quality index. EKF algorithm is used for learning the system. In [15] the authors used an online fractional-order training algorithm for turning the parameters of consequent parts.

As shown, the type-3 fuzzy system is being widely used for solving different problems. In this paper, we are considering the development of type-3 FLS for the control of dynamic plants. The contributions of the paper include:

- Using type-3 fuzzy logic, the structure of the type-3 fuzzy controller is proposed.
- The inference mechanism of type-3 FLS is developed.
- Using type-3 fuzzy logic the controller's rule base is developed.
- The design stages type-3 fuzzy control system were presented. The proposed system is used for the control of nonlinear dynamic plants.
- The modelling of the designed system is carried out and transient response is derived. A comparison of the designed system with the type-1 FL-based system is provided to show the efficiency of using the proposed control structure.

The paper is organized as follows. Section 2 includes the theory and inference mechanism of type-3 FLS. The structure and design algorithm of type-3 FLS were presented. Section 3 presents the design of the type-3 fuzzy control system for dynamic plants. The simulation of the control system has been carried out. Section 4 gives the conclusions.

2 Type-3 fuzzy control

Since the type-3 FLS employs a three-dimensional membership function, it can effectively handle a higher level of uncertainty compared to the Type-2 fuzzy system. The membership function of the type-3 FLS in three-dimensional space is illustrated in Figure 1. The cut of cross-sections of the membership function (MF) is given to depict uncertainty levels (see Figure 1(a)). The type-3 fuzzy MFs are represented as $\Omega = (x, y(x), z(x, y), \mu_\Omega(x, y, z))$, where, $x \subset X, y \subset Y \in [0, 1], z \subset Z \in [0, 1]$. Here $\mu_\Omega(x, y, z)$ is the type-3 MF. The type-3 MFs are represented using Gaussians which are described by the mean c and width σ parameters. The uncertainties can be assigned to the center or width. In the paper, we are considering the case when uncertainties are assigned to the width.

In Figure 1(b), we used α slices to represent the third dimension. Each α slice is presented by four membership functions that are two interval type-2 fuzzy sets. These type-2 fuzzy sets represent lower and upper levels of uncertainties of membership functions. Here, the type-3 fuzzy sets are utilized for the construction of the control system of a dynamic plant.

The structure of the controller based on type-3 FLS is given in Figure 2. The system includes a rule base that includes Mamdani type type-3 fuzzy rules. Here, type-3 fuzzy sets are utilized to describe the values of input-output variables. These rules and type-3 fuzzy logic-based reasoning are employed for the design of the controller. Assume that fuzzy type-3 rules are given

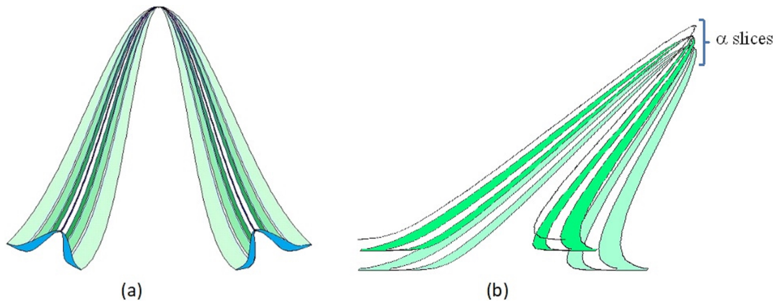

Figure 1: Type-3 MF. (a) Cross-sections of Type-3 MF, (b) α slices of MF (scaled for clear visibility).

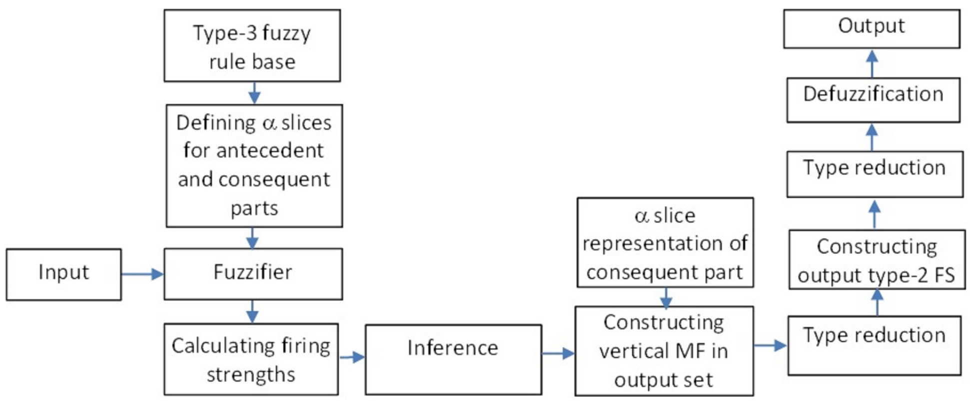

Figure 2: The structure of the system.

$$\text{If } \mathrm{x}_1 \text{ is } A_1 \text{ and } \cdots \text{ and } \mathrm{x}_m \text{ is } A_m \text{ Then } y \text{ is } B_m \tag{1}$$

Here A_i and B_j are type-3 fuzzy sets. After designing the rule base, the α slices are employed to present type-3 fuzzy membership functions using a set of interval type-2 fuzzy sets. The input signals enter into the fuzzification block and membership degrees are calculated. Using fuzzified signals the firing strength of each rule is computed. The inference block determines the recommendation of the antecedent part (see Figure 2). This recommendation is combined with the fuzzy values of the output variables of the rules. The rules' output fuzzy sets are also represented by α slices. Using the recommendation degrees of the antecedent part, the output implied fuzzy sets are derived and the vertical MF is constructed. The type reduction is applied in order to derive type-2 fuzzy sets. Afterwards, type-reduction and defuzzification are utilized in order to drive the output signal of the type-2 fuzzy system. The above-mentioned operations are carried out using the following steps.

Assume that, input signal x_k is entering the fuzzy rule-based system input. In the rule base, we used MFs with uncertain widths. The MFs of each horizontal α_i slice will be represented by the lower and upper bounds. For the input x_i lower and upper membership functions will be presented by the following formulas.

$$\underline{\mu}_{k,\underline{\alpha}^i}(x_j) = \exp\,(-\frac{(x_k - c_i)^2}{\underline{\sigma}^2_{\underline{\alpha}_i}});\;\; \overline{\mu}_{k,\underline{\alpha}^i}(x_j) = \exp\,(-\frac{(x_k - c_i)^2}{\overline{\sigma}^2_{\underline{\alpha}_i}}); \tag{2}$$

$$\underline{\mu}_{k,\overline{\alpha}^i}(x_j) = \exp\,(-\frac{(x_k - c_i)^2}{\underline{\sigma}^2_{\overline{\alpha}_i}});\;\; \overline{\mu}_{k,\overline{\alpha}^i}(x_j) = \exp\,(-\frac{(x_k - c_i)^2}{\overline{\sigma}^2_{\overline{\alpha}_i}}); \tag{3}$$

$\underline{\mu}_{k,\underline{\alpha}^i}(x_j)$, $\overline{\mu}_{k,\underline{\alpha}^i}(x_j)$ and $\underline{\mu}_{k,\overline{\alpha}^i}(x_j)$, $\overline{\mu}_{k,\overline{\alpha}^i}(x_j)$ are lower and upper membership functions for each input x_k obtained from α_i horizontal slice. c_i is the center, $\underline{\sigma}^2_{\underline{\alpha}_i}$, $\overline{\sigma}^2_{\underline{\alpha}_i}$ and $\underline{\sigma}^2_{\overline{\alpha}_i}$, $\overline{\sigma}^2_{\overline{\alpha}_i}$ are widths of lower and upper MFs of α_i slice.

In the rule base, each linguistic term is represented using three-dimensional membership functions. The formulas (2) and (3) are used to represent these membership functions. The inference is based on max-min composition using three-dimensional membership functions represented by means of horizontal α slices. As shown in Figure 1, using α

slices, the three-dimensional MF is represented with interval type-2 membership functions. For simplicity, we used three α slices to describe the type-3 fuzzy membership function. For each α slice, we used the min operation to find the firing strength of each rule. For each lower and upper side of the slice, the lower and upper rule firings are determined using the t-norm *min* operation

$$\underline{\mu}_{k,\underline{\alpha}^i}(x^1) = min(\underline{\mu}_{k,\underline{\alpha}^i}(x_1^1), \underline{\mu}_{k,\underline{\alpha}^i}(x_2^1), \ldots, \underline{\mu}_{k,\underline{\alpha}^i}(x_m^1)); \tag{4}$$

$$\underline{\mu}_{k,\overline{\alpha}^i}(x^2) = min(\underline{\mu}_{k,\overline{\alpha}^i}(x_1^2), \underline{\mu}_{k,\overline{\alpha}^i}(x_2^2), \ldots, \underline{\mu}_{k,\overline{\alpha}^i}(x_m^2)); \tag{5}$$

$$\overline{\mu}_{k,\underline{\alpha}^i}(x^3) = min(\overline{\mu}_{k,\underline{\alpha}^i}(x_1^3), \overline{\mu}_{k,\underline{\alpha}^i}(x_2^3), \ldots, \overline{\mu}_{k,\underline{\alpha}^i}(x_m^3)); \tag{6}$$

$$\overline{\mu}_{k,\overline{\alpha}^i}(x^3) = min(\overline{\mu}_{k,\overline{\alpha}^i}(x_1^3), \overline{\mu}_{k,\overline{\alpha}^i}(x_2^3), \ldots, \overline{\mu}_{k,\overline{\alpha}^i}(x_m^3)); \tag{7}$$

After deriving the firing strange of the rules, the type reduction in output type-3 fuzzy set is performed. For this aim using α slices, we derived upper and lower MFs of output vertical fuzzy sets (Figure 3).

$$\underline{\phi}(\mu_i) = \begin{cases} \exp(-\frac{(\overline{\mu}_{k,\underline{\alpha}^i}-c)^2}{\overline{\zeta}^2_{k,\underline{\alpha}_i}}), \text{ if } \overline{\mu}_{k,\underline{\alpha}^i} < c, \\ \exp(-\frac{(\underline{\mu}_{k,\overline{\alpha}^i}-c)^2}{\underline{\zeta}^2_{k,\overline{\alpha}_i}}), \text{ if } \underline{\mu}_{k,\overline{\alpha}^i} \geq c; \end{cases} \tag{8}$$

$$\overline{\phi}(\mu_i) = \begin{cases} \exp(-\frac{(\underline{\mu}_{k,\underline{\alpha}^i}-c)^2}{\overline{\zeta}^2_{k,\underline{\alpha}_i}}), \text{ if } \underline{\mu}_{k,\underline{\alpha}^i} < c, \\ \exp(-\frac{(\overline{\mu}_{k,\overline{\alpha}^i}-c)^2}{\overline{\zeta}^2_{k,\overline{\alpha}_i}}), \text{ if } \overline{\mu}_{k,\overline{\alpha}^i} \geq c; \end{cases} \tag{9}$$

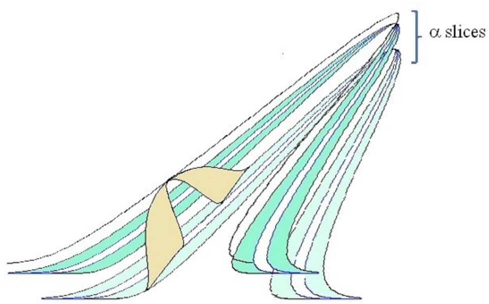

Figure 3: Vertical MF obtained from vertical cross-section.

The received membership functions are scaled in interval $[0, 1]$. As shown vertical MF is also represented by the lower and upper bounds. After obtaining vertical membership functions on the output fuzzy sets, type-reduction is applied to drive type-2 fuzzy sets (see Fig.3). For this purpose, using upper and lower membership functions, we applied the type-reduction for transforming type-3 fuzzy sets to type-2 fuzzy sets. For this purpose, the Karnik-Mendel (K-M) algorithm is employed

$$c_r = \frac{\sum_{i=1}^{R} x_i \underline{\phi}_{\overline{A}}(x_i) + \sum_{i=R+1}^{N} x_i \overline{\phi}_{\overline{A}}(x_i)}{\sum_{i=1}^{R} \underline{\phi}_{\overline{A}}(x_i) + \sum_{i=R+1}^{N} \overline{\phi}_{\overline{A}}(x_i)}; \;\; c_l = \frac{\sum_{i=1}^{L} x_i \overline{\phi}_{\underline{A}}(x_i) + \sum_{i=L+1}^{N} x_i \underline{\phi}_{\underline{A}}(x_i)}{\sum_{i=1}^{L} \overline{\phi}_{\underline{A}}(x_i) + \sum_{i=L+1}^{N} \underline{\phi}_{\underline{A}}(x_i)}; \tag{10}$$

As a result, interval type-2 fuzzy sets will be derived on two-dimensional space (surface). Here L and R are the switch points which are determined by iterative procedure [3]. x_i is corresponding to the α slices of μ membership functions. We used α slices to implement type reduction operation for the vertical membership function. Here c_l and c_r will represent the left and right sides of vertical MF. Using c_l and c_r we can construct the lower and upper sides of the type-2 horizontal MF. The c_l ve c_r values also represent the defuzzified value of the firing strength of the antecedent part of the rules. The aggregation of interval type-2 sets of the outputs of the active rules is performed. In the next step, type reduction and defuzification are employed to calculate the output value of the system. The K-M algorithm is applied for type reduction of the type-2 fuzzy sets. After type reduction, defuzzification is applied to calculate the controller's output signal. The implementation stages of the type-3 fuzzy controller are given as follows.

1. Use input signals error, change-in-error, output control signal and Table 1 to design a fuzzy rule base for the controller to control the dynamic system.

2. Enter plant coefficients, set-point signal, the current value of the plant output and sampling time. Set the number of α_i slices.

3. Perform α_i slice for each term set of variables of antecedent and consequent parts of the rule base and save them in memory. Here i is the number of slices set by the programmer. Using the centers of fuzzy sets, generate the width of Gaussian membership functions. For each MF, one center and four widths are generated. Initially, this is done for α=0 level. By decreasing width, we generated other $\alpha_i \in [0, 1]$ levels. In this way, each linguistic term in the rule base is represented with type-3 fuzzy sets.

4. Use the set-point signal and current plant output to calculate error e and change-in-error Δe signals.

5. Use input signals error e and change-in-error Δe and fuzzy rule base do determine active rules.

6. Use current inputs to calculate the membership functions $\underline{\mu}_{k,\underline{\alpha}^i}(x_j)$, $\underline{\mu}_{k,\overline{\alpha}^i}(x_j)$ and $\overline{\mu}_{k,\underline{\alpha}^i}(x_j)$, $\overline{\mu}_{k,\overline{\alpha}^i}(x_j)$ for each α_i using the formulas (2) and (3). These values will represent the membership degrees of the input signals to a given term set. The MFs are calculated for each α_i slice (see Figure 1). Here k is the number of MFs, and i is the number of slices.

7. Using T norm min operation, calculate the firing strengths of each rule for each α_i slice. Formulas $(4-7)$ are employed for this purpose. For each α_i slice, four firing strengths are determined using the lower and upper sides of α_i slices.

8. Find intersection points of MFs $\underline{\mu}_{k,\underline{\alpha}^i}(x_j)$, $\underline{\mu}_{k,\overline{\alpha}^i}(x_j)$ and $\overline{\mu}_{k,\underline{\alpha}^i}(x_j)$, $\overline{\mu}_{k,\overline{\alpha}^i}(x_j)$ determined in step (7) with the MFs of the consequent parts of the rules for all α_i slices (see Figure 3).

9. The lower side of the vertical membership function is obtained by connecting the intersection points of the $\underline{\mu}_{k,\overline{\alpha}^i}(x_j)$ and $\overline{\mu}_{k,\underline{\alpha}^i}(x_j)$ with the corresponding α layers of the membership function of the fuzzy output value of the consequent part. The upper value of the vertical membership function is obtained by connecting the intersection points of the $\underline{\mu}_{k,\underline{\alpha}^i}(x_j)$ and $\overline{\mu}_{k,\overline{\alpha}^i}(x_j)$ with the corresponding α layers of the membership function of the fuzzy output value of the consequent part.

10. Calculate the centers of the lower and upper sides of the vertical membership function. Using these centers, firing strengths (membership functions in step (8)) and formulas (8) construct the lower and upper sides of the vertical membership function.

11. Apply type reduction formula (10) to the vertical membership function. During this operation, the c_l and c_r centers will be determined. Use these c_l and c_r values and the center of the type-3 membership function to determine the widths of the implied type-2 fuzzy sets.

12. Performs aggregation of type-2 fuzzy sets for active rules to determine the output of the fuzzy system.

13. Implement type-reduction and defuzzification to find the output of type-3 FLS.

3 Simulation

In industry, some dynamic plants are characterized by parametric or structural uncertainties. These plants are susceptible to external and internal disturbances. The existing deterministic models could not handle the uncertainties that existed in these processes and therefore they can not provide the required control performance. To solve such kinds of problems, one valuable alternative is the use of a type-3 fuzzy system. Type-3 fuzzy systems use three-dimensional membership functions, allowing them to describe uncertainties more effectively. In this section, we applied type-3 fuzzy sets for the dynamic plant's control. The control system's structure is given in Figure 4. Based on the desired (g) and current value of plant output (y) signals, the values of the error (e) and change of error (Δe) are calculated. Type-3 fuzzy controller uses these e and Δe signals to find the control signal u that enters the input of the plant. To measure the control system's performance we utilized root mean square error (RMSE). As an example, the controls of two different nonlinear dynamic plants are considered. The first one is presented by the following difference equation [23].

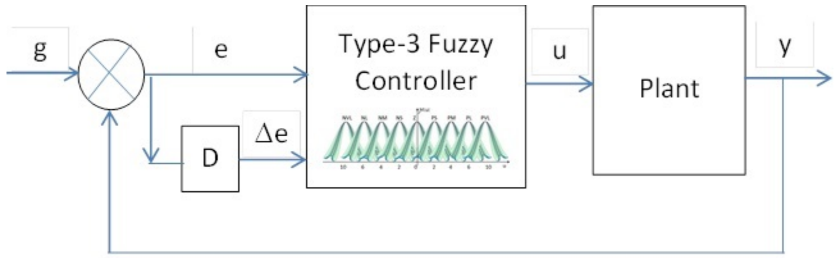

Figure 4: Type-3 fuzzy control system

$$y(k) = \frac{y(k-1)y(k-2)(y(k-1)+2.5)}{1+y(k-1)^2+y(k-2)^2} + u(k) \tag{11}$$

The reference signal used in the system is as follows

$$g(k) = \begin{cases} 10,\ 0 < k \le 25, \\ 15,\ 25 < k \le 50, \\ 10,\ 50 < k \le 75, \\ 15,\ 75 < k \le 100 \end{cases} \tag{12}$$

The rule base, describing the relationships between input signals (error and change-in-error) and the output control signal, is designed using human expert knowledge. The rule base consists of Mamdani-type rules of IF-Then form. The fuzzy rules present the associations between input variables "error" e and "change in error" Δe and output "control signal" u. The linguistic values are utilized to represent input-output variables in the rule base. The linguistic values of e and Δe were presented in the first column and first row of the table. The linguistic values inside the table describe the output control signal. Table 1 depicts the tabular representation of the If-Then fuzzy rule base of the controller. In this study, we used nine linguistic values. In the table, NVL, NL, NM and NS are "negative very large", "negative large", "negative medium" and "negative small", respectively. PVL, PL, PM and PS are "positive very large", "positive large", "positive medium" and "positive small", respectively. Z is "zero". The table shows how to control the plant using the error and change-in-error. The fragment of the knowledge base that represents control actions using if-then rules for the fuzzy controller is given below

Table 1. Rule base

		Δe								
		NVL	NL	NM	NS	Z	PS	PM	PL	PVL
	NVL	PVL	PVL	PVL	PVL	PL	PL	PM	PS	Z
	NL	PVL	PVL	PVL	PL	PL	PM	PS	Z	NS
	NM	PVL	PVL	PL	PM	PM	PS	Z	NS	NM
	NS	PVL	PL	PM	PM	PS	Z	NS	NM	NL
e	Z	PL	PM	PM	PS	Z	NS	NM	NL	NL
	PS	PM	PM	PS	Z	NS	NM	NL	NL	NVL
	PM	PM	PS	Z	NS	NM	NL	NL	NVL	NVL
	PL	PS	Z	NS	NM	NL	NL	NVL	NVL	NVL
	PVL	Z	NS	NM	NL	NL	NVL	NVL	NVL	NVL

If error is NL and change-in-error is PS Then control is PM
If error is PS and change-in-error is NM Then control is PS
... (13)
If error is PL and change-in-error is NM Then control is NS

The accurate evaluation of the associations between input and output variables presented by IF-Then rules affects the performance of the controller. Each input and output variable in the rule base was described by nine linguistic terms. Taking the number of linguistic values and the number of variables into account eighty-one rules will be used by the controller. The linguistic values of input-output variables used in IF-Then rules are presented by three-dimensional membership functions. In the paper, we used Gaussians to present linguistic values. The centers of these Gaussians are distributed in the interval $[-10, 10]$. Depending on the specific control example, we can scale this range $[-10, 10]$ to the desired range. The widths of Gaussians are determined using these centers. For each membership function, one center and four widths are generated. Initially, this is done for the $\alpha = 0$ level. By decreasing the width, we generate other $\alpha = [0, 1]$ levels. In this way, each linguistic term is represented with type-3 fuzzy sets in the rule base. Figure 5 presents linguistic values of input variables error and change-in-error and Figure 6 presents linguistic values of controller output signals. As shown in Figure 5 and Figure 6 the linguistic values are presented by three-dimensional Gaussian membership functions.

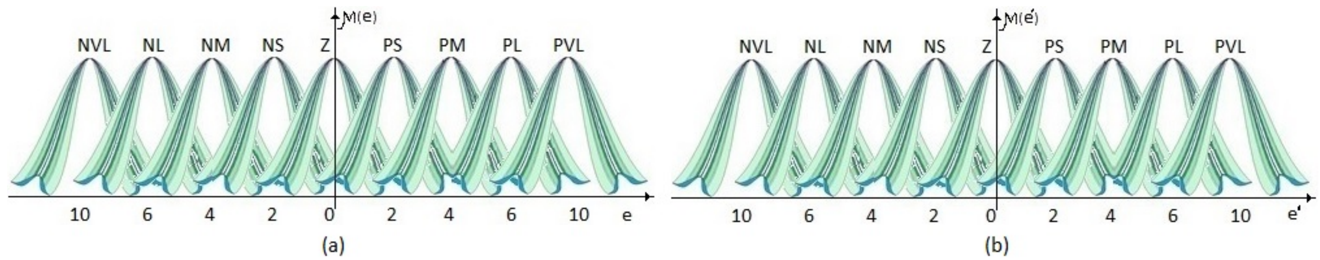

Figure 5: MFs of error (a) and change-in-error (b).

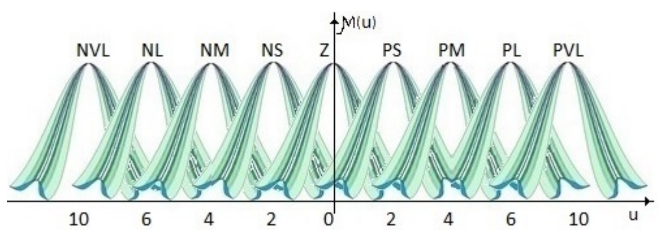

Figure 6: MFs of control signal.

The appropriate value of the control signal is determined using the current input values of error and change-in-error. Based on the type-3 fuzzy rule base and inference mechanism given in section 2, the corresponding control signal is calculated and sent to the plant. The T3FCS is simulated using Matlab. Figure 7 depicts the transient response characteristic of the T3FCS for the plant given in (11). The excitation signal presented in (12) is applied to the control system input. On the output of the system, the error between the current and desired output signals is determined. This error signal is used to calculate MSE(mean-square of errors) and RMSE (root-mean-square of errors) in order to estimate the performance of the control system. The values of MSE and RMSE were obtained as 0.185874 and 0.431131, respectively. The results obtained were averaged values of twenty simulations. The performance of the type-3 fuzzy controller is compared with the performance of the type-1 fuzzy control system using the same plant. MSE and RMSE for the type-1 fuzzy control system were obtained as 0.310784 and 0.557480, respectively, As shown the type-3 fuzzy controller has shown better performance than the other one.

In the next example, we consider the control of an inverted pendulum on a cart using type-3 FLS. The pendulum is hinged on the centre of the cart and can move forward and back parallel to the rail. The cart can also move forward and back on the rail in order to keep the pendulum in an upright position (Figure 8). The problem is controlling the angle θ of the pendulum from the upright position and maintaining its value at zero [19, 1]. The dynamic equation of the angle θ of the inverted pendulum can be expressed as

$$\ddot{\theta}(t) = \frac{g \cdot \sin\theta(t) - 0.5 \cdot P(t)\dot{\theta}^2(t)\sin(2\theta(t)) - a(t)\cos(\theta(t))u(t)}{\frac{4}{3}l - p(t)\cos^2(\theta(t))} \tag{14}$$

Here $g = 9.8m/s^2$ is the gravity acceleration, l is the half length of the pendulum (in meters), $\theta(t)$, $\dot{\theta}(t)$ and $\ddot{\theta}(t)$ are the angle, the velocity of the angle and the acceleration of the angle, $u(t)$ is the force (in the unit Newton) applied to the cart. Here

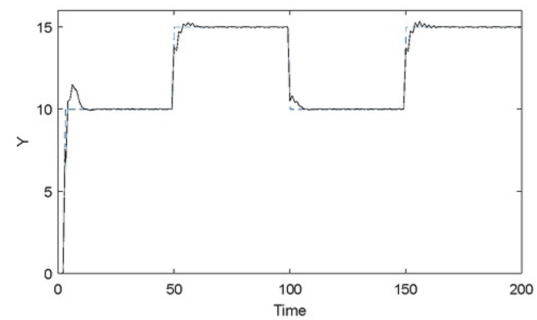

Figure 7: Transient response characteristic.

$$P(t) = a(t)(m + \Delta m(t)); \quad a(t) = \frac{1}{m + \Delta m(t) + M}; \quad \Delta m(t) = \begin{cases} 5 \cdot t \geq 2, \\ 0, otherwise. \end{cases} \tag{15}$$

where m and M are the mass of the pendulum and cart, respectively. $\Delta m(t)$ is the mass uncertainty. For a given plant $m = 2$ kg, $M = 8$ kg, $l = 2$m.

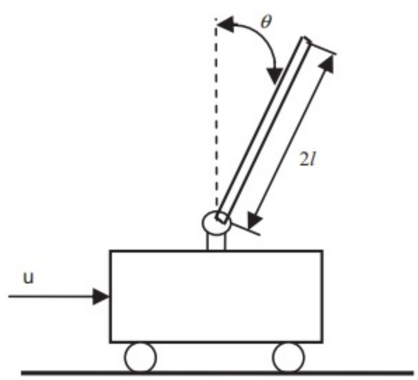

Figure 8: Inverted Pendulum on a cart.

To control the inverted pendulum the type-3 fuzzy rule base of the controller is constructed using Table 1. The fuzzy terms used in the rule base are presented in Fig. 5 and Fig.6. In T3FCS the fuzzy sets are initialised using Gaussinas. The centers and widths of the Gaussians for error e, change of error Δe and control u are generated as (-10,5), (5,3), (-2,1), (-1,1), (0,0.5), (1,1), (2,1), (5,3), (10,5). In the parenthesis, the first value is the center, and the second value is the width of Gaussians. For the given problem, we scaled the range of the input and output signals of the rule base to $[-1, 1]$. The output signal of the controller, after being multiplied by the coefficient ($K = 220$), is entered into the plant input. The stages given at the end of previous section 3 are utilized for simulating the control system. Initially, the simulation of the control system was carried out without any disturbance. The set-point signal for the pendulum is set to zero (0) and the initial value of the angle is 0.3 radians. Figure 9 depicts the response characteristics of control systems. Here solid line depicts the response characteristics of the control system based on T3FCS. For comparative analysis, a similar simulation was performed using the type-1 fuzzy controller. The dashed line depicts the response characteristics of the control system of the inverted pendulum using T1FCS. The angular velocity and the phase plane trajectory are presented in Figure 10 and Figure 11, respectively

We measured the performance of the control system using MSE and RMSE. The MSE and RMSE for T3FCS were obtained as 0.002126 and 0.046112, respectively (Table 2). For the T1FCS, the MSE and RMSE were obtained as 0.009082 and 0.095298, respectively. In the second simulation, mass disturbances are added to the inverted pendulum model. The same T3FCS is employed to control the inverted pendulum plant with mass uncertainties. The control system efficiently handled the uncertainty and controlled the pendulum to the desired position. Figure 12 depicts the response characteristics of the control system using T3FCS (solid line) and T1FCS (dashed line). The MSE and RMSE

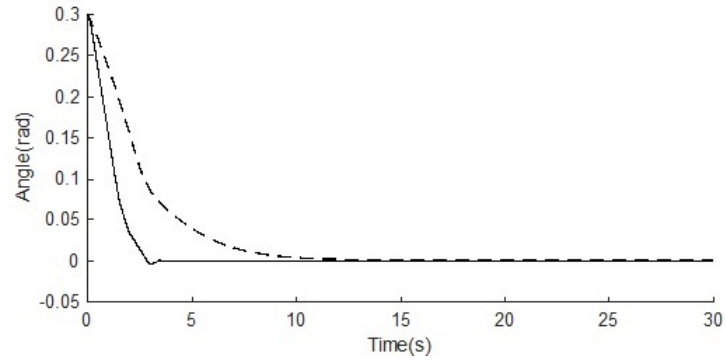

Figure 9: Response characteristics of the control system. T3FCS- solid line, T1FCS- dashed line.

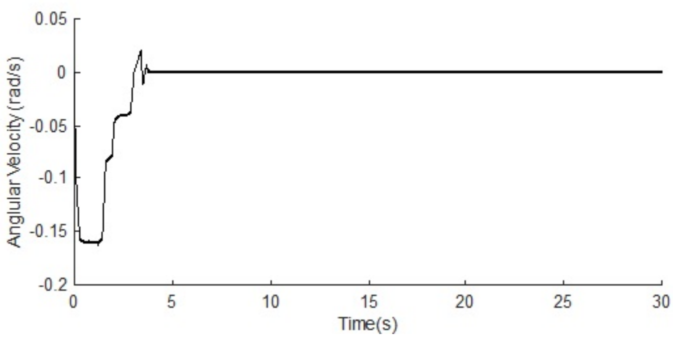

Figure 10: The plot of angular velocity.

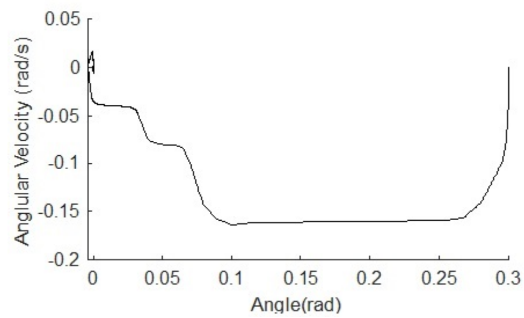

Figure 11: The phase plane trajectory.

results for T3FCS are nearly the same as in the normal case (without uncertainties) with small differences. The values of MSE and RMSE were 0.002127 and 0.046115, respectively. In this simulation, due to the small differences in the performances of T3FCS, the plots of angular velocity and the phase plane trajectory are similar to the normal case (without uncertainties) given in previous simulations, as depicted in Figure 10 and Figure 11.

Table 2. Performances of the control systems

	Normal Case		Mass uncertainties		External disturbances	
	MSE	RMSE	MSE	RMSE	MSE	RMSE
T1FCS	0.009082	0.095298	0.010344	0.10703	0.007943	0.089122
T3FCS	P0.002126	0.046112	0.002127	0.046115	0.004412	0.066425

The obtained results demonstrate that T3FCS can efficiently handle uncertainties that exist in model dynamics. For comparative analysis, the same simulations were performed using the T1FCS. The values of MSE and RMSE for the control of the inverted pendulum with mass uncertainty were obtained as 0.010344 and 0.101703, respectively. Table 2 depicts the performances of the control systems with and without mass uncertainties. In the next simulations, external disturbances are added to the input of the inverted pendulum. The same T3FCS is employed to control the inverted pendulum on a cart when external disturbances are added to the plant input (to the control signal u). The following external disturbance signal is used.

$$d(t) = \begin{cases} 150, t \in [2,3], \\ -150, t \in [5,6], \\ 150, t \in [7,8] \\ 0, otherwise. \end{cases} \tag{16}$$

The T3FCS employed a rule base given in Table 1 for the control of the inverted pendulum. Figure 13 depicts response characteristics obtained with T3FCS. As shown T3FCS can lead the pendulum to the set-point position. As shown, despite the varying values of external disturbances, T3FCS can guide the inverted pendulum to its desired

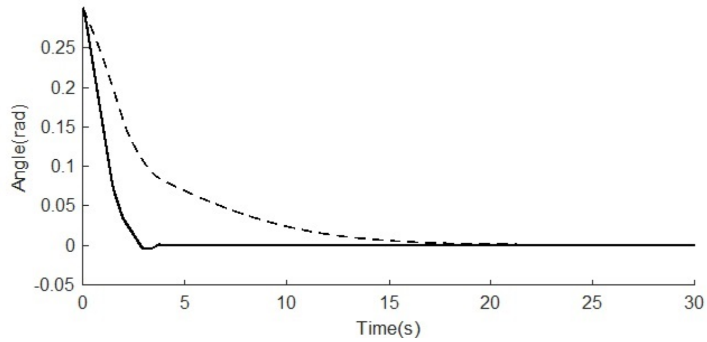

Figure 12: Response characteristics of the control system with mass uncertainty. T3FCS- solid line, T1FCS- dashed line.

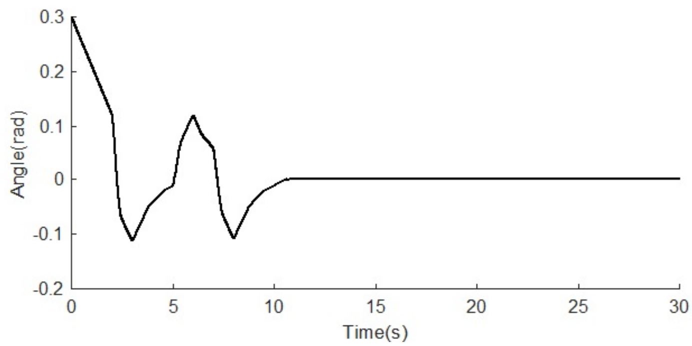

Figure 13: Response characteristics of the control system when external disturbances are added to the system.

state position after some time. The performances of the T3FCS and T1FCS for inverted pendulum when external disturbances are added to the system are presented in Table 2.

Type-1 and type-3 FLSs are applied to control nonlinear plants. The knowledge base of type-3 FLS describes the input-output relationship of the controller, constructed using type-3 fuzzy membership functions. As seen from the simulation results, T3FCS has demonstrated better performance than T1FCS in controlling the inverted pendulum on the cart, both in the normal case (without uncertainties) and when the control system has uncertainties. T3FCS efficiently handle uncertainties that exist in the dynamics of the plant. T1FCS can also handle uncertainties related to mass and external disturbances. However, there are noticeable differences in the performances of the T1FCSs when controlling plants with and without uncertainties. T3FCS has shown better performance than the type-1 controller in controlling the inverted pendulum in both cases. However, the main disadvantage of the type-3 FLS is its runtime complexity. Because the fuzzy rule base of the T3FCS includes three-dimensional membership functions, this control system performs more calculations in order to determine the control output. In the PC with I7 and 32RAM, the time required for the control of the inverted pendulum for 6000 iterations with T3FCS was obtained as 93.21s, while with T1FCS the run time was obtained as 0.0156s. Nevertheless, the use of three-dimensional fuzzy sets in the rule base allows us to describe the deep uncertainties and provide a more adequate description of the considered problem. By employing a type-3 fuzzy system, we can improve the performance of the system. The obtained simulation results have demonstrated the advantages of T3FCS over its type-1 counterpart. The obtained results proved the suitability of using type-3 fuzzy logic in the control of dynamic plants characterized by uncertainties.

4 Conclusions

A type-3 FLS-based controller is designed for the control of dynamic plants characterized by uncertainties. Using type-3 membership functions and expert knowledge the controller's rule base is developed. Here, the type-3 membership functions are employed to describe the linguistic values of input-output variables. The design stages of the controller are presented and the inference mechanism of type-3 FLS is developed. The inference mechanism is based on interval type-3 fuzzy logic. The proposed typ-3 FLS algorithm is applied for the control of nonlinear dynamic plants. The simulation of the type-3 FLS-based control system is performed using Matlab programming. The transient characteristic of a control system is derived using different values of the excitation signal. The proposed T3FCS was tested on an inverted pendulum on a cart using three simulation tasks. The first task was conducted without any internal and external disturbances, the second task tested with mass uncertainties, and the third task tested with external disturbances. In the first two cases, T3FCS demonstrated very good response characteristics. In the third case, T3FCS guided the plant to the set point position after a few oscillations. Simulation results of T2FCS were compared with those of T1FCS in all simulations, with the former showing better performance. The comparative results indicate the efficiency of type-3 FLS in control of the dynamic system characterized by uncertainties.

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