

A novel extended approach to evaluate criteria weights in MADM problems in fuzzy framework

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Abstract

Determining weights of criteria is a pivotal challenge that arises in Multi-Attribute Decision Making (MADM) problems. Different methods have been suggested in the literature which can be classified into three main categories: subjective, objective and integrated. Especially when the decision maker does not have a specific judgment regarding the weights of the criteria of the decision problem or the number of criteria is large, the methods based on pairwise comparisons are not effective due to the large number of required judgments as well as the natural increase in inconsistency in the judgments. In this paper, we propose an integrated model to determine criteria weights in MADM problems while combining the Ordered Weighted Average (OWA) Yager, entropy, fuzzy/crip initial decision maker's judgments about the preferences of alternatives, and the information of decision matrix. In this regard, by considering a decision matrix, we formulate the idea as an optimization problem including an extended TOPSIS, L-p metric and $\widetilde{L-p}$ metric, or goal programming model in order to provide an extension of OWA operator and entropy method, simultaneously. Then, we use the proposed method in a real-world dataset to evaluate the priorities of mining opportunities in 17 provinces of Iran (including 235 alternatives by considering 48 criteria). In this regard, a comprehensive list of economic, political, social, strategic and environmental criteria has been used. A full analysis is performed to illustrate the application of the technique that stems from our approach. Finally, we compare the results that we obtain with the results from existing approaches, including Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Simple Additive Weighting (SAW) method and Yager. In this way the accuracy and effectiveness of the presented work is conclusively validated.

Keywords: OWA operators, fuzzy preferences, constrained OWA optimization, L-p metric and goal programming, multi-attribute decision making (MADM).

1 Introduction

In this paper, the problem of evaluating the weights of criteria in MADM problems is investigated in the form of an optimization problem, in which a combination of $l-p$ -metric and $\widetilde{l-p}$ -metric methods with generalized TOPSIS multi-criteria decision-making approaches is used. In the research literature, several approaches for evaluating the weights of criteria have been offered, including pairwise comparison, Stepwise Weight Assessment Ratio Analysis (SWARA) method, Best Worst Method (BWM), Yager (OWA), and etc. One of the drawbacks of these approaches is that the weights of the criteria are determined without considering the data and the details of the decision matrix. This will in fact lead to weights that are not only consistent with the decision maker's preferences but also more consistent with the state of the alternatives.

The approach in this paper is applied to a case study in the real world. As we know, the minerals play a key role in the human life and they are present in almost everything that is used by humans. This key role is increasingly being strengthened in the real world, in such a way that all medical and pharmaceutical equipment, electronic equipment,

clean energy, military weapons, batteries, space equipment, etc. are highly dependent on minerals. Therefore, the mining sector is one of the basic areas of the dynamic global economy, and therefore, investing and researching in this area leads to economic dynamism in other economic sectors. Given that large-scale projects in the mining and mineral industries are very costly, any incorrect prioritization in their implementation will result in significant financial waste. Additionally, in studies in this field, there are always numerous fuzzy uncertainties related to estimations and future predictions. In fact, the analysis of such issues is exposed to the concept of risk. These conditions led to the OWA Yager decision-making approach. Therefore, prioritization related to these fuzzy uncertainties should be carried out using models that have the capability to process and consider all of these uncertainties.

In this real study, the mining opportunities of 17 provinces of Iran (including 235 alternatives) are ranked based on 48 criteria on a comprehensive list of economic, political, social, strategic and environmental criteria during a 3-year time span from 2019 to 2022. On the other hand, we know that mining projects usually require huge investments and any wrong prioritization in this field will cause a lot of damage, especially in the environment. The high sensitivity and seriousness of this issue requires maximum accuracy and attention in determining the decision-making criteria in the ranking of mining activities. A very important point that is the main motivation for this research is the fact that the criteria of a decision problem do not have the same level of importance in different situations. For example, here, in the ranking of mining opportunities, the employment index in a province with a high unemployment rate is more important than the same index in a province with a low unemployment rate. As another example, in a province that is adjacent to the sea, the criterion of water access is less important than the same criterion in a desert province. Meanwhile, in Yager's model, the only factors for determining the weights of the criteria are the number of criteria, and the risk-level of the decision maker, while the status of alternatives is ignored in this model's evaluations. In this article, this deficiency is investigated, addressed and resolved.

The remaining sections are arranged as follows: In Section 2 some literature review is provided. MODM is considered in Section 4. In Section 5, our proposed approach of derivations of the OWA operator weights in the context of MODM are presented. In Section 6, we analyze real data collected in a case study in the Iranian Ministry of Industry, Mine and Trade during the years 2019 to 2022. Finally, the concluding remarks are presented in Section 7.

2 Literature review

The fuzzy-preference ranking organization method for the enrichment of evaluations constitutes a family of outranking Multiple-Attribute Decision-Making (MADM) methods that has been adopted by researchers from many areas during the last years [7, 49]. The OWA operator has been used to structure the optimization problem to derive the optimal weight values of the criteria in an MADM problem [65, 66]. Therefore, several researches have investigated the issue of obtaining the weights associated with the OWA operator. To do so, one of the main concerns is how to generate OWA operator weights and, as a result, numerous weight generation methods have been advanced, for instance, Ahn [1, 2] proposed approaches to solve the constrained OWA aggregation problem. García-Lapresta et al. [23] generate OWA weights from individual assessments. Beliakov [6] proposed a method that introduced weights into OWA Operators and other symmetric functions. Miguel et al. [18] proposed an algorithm for group decision making using n -dimensional fuzzy sets, admissible orders and OWA operators. Zarghami and Szidarovszky [72] revised the OWA operator for multi criteria decision making problems under uncertainty. Wang and Mendel [60] investigated the group decision making problems in which all the linguistic information provided by the decision makers was expressed as interval type-2 fuzzy decision matrices where each of the elements was characterized by interval type-2 fuzzy set, and the information about attribute weights was completely unknown. Dai [16] introduced the complex fuzzy ordered weighted distance measure that used the main characteristics of the OWA operator and uncertain information represented as complex fuzzy values. Zhou and Chen [75] investigated multiple criteria group decision analysis using a Pythagorean fuzzy programming model for the multidimensional analysis of preference based on novel distance measures. Arya and Yadav [5] proposed a new approach to rank the decision making units in presence of infeasibility in intuitionistic fuzzy environment. Akram and Shumaiza [3] used q-rung orthopair fuzzy numbers to render the ratings of alternatives, which allowed them to express uncertain and vague information. Liu et al. [42] investigated the ranking range comparisons for the selected seven popular MADM approaches while the attribute weights were manipulated. He et al. [32] proposed an induced OWA operator for group decision making which dealt with extended comparative linguistic expressions with symbolic translation. Ji et al. [35] developed an induced ordered weighted averaging operator for expert opinions aggregation. Chachi et al. [11] investigated a multi-attribute assessment of fuzzy regression models and proposed a new approach to estimate and to obtain fuzzy regression model. Kazemifard and Chachi [37] employed MADM techniques to analyse the performance of fuzzy regression models. Chaji [12] investigated an analytic approach to maximum Bayesian entropy of OWA operators (see also [25, 68, 69]). Chaji et al. [13] selected a model for generating OWA operator weights in

MAGDM problems by maximum entropy membership function. Chachi and Chaji [10] proposed and investigated an OWA-based approach to quantile fuzzy regression. D'Urso and Chachi [20] investigated OWA fuzzy regression. In the context of MADM, Kazemifard [36] proposed an extension of TOPSIS model based on monotonic utility of criteria (see also [14]). D'Urso et al. [21] proposed an OWA-based multi-criteria decision making based on fuzzy methods.

Žižović and Pamučar [77] presented a subjective model for determining weight coefficients in multi-criteria decision-making models, called Level Based Weight Assessment (LBWA), which enabled the involvement of experts from different fields with the purpose of defining the relations between criteria and providing rational decision making. Pamučar et al. [46] proposed a multi-criteria problem solving method, known as FULL Consistency Method (FUCOM), which imply the definition of two groups of constraints that need to satisfy the optimal values of weight coefficients. Ilieva [34] extended FUCOM to solve a fuzzy optimization problem in order to obtain the fuzzy weights of criteria. Hatefi [26] proposed a new method called Indifference Threshold-based Attribute Ratio Analysis (ITARA) to assign weights to the attributes in MADM problems. Hatefi [27] presented a trade-off method, entitled Block-wise Rating the Attribute Weights (BRAW). Hatefi [28] developed a revised version of the Rank Order Centroid (ROC) method called Improved ROC (IROC) method which deals with criteria weight approximation in Multiple Criteria Decision Making (MCDM) problems. In competition with the ROC method, Hatefi [30] developed another method, called Rank Order Logarithm (ROL), that is based on a justifiable and well-founded concept. Hatefi et al. [31] established a set of reasonable and well-founded attributes to gauge different features of the Surrogate Weighting Method (SWM) which is a type of the techniques to determine quantitative weights of the criteria in decision making. Ginevičius [24] proposed a method of determining the criteria weights, called FARE (Factor Relationship) method, based on the relationships between all the criteria describing the phenomenon under consideration. Chakraborty et al. [15] investigated Simultaneous Evaluation of Criteria and Alternatives (SECA) method to deal with evaluating a multi-objective non-linear mathematical model to determine the overall performance scores of the alternatives and the weights of the criteria simultaneously. Krylovas et al. [38] proposed Kemeny Median Indicator Ranks Accordance (KEMIRA) method for determining criteria priority and weights in solving MCDM problem. Soltanifar et al. [58] and Soltanifar [57] provided an improved version of KEMIRA, by utilizing the concept of preferential voting. Diakoulaki et al. [19] proposed the CRITIC method for the determination of objective weights in MCDM problems. Lotfi et al. [33] introduced the fuzzy extension of the CRITIC method. Zavadskas and Podvezko [74] combined the best features of the entropy method and the Criterion Impact Loss (CILOS) approach to obtain a new method – Integrated Determination of Objective CRITeria Weights (IDOCRIW). Podvezko et al. [48] extended an objective weight assessment methods CILOS and IDOCRIW to fuzzy MCDM. Naz et al. [44] employed the CILOS with Weighted Aggregated Sum Product Assessment (WASPAS) for evaluating cosmetic brands in a group decision-making environment. Rani et al. [51] proposed a weight-determining method to derive attributes' weights based on the standard deviation-based method with Pythagorean fuzzy information. Yang et al. [67] proposed a three-stage hybrid approach for weight assignment in MADM problems. Roselli et al. [54] used the success-based decision rule to support the holistic evaluation process in Flexible and Interactive Tradeoff (FITradeoff) method. Almeida et al. [4] presented a broad overview of main contributions related to the FITradeoff method. For a quick access to the relevant literature, better capabilities to address suitable CW methods in real-world problems, and better communication among the MADM/CW researchers read the paper by Hatefi [29].

3 Fuzzy set theory: Preliminary concepts

A fuzzy set \tilde{A} on the universal set \mathbb{X} is described by its membership function $\tilde{A}(x) : \mathbb{X} \rightarrow [0, 1]$ [71]. Through this paper we consider $\mathbb{X} = \mathbb{R}$ (the real line). The α -cut of a fuzzy set \tilde{A} is defined as the crisp set $A_\alpha = \{x \in \mathbb{R} : \tilde{A}(x) \geq \alpha\}$, $\alpha \in (0, 1]$. For $\alpha = 0$, define A_0 as the closure of the set $\{x \in \mathbb{R} : \tilde{A}(x) > 0\}$. A fuzzy set \tilde{A} of \mathbb{R} is called a fuzzy number if A_α is a non-empty compact interval, for any $\alpha \in [0, 1]$. Such interval is represented by $A_\alpha = [A_\alpha^l, A_\alpha^r]$. A specific type of fuzzy number, which is rich and flexible enough to cover most of the applications, is the so-called LR -fuzzy number denoted by $\tilde{N} = (n, l, r)_{LR}$ with central value $n \in \mathbb{R}$, left and right spread values $l, r \in \mathbb{R}^+$, decreasing left and right shape functions $L, R : \mathbb{R}^+ \rightarrow [0, 1]$, with $L(0) = R(0) = 1$. Such LR -fuzzy numbers have the following membership function [76]

$$\tilde{N}(x) = \begin{cases} L\left(\frac{n-x}{l}\right) & \text{if } x \leq n, \\ R\left(\frac{x-n}{r}\right) & \text{if } x > n. \end{cases}$$

A special type of LR -fuzzy number is the so-called triangular fuzzy number, denoted by $\tilde{N} = (n, l, r)_T$. For $l = r$, the triangular fuzzy number \tilde{N} is called symmetric and is abbreviated by $\tilde{N} = (n, l)_T$. The membership function and the

α -cut of the triangular fuzzy number \tilde{N} are as follows

$$\begin{aligned}\tilde{N}(x) &= \frac{x - (n - l)}{l} \mathcal{I}_{[n-l, n]}(x) + \frac{(n + r) - x}{r} \mathcal{I}_{(n, n+r]}(x), \quad x \in \mathbb{R}, \\ N_\alpha &= [N_\alpha^l, N_\alpha^r] = [n - (1 - \alpha)l, n + (1 - \alpha)r], \quad \alpha \in [0, 1],\end{aligned}$$

where \mathcal{I}_A stands the characteristic function of a crisp set A [43].

Theorem 3.1. [76] *Let $\tilde{M} = (m, l_m, r_m)_{LR}$ and $\tilde{N} = (n, l_n, r_n)_{LR}$ be two LR fuzzy numbers, and λ be a real number. Then*

$$\begin{aligned}\lambda \otimes \tilde{M} &= \begin{cases} (\lambda m, \lambda l_m, \lambda r_m)_{LR} & \text{if } \lambda > 0, \\ \mathcal{I}_{\{0\}} & \text{if } \lambda = 0, \\ (\lambda m, |\lambda| r_m, |\lambda| l_m)_{RL} & \text{if } \lambda < 0, \end{cases} \\ \tilde{M} \oplus \tilde{N} &= (m + n, l_m + l_n, r_m + r_n)_{LR}.\end{aligned}$$

4 Multi-criteria decision making

Multiple criterion decision making (MCDM) refers to making decisions in the presence of multiple, usually conflicting criteria/constraints [52, 53]. The problems of MCDM can be broadly classified into two categories:

1. Multi-Attribute Decision Making (MADM),
2. Multi-Objective Decision Making (MODM),

In MADM problems, the objective is to prioritize a finite collection of alternatives say $\mathfrak{A} = \{\mathcal{A}_1, \dots, \mathcal{A}_m\}$ based on a finite number of criteria/attribute say $\mathfrak{C} = \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$. In this regard, the relevant information is organized in the form of a matrix $\mathbf{D} = [d_{ij}]_{m \times n}$, $i = 1, \dots, m$ and $j = 1, \dots, n$ called as the decision matrix, in which d_{ij} represents the status of alternative $\mathcal{A}_i \in \mathfrak{A}$ with respect to criterion $\mathcal{C}_j \in \mathfrak{C}$.

In MODM problems, the objective is to find the optimum solution for one or more objective functions while considering a set of constraints, i.e.

$$\begin{aligned}\text{optimize} \quad & f_1(\mathbf{x}), \dots, f_n(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{x} = (x_1, \dots, x_k) \in \mathbb{D} \subseteq \mathbb{R}^k, n \geq 2 \text{ and } k \geq 1.\end{aligned} \tag{1}$$

In literature, there are multiple methods for solving MADM problems, and one of the most important and practical ones is the TOPSIS method [14, 36], which will also be used in this paper. In addition, among the various methods in domain MODM [39, 73], the methods L-p metric and Goal Programming (GP) play a key role in this paper.

4.1 Entropy measure

Without loss of generality, suppose the matrix $\mathbf{D} = [d_{ij}]_{m \times n}$ is scaleless. Then we set $\mathbf{D}^{\text{owa}} = [\zeta_{ij}]_{m \times n}$, where for each $i = 1, \dots, m$, ζ_{ij} be the j^{th} largest value of d_{i1}, \dots, d_{in} , i.e. $\zeta_{i1} \geq \dots \geq \zeta_{in}$. In the following, we recall Entropy measure [56] that can be used for determining weight values. The entropy weight values can then be used to modify the optimal weight values obtained from the optimization problems introduced in this paper, in order to introduce new modified weight values. To obtain entropy weight values, the following Steps 1, 2, 3 are done for matrix \mathbf{D}^{owa} .

1. Transform matrix \mathbf{D}^{owa} into matrix $\mathcal{E} = [\varepsilon_{ij}]_{m \times n}$ by using the normalization formula

$$\varepsilon_{ij} = \frac{\zeta_{ij}}{\sum_{i=1}^m \zeta_{ij}}, \quad i = 1, \dots, m, \text{ and } j = 1, \dots, n.$$

notice each column can be considered as a probability distribution.

2. Calculate the information Entropy of each column as

$$E_j = -\frac{1}{\ln(m)} \sum_{i=1}^m \varepsilon_{ij} \ln(\varepsilon_{ij}), \quad j = 1, \dots, n.$$

note that $\varepsilon_{ij} \ln(\varepsilon_{ij})$ is defined as 0 if $\varepsilon_{ij} = 0$, for some i and j [56].

3. Derive the weight vector $\mathbf{e} = [e_1, \dots, e_n]_{1 \times n}$, where

$$e_j = \frac{1 - E_j}{\sum_{l=1}^n (1 - E_l)}, \quad j = 1, \dots, n.$$

4.2 TOPSIS

TOPSIS is a multi-criteria decision analysis method, which was originally developed by Yoon and Hwang [70]. TOPSIS is based on the assumption that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution [22, 47]. It is a method of compensatory aggregation that compares a set of alternatives, normalizing scores for each criterion and calculating the geometric distance between each alternative and the ideal alternative, which is the best score in each criterion. The TOPSIS process is carried out as follows [14].

Algorithm: TOPSIS procedure

Do the following Steps 1, 2, 3 for matrix $D = [d_{ij}]_{m \times n}$.

1. Euclidian normalization: Transform each column of matrix D into matrix $\Delta = [\delta_{ij}]_{m \times n}$ using the following formula

$$\delta_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^m d_{ij}^2}}, \quad i = 1, \dots, m, \text{ and } j = 1, \dots, n.$$

2. Calculate the weighted normalized matrix $\Lambda = \Delta \times \mathbf{e} = [\lambda_{ij}]_{m \times n}$, where $\mathbf{e} = \text{diag}[e_1, \dots, e_n]_{n \times n}$. Here, that the weighting vector $[e_1, \dots, e_n]_{1 \times n}$ could be obtained running the Entropy Measure [56].
3. Determine the positive ideal alternative and negative ideal alternative as

$$R^+ = [\lambda_1^+, \dots, \lambda_n^+] = \{(\max_i \lambda_{ij} | j \in B), (\min_i \lambda_{ij} | j \in B')\},$$

$$R^- = [\lambda_1^-, \dots, \lambda_n^-] = \{(\min_i \lambda_{ij} | j \in B), (\max_i \lambda_{ij} | j \in B')\}.$$

note that $B \subseteq \{1, \dots, n\}$, $B' \subseteq \{1, \dots, n\}$, $B \cup B' = \{1, \dots, n\}$ and $B \cap B' = \emptyset$ are associated with the index of the benefit criteria and the index of the cost criteria, respectively.

4. Using the n -dimensional Euclidean distance, calculate the separation measures as follows

$$d_i^+ = \sqrt{\sum_{j=1}^n (\lambda_{ij} - \lambda_j^+)^2}, \quad i = 1, \dots, m,$$

$$d_i^- = \sqrt{\sum_{j=1}^n (\lambda_{ij} - \lambda_j^-)^2}, \quad i = 1, \dots, m,$$

5. Calculate the relative closeness to the ideal alternative as follows

$$L_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad i = 1, \dots, m.$$

6. Rank the preference order $\{L_1, \dots, L_m\}$ and select the best alternative as $i^* = \text{argmax}_{i \in \{1, \dots, m\}} \{L_1, \dots, L_m\}$.

4.3 $L - p$ metric and $\tilde{L} - p$ metric

$L - p$ metric and $\tilde{L} - p$ metric are two popular algorithms whose objectives are the minimization and maximization of deviations of the target functions from ideal and anti-ideal solution(s), respectively [8, 55]. Here, it is supposed that we have a multi-objective optimization problem as (1). Here, effective solutions induced in the method $L - p$ metric are obtained by solving the following optimization problem

$$\min_{\mathbf{w}} \quad \left[\sum_{j=1}^n w_j \left(\frac{f_j^+ - f_j}{f_j^+ - f_j^-} \right)^p \right]^{\frac{1}{p}},$$

$$s.t. \quad \mathbf{x} \in \mathbb{D}, \quad w_j \geq 0 \text{ for } j = 1, \dots, n, \quad \sum_{j=1}^n w_j = 1.$$

While, effective solutions induced in the method $\tilde{L}-p$ metric are obtained by solving the following optimization problem

$$\begin{aligned} \max_{\tilde{\mathbf{w}}} \quad & \left[\sum_{j=1}^n \tilde{w}_j \left(\frac{f_j - f_j^-}{f_j^+ - f_j^-} \right)^p \right]^{\frac{1}{p}}, \\ \text{s.t.} \quad & \mathbf{x} \in \mathbb{D}, \tilde{w}_j \geq 0 \text{ for } j = 1, \dots, n, \sum_{j=1}^n \tilde{w}_j = 1, \end{aligned}$$

where $1 \leq p \leq \infty$ and for each $j = 1, \dots, n$, $f_j^+ = \max_{\mathbf{x} \in \mathbb{D}} f_j(\mathbf{x})$, and $f_j^- = \min_{\mathbf{x} \in \mathbb{D}} f_j(\mathbf{x})$, and w_j is the relative importance of the objective function f_j , as well as \tilde{w}_j .

$L-p$ metric ($\tilde{L}-p$ metric) method is used when we are emphasizing to the closeness (farness) to the ideal (anti-ideal) solution completely. Now, let the emphasize to the closeness to the ideal solution and farness from the anti-ideal solution is considered as λ and $1-\lambda$, respectively. Here, consider a multi-objective optimization problem as follows

$$\begin{aligned} \max \quad & \mathfrak{S} = \frac{\lambda \left[\sum_{j=1}^n \tilde{w}_j \left(\frac{f_j - f_j^-}{f_j^+ - f_j^-} \right)^p \right]^{\frac{1}{p}}}{\lambda \left[\sum_{j=1}^n \tilde{w}_j \left(\frac{f_j - f_j^-}{f_j^+ - f_j^-} \right)^p \right]^{\frac{1}{p}} + (1-\lambda) \left[\sum_{j=1}^n w_j \left(\frac{f_j^+ - f_j}{f_j^+ - f_j^-} \right)^p \right]^{\frac{1}{p}}}, \\ \text{s.t.} \quad & p \geq 1, 0 < \lambda < 1, \mathbf{x} \in \mathbb{D}, w_j \geq 0, \tilde{w}_j \geq 0, \text{ for } j = 1, \dots, n, \sum_{j=1}^n w_j = 1, \sum_{j=1}^n \tilde{w}_j = 1. \end{aligned} \quad (2)$$

Remark 4.1. 1. Notice $0 \leq \mathfrak{S} \leq 1$ and \mathfrak{S} has increasing utility.

2. Note that, if $\lambda = 0$ then the solutions are obtained as $L-p$ metric method. While, if $\lambda = 1$ then the solutions are obtained as $\tilde{L}-p$ metric method.

4.4 GP method

Supposed that f_j , $j = 1, \dots, n$, are objective functions and L_j , $j = 1, \dots, n$, denote the target or goal set by DM for j th objective function f_j and \mathbb{D} represents the feasible region from which the choices of vector (f_1, \dots, f_n) must be effected. A goal-programming model can be stated as follows:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \sum_{j=1}^n (\lambda_j^+ d_j^+ + \lambda_j^- d_j^-) \\ \text{s.t.} \quad & \mathbf{x} \in \mathbb{D}, f_j - d_j^+ + d_j^- = L_j, d_j^+ \geq 0, d_j^- \geq 0, \lambda_j^+ \geq 0, \lambda_j^- \geq 0, \text{ for } j = 1, \dots, n, \sum_{j=1}^n \lambda_j^+ = 1, \sum_{j=1}^n \lambda_j^- = 1. \end{aligned}$$

The parameters λ_j^+ and λ_j^- , $j = 1, \dots, n$ are positive weights that reflect the decision maker's fuzzy preferences regarding the relative importance of each goal [45]. A major strength of goal programming is its simplicity and ease of use. This accounts for the large number of goal programming applications in many and diverse fields.

5 Derivation of the OWA operator weights in the context of MOMD

5.1 The $L-p$ metric and $\tilde{L}-p$ metric for derivation of the weight in OWA operators

To model the problem by the approach $L-p$ metric and $\tilde{L}-p$ metric (2), we are considering the following goals, simultaneously:

1. Maximum Entropy of w_1, \dots, w_n must be achieved.

2. The minimization of $\frac{\sum_{j=1}^n w_j \zeta_{tj}}{\sum_{j=1}^n w_j \zeta_{sj}}$ for each $((A_s, A_t), \wp_{(s,t)}) \in \tilde{\Theta}_1$, where

$$\tilde{\Theta}_1 = \{((A_s, A_t), \wp_{(s,t)}) \mid (A_s, A_t) \in \mathfrak{A} \times \mathfrak{A} \text{ and } \wp_{(s,t)} \in [0, 1]\},$$

and $\wp_{(s,t)}$ indicates the degree of fuzzy preference of A_s over A_t from the decision-maker's perspective.

3. The maximization of $\sum_{j=1}^n w_j \zeta_{ij}$ for each $(A_i, \nu_i) \in \tilde{\Theta}_2$, $i = 1, \dots, m$, where

$$\tilde{\Theta}_2 = \{(A_i, \nu_i) \mid A_i \in \mathfrak{A} \text{ and } \nu_i \in [0, 1]\},$$

where the value of ν_i reflects the degree of decision makers' emphasise on the closeness of the value of $\sum_{j=1}^n w_j \zeta_{ij}$ to ζ_{i1} . Notice the degree of fuzzy preferences can also be defined as such as Likert scale [40], Saaty's spectrum [17], and so on.

4. Considering the risk level (orness α).

Therefore, for any fixed value of $0 \leq \alpha \leq 1$, we take orness as LR-fuzzy number $\tilde{\alpha} = (\alpha, l, r)_{LR}$ for small value of l and r and $0 < \alpha - l < \alpha < \alpha + r < 1$, and ε is a fixed small value determined priory by the decision maker. Now considering above items, Equation (2) can be formulated as follows to evaluate w_1, \dots, w_n :

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{B}{A+B} \\ \text{s.t.} \quad & \end{aligned} \tag{3}$$

$$\begin{aligned} A = & \left[\delta_1 \left(1 + \frac{1}{\log(n)} \sum_{j=1}^n w_j \log(w_j) \right)^p + \delta_2 \sum_{(A_s, A_t) \in \tilde{\Theta}_1} \wp_{(s,t)} \left(\frac{\zeta_{s1}}{\zeta_{tn}} - \frac{\sum_{j=1}^n w_j \zeta_{tj}}{\sum_{j=1}^n w_j \zeta_{sj}} \right)^p + \right. \\ & \left. + \delta_3 \sum_{A_i \in \tilde{\Theta}_2} \nu_i \left(\frac{\zeta_{i1} - \sum_{j=1}^n w_j \zeta_{ij}}{\zeta_{i1} - \zeta_{in}} \right)^p \right]^{\frac{1}{p}}, \\ B = & \left[\delta'_1 \left(-\varepsilon - \frac{1}{\log(n)} \sum_{j=1}^n w_j \log(w_j) \right)^p + \delta'_2 \sum_{(A_s, A_t) \in \tilde{\Theta}_1} \wp_{(s,t)} \left(\frac{\sum_{j=1}^n w_j \zeta_{tj}}{\sum_{j=1}^n w_j \zeta_{sj}} - \frac{\zeta_{sn}}{\zeta_{t1}} \right)^p + \right. \\ & \left. + \delta'_3 \sum_{A_i \in \tilde{\Theta}_2} \nu_i \left(\frac{\sum_{j=1}^n w_j \zeta_{ij} - \zeta_{in}}{\zeta_{i1} - \zeta_{in}} \right)^p \right]^{\frac{1}{p}}, \end{aligned}$$

$$\text{orness}(\mathbf{w}) = \sum_{j=1}^n \frac{n-j}{n-1} w_j \in [\alpha - l + \alpha + r],$$

$$w_j \in [0, 1] \text{ for } j = 1, \dots, n, \quad \sum_{n=1}^n w_j = 1,$$

$$\delta_1 \geq 0, \delta_2 \geq 0, \delta_3 \geq 0, \quad \delta_1 + \delta_2 + \delta_3 = 1,$$

$$\delta'_1 \geq 0, \delta'_2 \geq 0, \delta'_3 \geq 0, \quad \delta'_1 + \delta'_2 + \delta'_3 = 1,$$

$$p \geq 1, \nu_i \in [0, 1] \text{ for } i = 1, \dots, m,$$

Remark 5.1. If $\mathbf{w}^* = [w_1^*, \dots, w_n^*]$ are the optimal values for $\mathbf{w} = [w_1, \dots, w_n]$, then $w_j'^* = \frac{e_j w_j^*}{\mathbf{e} \mathbf{w}^*}$, for $j = 1, \dots, n$ is the modified weight value of the optimal weight values w_j^* by the entropy weight vector \mathbf{e} .

Remark 5.2. 1. If in the above optimization problem, $\delta_1 = 1$ and $p = 1$ then the proposed method coincides with the OWA operator proposed by [65].

2. We can consider the decision-level risk deviation from α as a goal instead of a constraint in the optimization problem. This makes it possible to achieve optimal weights for an acceptable amount of tolerance of orness value. So, it can be used when the decision maker may consider the amount of tolerance about orness into the optimization

problem. Therefore, the optimization problem (5) can be rewritten as

$$\begin{aligned}
\min_{\mathbf{w}} \quad & \left[\delta_1 \left(1 + \frac{1}{\log(n)} \sum_{j=1}^n w_j \log(w_j) \right)^p + \delta_2 \sum_{(A_s, A_t) \in \tilde{\Theta}_1} \wp_{(s,t)} \left(\frac{\sum_{j=1}^n w_j \zeta_{tj}}{\sum_{j=1}^n w_j \zeta_{sj}} \right)^p + \right. \\
& \left. \delta_3 \sum_{A_i \in \tilde{\Theta}_2} \nu_i \left(\frac{\zeta_{i1} - \sum_{j=1}^n w_j \zeta_{ij}}{\zeta_{i1} - \zeta_{in}} \right)^p \right] + \delta_4 |\text{orness}(\mathbf{w}) - \alpha|^p]^{\frac{1}{p}}, \quad (4) \\
\text{s.t.} \quad & w_j \in [0, 1] \text{ for } j = 1, \dots, n, \quad \sum_{i=1}^n w_j = 1, \\
& \delta_l \in [0, 1] \text{ for } l = 1, \dots, 4, \quad \sum_{l=1}^4 \delta_l = 1, \quad p \geq 1, \quad \nu_i \in [0, 1] \text{ for } i = 1, \dots, m.
\end{aligned}$$

3. The values of $p, \delta_1, \dots, \delta_4$, and ν_1, \dots, ν_m can be priori determined by the decision maker.

Remark 5.3. The decision-maker may express the priority of A_s over A_t in the form of a fuzzy number $\widetilde{\wp_{(s,t)}}$. In particular if $\widetilde{\wp_{(s,t)}}$ is a triangular fuzzy number such as $\widetilde{\wp_{(s,t)}} = (\wp_{(s,t)}, l_{(s,t)}, r_{(s,t)})$, then the proposed model can be rewritten as

$$\begin{aligned}
\max_{\mathbf{w}} \quad & \frac{B}{A+B} \quad (5) \\
\text{s.t.} \quad & A = \left[\delta \left(1 + \frac{1}{\log(n)} \sum_{j=1}^n w_j \log(w_j) \right)^p + (1-\delta) \sum_{A_i \in \tilde{\Theta}_2} \nu_i \left(\frac{\zeta_{i1} - \sum_{j=1}^n w_j \zeta_{ij}}{\zeta_{i1} - \zeta_{in}} \right)^p \right], \\
& B = \left[\delta' \left(-\varepsilon - \frac{1}{\log(n)} \sum_{j=1}^n w_j \log(w_j) \right)^p + (1-\delta') \sum_{A_i \in \tilde{\Theta}_2} \nu_i \left(\frac{-\zeta_{in} + \sum_{j=1}^n w_j \zeta_{ij}}{\zeta_{i1} - \zeta_{in}} \right)^p \right], \\
\text{orness}(\mathbf{w}) \quad & = \sum_{j=1}^n \frac{n-j}{n-1} w_j \in [\alpha - l + \alpha + r], \\
& \wp_{(s,t)} - l_{(s,t)} \leq \frac{\sum_{j=1}^n w_j \zeta_{tj}}{\sum_{j=1}^n w_j \zeta_{sj}} \leq \wp_{(s,t)} + r_{(s,t)}, \quad \forall (A_s, A_t) \in \tilde{\Theta}_1, \\
& w_j \in [0, 1] \text{ for } j = 1, \dots, n, \quad \sum_{j=1}^n w_j = 1, \\
& \delta \in [0, 1], \delta' \in [0, 1], \quad p \geq 1, \quad \nu_i \in [0, 1] \text{ for } i = 1, \dots, m,
\end{aligned}$$

Remark 5.4. In addition to the alternation of the Entropy objective function, which was an efficient method proposed by Yager to solve the constrained OWA optimization problem, other objective functions were proposed by some researchers. Indeed, instead of the maximal entropy objective function in the above optimization problem, other objective functions can be considered as well. Depending on the goal to attain some of them are listed below, but are not limited to [6]:

1. minimize: $\sum_{j=1}^n (w_j - w_{j+1})^2$ for a minimum variance approach,
2. minimize: $\max_j |w_j - w_{j+1}|$ for a minimax disparity approach,
3. minimize: $\sum_{j=1}^n (w_j - \frac{1}{n})^2$ for a least square OWA approach,
4. minimize: $\max_j w_j$ for another expression of a measure of entropy.
5. maximize: $\sum_{j=1}^n w_j e^{1-w_j}$ known as Lambert Function Method.

5.2 The GP approach for derivation of the weight in OWA operators

Considering the decision matrix $\mathbf{D} = [d_{ij}]_{m \times n}$, the GP approach for derivation of the weights is formulated as follows

$$\begin{aligned}
\min_{\mathbf{w}} \quad & \left(\omega_D D + w_\alpha^+ d_\alpha^+ + w_\alpha^- d_\alpha^- + \sum_{(A_s, A_t) \in \tilde{\Theta}_1} \wp_{(s,t)} d_{(s,t)}^+ + \sum_{A_i \in \tilde{\Theta}_2} \nu_i d_i^+ \right), \\
\text{s.t.} \quad & D - \frac{1}{\log(n)} \sum_{j=1}^n w_j \log(w_j) = 1, \\
& \sum_{j=1}^n \left(\frac{n-j}{n-1} w_j \right) + d_\alpha^+ - d_\alpha^- = \alpha, \\
& d_{(s,t)}^+ + \frac{\sum_{j=1}^n w_j \zeta_{sj}}{\sum_{j=1}^n w_j \zeta_{tj}} = \frac{\zeta_{s1}}{\zeta_{tn}}, \quad \forall (A_s, A_t) \in \tilde{\Theta}_1, \\
& d_1^+ + \sum_{j=1}^n w_j \zeta_{1j} \geq \zeta_{11}, \\
& \vdots \\
& d_m^+ + \sum_{j=1}^n w_j \zeta_{mj} \geq \zeta_{m1}, \\
& \omega_D, D, w_\alpha^+, w_\alpha^-, w_j, d_\alpha^+, d_\alpha^-, d_i^+, d_{(s,t)}^+ \geq 0, \forall i = 1, \dots, m, \forall A_i \in \tilde{\Theta}_2, \forall (A_s, A_t) \in \tilde{\Theta}_1,
\end{aligned} \tag{6}$$

6 Applied numerical example

Mineral sector as the essential part of the economy has a crucial role in supplying industries' raw materials. Investment promotion in this sector can earn high value. Consequently, it is necessary to recognize high priority minerals for future investment. The issue of evaluating investment priorities in the mineral sector was defined and implemented by the Iranian Ministry of Industry, Mine and Trade in 17 developing regions of a comprehensive project during 14 months from January 2021 to February 2022 (Table 1).

This project was implemented with the aim of reviewing, evaluating and prioritizing investments by identifying 235 alternatives. Notice, in each province, there were a large number of investment opportunities in the mining industry that were filtered in the prefeasibility study of the project. A number of them were then selected for further studies. For this purpose, we formulated and solved 17 separated MADM problems based on 48 criteria. The main reason for dividing the main problem into 17 separated sub-problems was that the decision maker's preferences were different from one province to another and were completely dependent on the situations of each province. For example, Kurdistan province, the focus of this section, was candidate for more than 450 items, which have been then reduced to 5 after prefeasibility study as follows (see Table 5 for Kurdistan recorded data):

1. Fluorspar.
2. Ferrosilicon.
3. Gold.
4. Barite.
5. Hematite.

Finally, the total number of alternatives, i.e. the total number of candidate projects requiring investment submitted by the 17 provinces, is equal to 235. To evaluate the priority of the alternatives, several criteria were identified and defined through literature review and interviews with mining experts. Then based on the current situation in Iran, 8 criteria and 48 sub-criteria were selected (Table 2). The status of these 235 alternatives in relation to these 8 criteria and 48 sub-criteria was extracted through library studies and field studies which leads to the decision matrix given in Table 3.

Our aim for analysing the results is twofold. Firstly, the output of this case study should be to sort out the alternatives, which will be done in the following cases:

Table 1: Province list and Mineral list

No.	Province Name	No.	Minerals
1	Ardabil	1	Barite
2	Buschehr	2	Bauxite
3	Fars	3	Bismuth
4	Golestan	4	Chromite
5	Hamadan	5	Cobalt
6	Hormozgan	6	Ferromanganese
7	Ilam	7	Ferrosilicon
8	Kermanshah	8	Fluorspar
9	Kohgiluyeh and Boyer-Ahmad	9	Gilsonite
10	Kurdistan	10	Gold
11	Lorestan	11	Graphite
12	North Khorasan	12	Hematite
13	Qom	13	Iron ore
14	Semnan	14	Magnesium
15	Sistan and Baluchestan	15	Molybdenum
16	South Khorasan	16	Nickel
17	Yazd	17	Potash
		18	Silver
		19	Titanium
		20	Zinc

1. Sort the alternatives in each province.
2. Sort all 235 alternatives (regardless of the provinces).

Secondly, a statistical study is therefore conducted to make a coherent comparative study between the method proposed in this paper and TOPSIS method, SAW method and OWA operator [14, 65], which are well-known and widely used methods in multi-criteria decision making.

6.1 Comparison of competitive methods based on the provinces

6.1.1 Computations of the competitive methods according to each province

One of the most important steps in this project was to determine the weights of the criteria. To do so, two different parallel paths were considered in making more than 150 experts' judgment in various fields of economics, technology and mining engineering, clean energy sectors, and so on, as follows:

1. The weights of the alternatives were evaluated in the form of a pairwise comparison matrix about the criteria as well as the sub-criteria. Then, based on this, the TOPSIS method and SAW method were used to rank the alternatives (for instance, see the first row of Table 4 which shows the criteria wights for the Kurdistan dataset given in Table 5).
2. By conducting a statistical study on the set of experts' judgments, their risk level followed a normal distribution with mean 0.57 and variance 0.2. Then the weights of the criteria were obtained based on the methods proposed in this paper.

Table 4 shows the results of approximated weights of the criteria based on the proposed method in this paper along with the results of TOPSIS, SAW and OWA. For visual comparison, Figure 1a and Figure 1b show the graph values of the weights obtained from different methods in Table 4 from different perspective to better see their differences and similarities. Now, using these approximated weights, the alternatives in Table 5 are ranked. Their corresponding preference values as well as their ranks are given in Table 6.

6.1.2 Similarity ratio of ranks as a measure of comparison

Assume that in each province the sets M_1 and M_2 are the three superior alternatives obtained by both competitive Method 1 and Method 2, respectively. In this case, the ratio of their overlap in that province is as follows

$$P = \frac{\#(M_1 \cap M_2)}{3},$$

where $\#A$ is the total number of elements in a set A . The results of Table 8 show the comparison of the same ranking ratio of the top 3 alternatives in different methods. It can be concluded that the top 3 ranked al Three top ranked alternatives are the same in methods TOPSIS, SAW and our proposed method. But, 2 of the 3 top ranked alternatives are the same in methods TOPSIS, SAW and our proposed method while compared to OWA method.

Table 2: Criteria and sub-criteria in case study

Criteria	Sub-Criteria
C.1: Compliance with Strategies	$S_1^{C.1}$: Alignment with organization's vision, mission and objectives $S_2^{C.1}$: Executive Preferences (The degree of acceptability in C-level executive) $S_3^{C.1}$: Preferences of off-organization officials and benefactors $S_4^{C.1}$: Alignment with provincial/state investment priorities
C.2: Cost Factors	$S_1^{C.2}$: Capital expenditure $S_2^{C.2}$: Working capital expenditure $S_3^{C.2}$: Foreign expenditure share in CAPEX $S_4^{C.2}$: Foreign expenditure share in OPEX $S_5^{C.2}$: Financial progress
C.3: Market Factors	$S_1^{C.3}$: Local demand $S_2^{C.3}$: Import demand of the region countries $S_3^{C.3}$: Rivals in local market $S_4^{C.3}$: Rivals in region countries $S_5^{C.3}$: International trade share of the relative global market $S_6^{C.3}$: Vertical integration and well-distributed supply through the chain $S_7^{C.3}$: The opportunity to create local and regional market $S_8^{C.3}$: Meeting major industries' demands (especially IMIDRO subsidiaries) and guarantee to purchase
C.4: Economic and Profitability Factors	$S_1^{C.4}$: Return of capital (including construction duration) $S_2^{C.4}$: Profit Margin $S_3^{C.4}$: Export sale $S_4^{C.4}$: Meeting import demand
C.5: Technical and Operating Factors	$S_1^{C.5}$: Skilled labor force $S_2^{C.5}$: Similar experiences in local market $S_3^{C.5}$: Technology Accessibility $S_4^{C.5}$: Energy and Fuel Accessibility $S_5^{C.5}$: Water demand $S_6^{C.5}$: Ease of water supply $S_7^{C.5}$: Self-sufficiency in supplying localized equipment $S_8^{C.5}$: Physical progress $S_9^{C.5}$: Contribution to developing other projects $S_{10}^{C.5}$: Access to sufficient and inexpensive transportation facilities
C.6: Risk Factors	$S_1^{C.6}$: Substitution Risk $S_2^{C.6}$: Risk of technology and equipment import $S_3^{C.6}$: Price volatility risk $S_4^{C.6}$: Local community risk (social responsibility and local protestors) $S_5^{C.6}$: Environment risks $S_6^{C.6}$: Raw material supply dependency to neighboring towns and cities $S_7^{C.6}$: Reserve sustainability risk $S_8^{C.6}$: Investment participation risk $S_9^{C.6}$: Natural disasters risk
C.7: Social Responsibility Factors	$S_1^{C.7}$: Employment $S_2^{C.7}$: Impact on local economy $S_3^{C.7}$: Alignment with national needs $S_4^{C.7}$: Severity of deprivation in the community $S_5^{C.7}$: Pollution Intensity and Irreversible environmental effects
C.8: Ease of Finance Factors	$S_1^{C.8}$: Ability to finance the project via modern methods such as BOT and BOO $S_2^{C.8}$: Ease of finance by stakeholders resources and major off-organizational supporters $S_3^{C.8}$: Ability to finance by organizational resources $S_4^{C.8}$: Ease of finance by financial institutions and funds' facilities and mining investment hedge funds

6.2 Rank correlation coefficient tests as measures of comparison

Both Spearman's rank correlation coefficient (ρ) and Kendall's rank correlation coefficient (τ) are nonparametric measures of rank correlation (statistical dependence between the rankings of two variables) [61]. It assesses how well the relationship between two variables can be described by using a monotonic function. For example, a perfect Spearman correlation of +1 or -1 occurs when each of the variables is a perfect monotone function of the other. Intuitively, the Spearman correlation between two variables will be high when observations have a similar rank, and low when observations have a dissimilar rank between the two variables. The Spearman's rank correlation coefficient test results as well as Kendall's rank correlation test results for different comparative methods are shown in Table 7, as well as the

Table 3: Decision matrix used in case study

Province	Alternatives		Criteria				Sub-Criteria	
	Totally	In Province	C.1	...	C.8	C.1	...	C.48
Ardabil	A ₁	A.1	0.5282		0.7125	0.1256		0.2125
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	A ₆	A.6	0.4578		0.6253	0.3678		0.4125
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Kurdistan	A ₁₅₆	A.1	0.4036	...	0.5501	0.1102	...	0.1365
	A ₁₅₇	A.2	0.3229	...	0.4126	0.1325	...	0.2548
	A ₁₅₈	A.3	0.4843	...	0.4813	0.2569	...	0.0895
	A ₁₅₉	A.4	0.3632	...	0.4469	0.2198	...	0.2145
	A ₁₆₀	A.5	0.6054	...	0.3094	0.3045	...	0.3598
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Yazd	A ₂₂₅	A.1	0.9845		0.3526	0.0958		0.0258
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	A ₂₃₅	A.9	0.2534		0.4425	0.1458		0.1198

Table 4: Weights values obtained by the competitive methods

Weight generator method	Criteria weights							
	w _{C.1}	w _{C.2}	w _{C.3}	w _{C.4}	w _{C.5}	w _{C.6}	w _{C.7}	w _{C.8}
TOPSIS and SAW Weights (given by experts)	0.2914	0.1609	0.1286	0.1212	0.1201	0.0846	0.0649	0.0281
OWA Weights by [65] (α = 0.38)	0.0654	0.0772	0.0911	0.1073	0.1265	0.1491	0.1758	0.2073
L-p Weights by Eq. (3) (α = 0.38, δ ₃ = 0, Θ = ∅)	0.5045	0.2274	0.0365	0.0100	0.0450	0.1366	0.2820	0.2076
L-p Weights by Remark 5.1 (α = 0.38, $\hat{p} = 585.9$, δ ₃ = 0, Θ = ∅)	0.0396	0.3156	0.0202	0.0055	0.0290	0.0611	0.2711	0.2579
GP Weights by Eq. (6)	0.3440	0.2338	0.1436	0.0887	0.0724	0.0544	0.0389	0.0238

Table 5: Kurdistan decision matrix in case study

Alternatives	C.1	C.2	C.3	C.4	C.5	C.6	C.7	C.8
Fluorspar	0.4036	0.4325	0.3788	0.4659	0.3565	0.5445	0.3626	0.5501
Ferrosilicon	0.3229	0.4901	0.2841	0.4326	0.3921	0.4356	0.4533	0.4126
Gold	0.4843	0.4036	0.4262	0.4992	0.3208	0.4719	0.5742	0.4813
Barite	0.3632	0.5190	0.5682	0.4326	0.4990	0.3993	0.4231	0.4469
Hematite	0.6054	0.3748	0.5209	0.3993	0.6060	0.3630	0.3929	0.3094

Table 6: Rank results of the competitive methods based on the weight values given in Table 4

Alternatives	TOPSIS	SAW	OWA	L-p (Eq. (3))	L-p (Remark 5.1)	GP (Eq. (6))
	Preference value(rank)	Preference value(rank)	Preference value(rank)	Preference value(rank)	Preference value(rank)	Preference value(rank)
Fluorspar	0.3137(4)	0.4203(4)	0.4119(3)	0.4213(4)	0.4246(4)	0.4943(4)
Ferrosilicon	0.1928(5)	0.3869(5)	0.3794(5)	0.3721(5)	0.3703(5)	0.4460(5)
Gold	0.4948(2)	0.4507(2)	0.4325(2)	0.4271(3)	0.4305(3)	0.5071(2)
Barite	0.4046(3)	0.4486(3)	0.4338(1)	0.4369(1)	0.4391(2)	0.5066(3)
Hematite	0.7360(1)	0.4898(1)	0.4101(4)	0.4291(2)	0.4402(1)	0.5305(1)

Table 7: Results of Spearman's rank correlation coefficient test (ρ(pvalue)) and Kendall's rank correlation test (τ(pvalue)) for different comparative methods in Table 6

Methods	SAW		OWA		L-p (Eq. (3))		L-p (Remark 5.1)		GP (Eq. (6))	
	ρ(pvalue)	τ(pvalue)	ρ(pvalue)	τ(pvalue)	ρ(pvalue)	τ(pvalue)	ρ(pvalue)	τ(pvalue)	ρ(pvalue)	τ(pvalue)
TOPSIS	1.00(0.016)	1(0.0166)	0.30(0.683)	0.2(0.8166)	0.7(0.233)	0.6(0.2333)	0.9(0.083)	0.8(0.0833)	1.0(0.016)	1.0(0.0166)
SAW	x	x	0.30(0.683)	0.2(0.8166)	0.7(0.233)	0.6(0.2333)	0.9(0.083)	0.8(0.0833)	1.0(0.016)	1.0(0.0166)
OWA	x	x	x	x	0.7(0.233)	0.6(0.2333)	0.4(0.516)	0.4(0.4833)	0.3(0.683)	0.2(0.8166)

Table 8: Ratio of similarity between the first 3 top ranks in Table 6

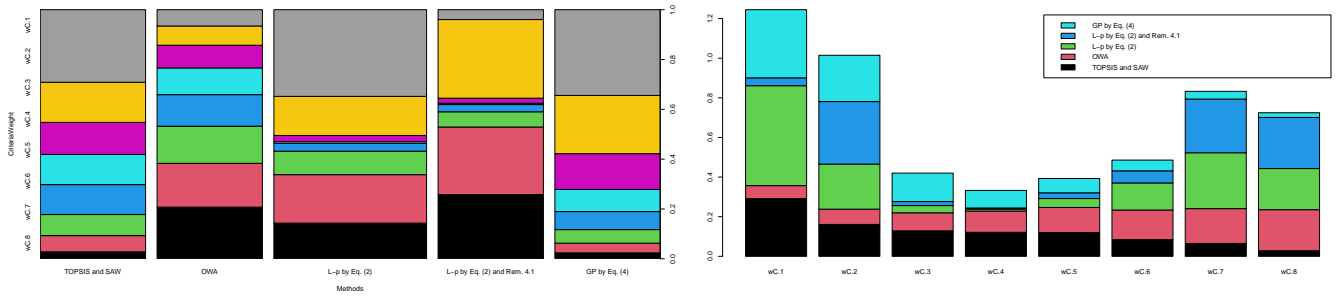
Methods	SAW	OWA	L-p (Eq. (3))	L-p (Remark 5.1)	GP (Eq. (6))
TOPSIS	$\frac{3}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
SAW	x	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
OWA	x	x	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{3}$

corresponding p-value for testing the hypotheses

$$H_0 : \text{correlation coefficient is } 0$$

$$H_1 : \text{correlation coefficient is not } 0.$$

(7)



(a) The diagram of the values of the different weight generator methods in Table 4 (b) The diagram of the values of the different weight generator methods in Table 4

Figure 1: Analysis of weights

Table 9: Ratio of similarity between the first 20 top ranks in the 235 alternatives

Methods	SAW	OWA	L-p (Eq. (3))	L-p (Remark 5.1)	GP (Eq. (6))
TOPSIS	$\frac{16}{20}$	$\frac{12}{20}$	$\frac{18}{20}$	$\frac{17}{20}$	$\frac{18}{20}$
SAW	x	$\frac{12}{20}$	$\frac{18}{20}$	$\frac{15}{20}$	$\frac{17}{20}$
OWA	x	x	$\frac{8}{20}$	$\frac{11}{20}$	$\frac{11}{20}$

Here, the correlation coefficient value close to zero shows that the correlation between method 1 and method 2 is very low. The positive value suggests that the longer preference values of method 1, the longer preference values of method 1 and vice versa. The negative value suggests that the longer preference values of method 1, the lower preference values of method 1 and vice versa. Considering the results of Table 7, it can be concluded that the ranked values obtained by our proposed methods are more correlated to the ranked values obtained by TOPSIS and SAW methods. Less correlation value obtained for the ranked values obtained by OWA method in comparison to the others.

6.3 Comparison of the competitive methods for ranking all of the alternatives

6.3.1 Computations

We showed the calculations in Sub-section 6.1.1 for the Kurdistan decision matrix as an example. In principle, such calculations were performed similarly for the main decision matrix with dimension 235×8 . In the sequel the results of the comparative methods are analyzed while ranking the 235 alternatives.

6.3.2 Similarity ratio

For the priority list of the 235 alternatives, here, 20 primary alternatives were identified based on these six methods. The percentage of similar rankings of these methods is given in Table 9. For example, a comparison of the ranking of alternatives using OWA method with the TOPSIS method confirmed that about $\frac{12}{20} \times 100\%$ of the rankings obtained from both methods are the same, while the value of similarity between SAW and TOPSIS method is $\frac{16}{20} \times 100\%$. The results of Table 9 show that the method proposed in this paper is relatively more consistent than SAW and than OWA method with the results provided with the conventional TOPSIS, in ranking the alternatives.

Note that, as it is shown in Table 9, the results obtained from the proposed method in this paper, which is a generalization of the Yager method, differ from the results of Yager method. This indicates that incorporating decision matrix information in evaluating the weights of the criteria impacts the ranking of the alternatives. On the other hand, the rankings obtained from the proposed method are closer to the results of the TOPSIS method compared to the SAW method, which is an advantage because the TOPSIS method, utilizes more information in its algorithm compared to other methods such as SAW method.

6.3.3 Testing hypothesis

Both Spearman’s rank correlation coefficient test and Kendall’s rank correlation test are done as measures of comparison. The results given in Table 10 indicate that the rank values obtained from TOPSIS and our proposed method are more correlated than the SAW’s rank values and OWA’s rank values. Therefore, some reasons for the similarity of the ranking

Table 10: Results of Spearman’s rank correlation coefficient test (ρ (pvalue)) and Kendall’s rank correlation test (τ (pvalue)) for different comparative methods

Methods	SAW		OWA		L-p (Eq. (3))		L-p (Remark 5.1)		GP (Eq. (6))	
	ρ (pvalue)	τ (pvalue)	ρ (pvalue)	τ (pvalue)	ρ (pvalue)	τ (pvalue)	ρ (pvalue)	τ (pvalue)	ρ (pvalue)	τ (pvalue)
TOPSIS	0.9895(0.000)	0.9149(0.000)	0.7733(0.000)	0.5931(0.000)	0.9968(0.000)	0.9514(0.000)	0.9965(0.000)	0.9480(0.000)	0.9928(0.000)	0.9266(0.000)
SAW	x	x	0.7746(0.000)	0.5910(0.000)	0.9874(0.000)	0.9033(0.000)	0.9854(0.000)	0.8967(0.000)	0.9847(0.000)	0.8917(0.000)
OWA	x	x	x	x	0.7745(0.000)	0.5929(0.000)	0.7714(0.000)	0.5878(0.000)	0.7708(0.000)	0.5900(0.000)

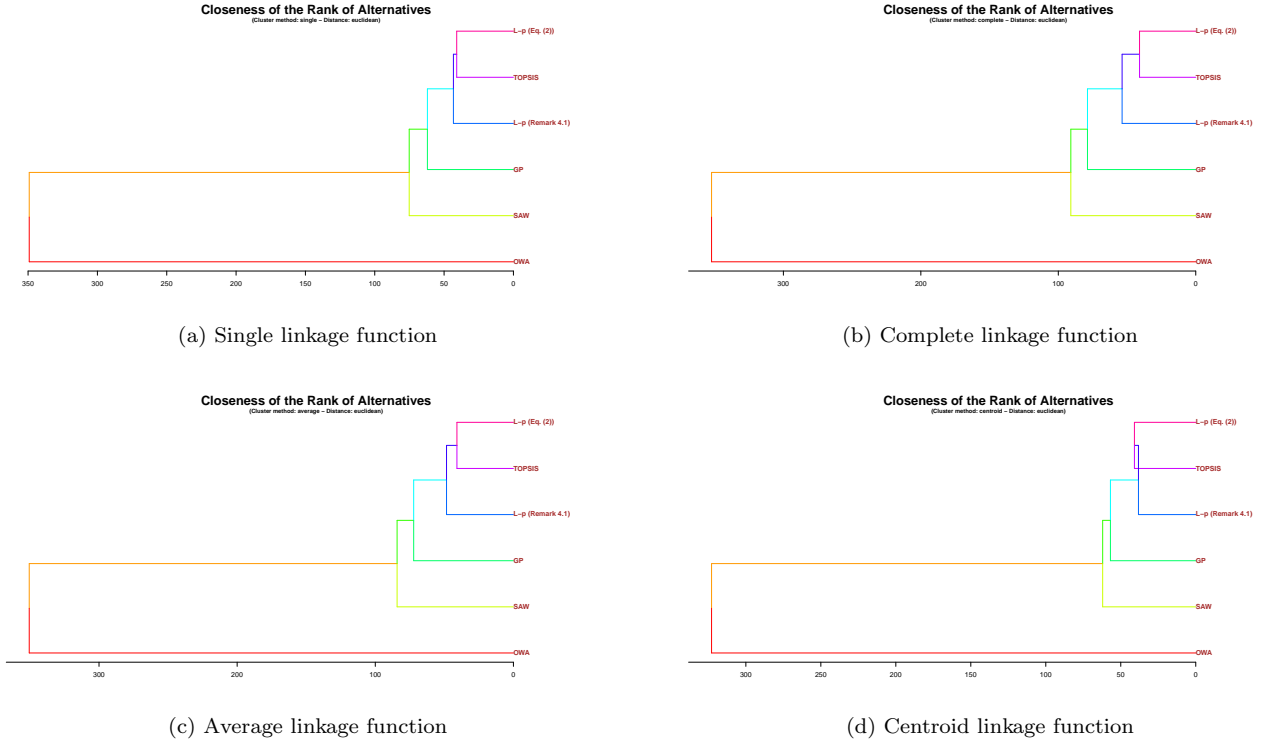


Figure 2: Dendrogram for different linkage functions and Euclidian distance

of alternatives based on the method proposed in this paper with the TOPSIS method can be concluded by the results of Table 10. These results also show that there is a greater correlation between the values of ranking created by TOPSIS and SAW methods and the method proposed in this paper. However, there is less correlated value between the OWA method and the others.

6.3.4 Clustering

Hierarchical clustering is a group of statistical techniques that measure the similarity among a group of entities [62]. These methods start with the calculation of the distances of each entity from all the other entities in a dataset. Following measures of the distance between entities are typically calculated using between given vector variables $\mathbf{x} = (x_1, \dots, x_n)$, and $\mathbf{y} = (y_1, \dots, y_n)$:

1. Euclidean distance: $d(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$.
2. Manhattan distance: $d(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^n |x_i - y_i|$.
3. Maximum distance: $d(\mathbf{x}, \mathbf{y}) = \max_{i=1, \dots, n} |x_i - y_i|$.
4. Correlation distance: $d(\mathbf{x}, \mathbf{y}) = 1 - \rho(\mathbf{x}, \mathbf{y})$ where $\rho(\mathbf{x}, \mathbf{y})$ is the Spearman or Kenedall correlation.
5. Absolute Correlation distance: $d(\mathbf{x}, \mathbf{y}) = 1 - |\rho(\mathbf{x}, \mathbf{y})|$.

R software has a function that computes distances between the columns of matrices and offers many different distance functions [50]. There are many choices for the linkage function that tells you how to measure the distance between clusters. Given \mathbf{x} and \mathbf{y} are in the same cluster, some linkage function are as follows:

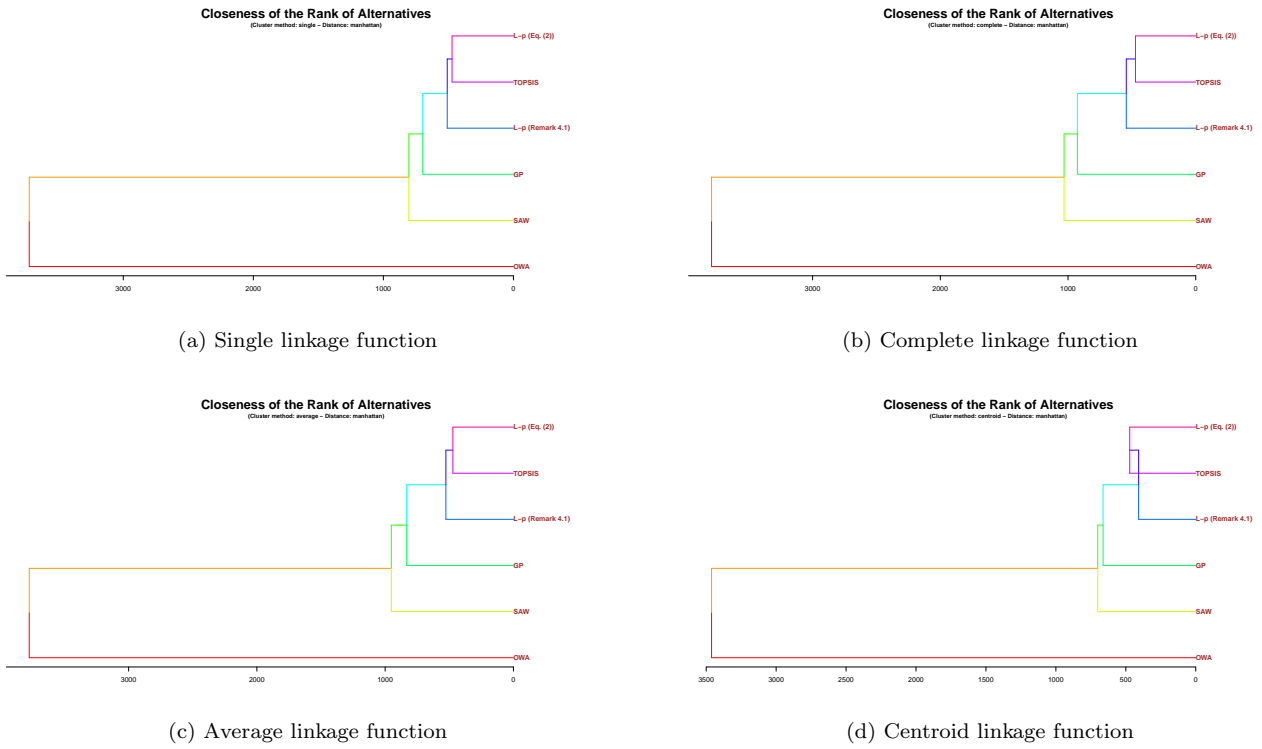


Figure 3: Dendrogram for different linkage functions and Manhattan distance

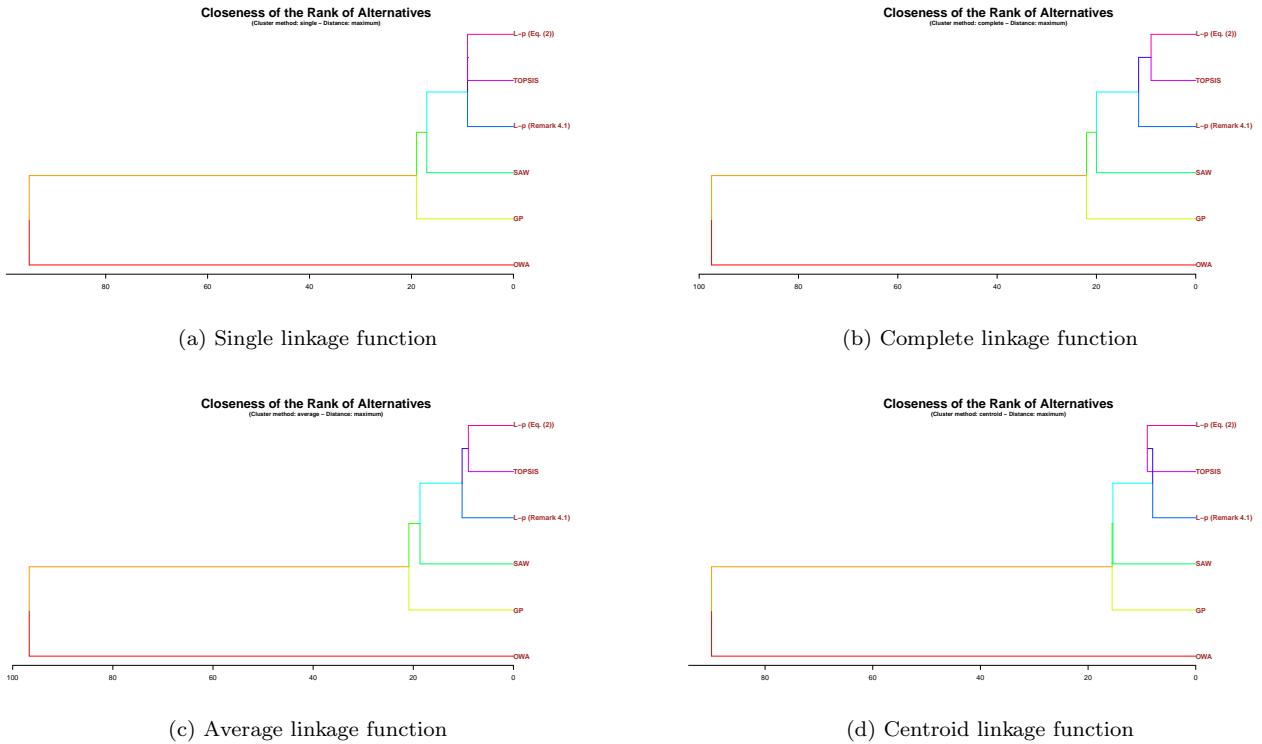


Figure 4: Dendrogram for different linkage functions and Maximum distance

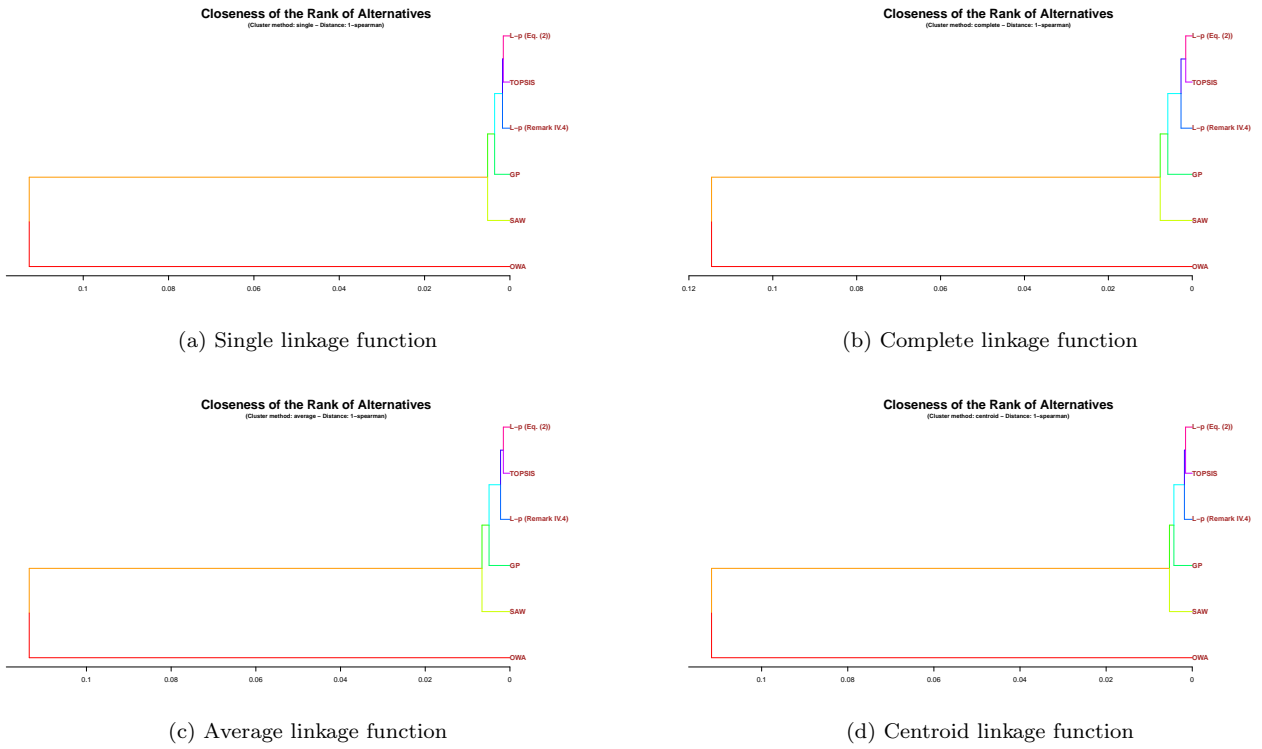


Figure 5: Dendrogram for different linkage functions and Spearman correlation distance

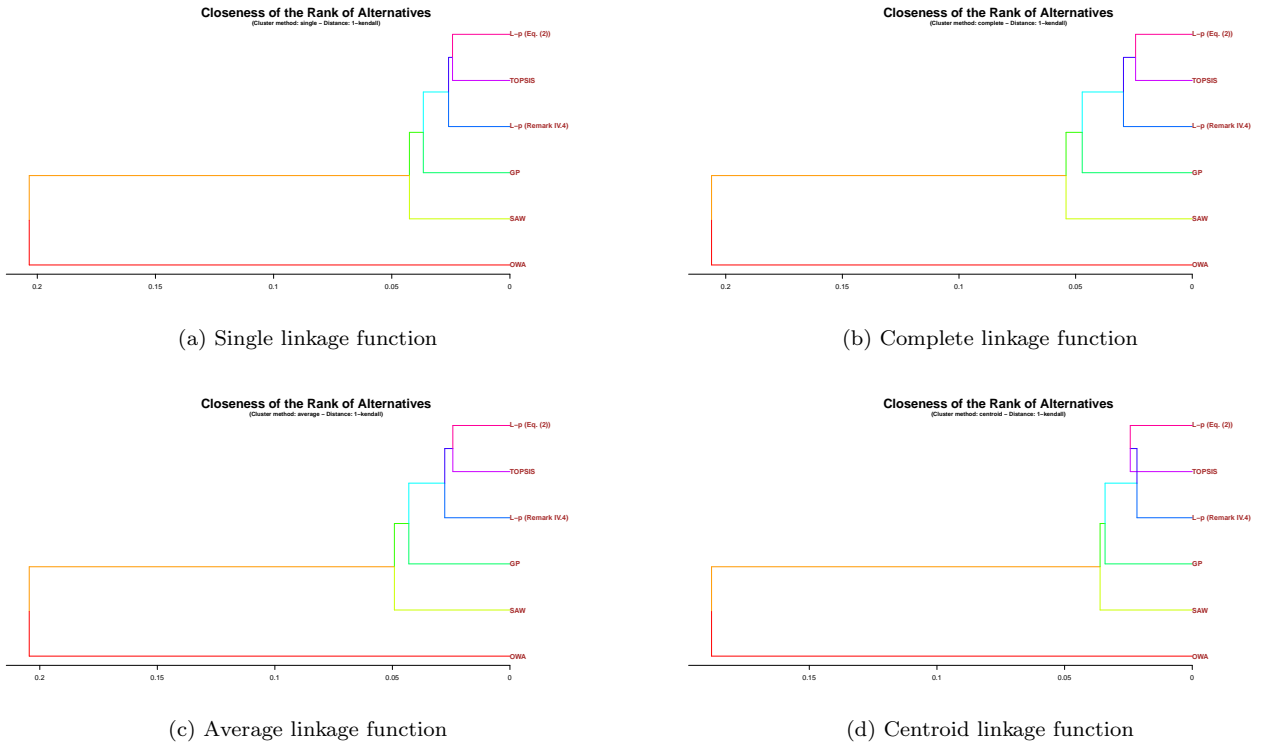


Figure 6: Dendrogram for different linkage functions and Kenedall correlation distance

1. Single linkage function: $\min(d(\mathbf{x}, \mathbf{y}))$.
2. Complete linkage function: $\max(d(\mathbf{x}, \mathbf{y}))$.
3. Average linkage function: $\text{average}(d(\mathbf{x}, \mathbf{y}))$.
4. Centroid linkage function: $d(\text{average}(\mathbf{X}), \text{average}(\mathbf{Y}))$ where we take the average over all items in each cluster.

Clusters are then formed usually by a process of agglomeration or division, when distance measure and linkage function being fixed. A dendrogram is a tree diagram used to display the groups formed by hierarchical clustering [62]. One of the major uses of dendrogram is to permit the viewer to perform intuitive visual cluster analysis, that is, to discern any subsets of observations whose members are both close to one another and/or isolated from the rest. Therefore, in order to provide a comparison study, dendrogram is used to show the differences and similarities of the proposed method in contrast to TOPSIS, SAW and OWA methods. Figure 2, Figure 3, Figure 4, Figure 5, and Figure 6 are dendrograms for different linkage functions as well as different distance measures. Although it is obvious each one of the competitive methods has different methodology, but it can be concluded our proposed method ultimately produce the same ranking as TOPSIS and SAW, while there is a difference between the rankings of OWA and the others.

7 Conclusions and recommendations for future research

One of the most important steps in solving MADM problems is determining the weights of criteria. For this purpose, various methods have been proposed by researchers, including pairwise comparisons, SWARA method, BWM method, and Yager method, etc. A common drawback of these methods is that they assess the weights of criteria independently of the decision matrix information. In this paper, by developing a generalization of the Yager method, we have incorporated the decision matrix information into estimating the weights of the criteria. In order to better clarify the issue, an example of prioritization in mining and mineral industries has been used in this paper. Prioritization in the mining and mineral industries is often filled with various probabilistic and possibilistic uncertainties. This is due to the fact that accurate evaluation of different indicators related to mining, such as reserve volume and mineral grade, is usually very costly, and therefore, approximate information needs to be relied upon in many cases. Additionally, many of these relevant indicators are associated with uncertainty because they pertain to the future (such as production forecasts, supply forecasts, demand forecasts, price forecasts, etc.), and it is better to consider their data in the form of fuzzy numbers. Furthermore, expert judgments are often accompanied by doubt and uncertainty in many cases. It should also be noted that large-scale projects in the mining and mineral industries (such as the present case study) are often very costly, so any incorrect prioritization resulted because of neglecting information and uncertainties can lead to the loss of millions of dollars. Therefore, it is essential to use an accurate model for evaluating these priorities that can take into account all of this information and uncertainties. The combination of these factors led us to use the approach introduced in this paper for prioritizing mining opportunities, and if we were to use other methods, we would have to disregard many of the actual information and uncertainties present in the problem, which could greatly affect the accuracy of the results.

Based on the numerical example, the results were compared between three specific modes of the methods proposed in this paper and the methods of TOPSIS, SAW and OWA. Our goal was to compare the ranking similarities of these methods. These comparisons were performed in three different ways, including:

1. Calculating the percentage of similarity between the top rankings,
2. A pairwise comparison between the correlation coefficients between the obtained rankings,
3. Clustering of obtained rankings.

Totally speaking, it can be concluded that first, the prioritization resulting from the proposed method is different from the prioritization resulting from the Yager method, meaning that if we intervene with the decision matrix data in the Yager method process, it will have an impact on the final results. Second, by comparing the results obtained from the proposed method with two well-known and important methods, namely SAW and TOPSIS, it can be concluded that the prioritization obtained from the proposed method is more similar to the prioritization from the TOPSIS method than the results of prioritization by the SAW method, and this is considered an advantage because the TOPSIS method algorithm is more comprehensive compared to the SAW method algorithm and more information is utilized in its algorithm.

Some of the advantages of the proposed model are as follows:

- The introduced approach is a mixed approach, which is a combination of a subjective method and an objective method.
- In the evaluation of criteria weights, the decision maker's preferences can be taken into account.
- It also uses information of the decision matrix.
- In addition to finding criteria weights, it prioritizes alternatives.
- Input data can be fuzzy and/or crisp.

The combination of the above items complicates the proposed model to some extent. However, in many cases, the formulas may be easier, for example, 1). the case where the decision maker does not provide any preference between alternatives or criteria, 2). all the information is crisp. Here, a suitable software can be developed to facilitate the complexity of calculations, which is offered by the authors as one of the fields of research for future. There are also some other future research areas in this field that can be proposed, as the following

- Other versions of uncertainty can be used [64], such as uncertainty theory [41], hesitant fuzzy sets [59], type-2 fuzzy sets [9, 64];
- Instead of $l-p$ -metric and $\widetilde{l-p}$ -metric and GP other optimization methods can be used such as Geoffrion method, Satisfactory method, Zionts-Wallenius method, Goal Attainment method, Step method, Stem method and so on [63, 64].
- Instead of the maximal entropy objective function in the optimization problems, other objective functions can be considered as well (See Remark 5.4).

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References

- [1] B. S. Ahn, *Preference relation approach for obtaining OWA operators weights*, International Journal of Approximate Reasoning, **47** (2008), 166-178. <https://doi.org/10.1016/j.ijar.2007.04.001>
- [2] B. S. Ahn, *A new approach to solve the constrained OWA aggregation problem*, IEEE Transactions on Fuzzy Systems, **25** (2017), 1231-1238. <https://doi.org/10.1109/TFUZZ.2016.2604006>
- [3] M. Akram, S. Shumaiza, *Multi-criteria decision making based on q-rung orthopair fuzzy prometee approach*, Iranian Journal of Fuzzy Systems, **18** (2021), 107-127. <https://doi.org/10.22111/IJFS.2021.6258>
- [4] A. T. Almeida, E. A. Frej, L. R. P. Roselli, A. P. C. S. Costa, *A summary on fitradeoff method with methodological and practical developments and future perspectives*, Pesquisa Operacional, **43** (2023), e268356. <https://doi.org/10.1590/0101-7438.2023.043spe1.00268356>
- [5] A. Arya, S. P. Yadav, *A new approach to rank the decision making units in presence of infeasibility in intuitionistic fuzzy environment*, Iranian Journal of Fuzzy Systems, **17** (2020), 183-199. <https://doi.org/10.22111/IJFS.2020.5228>
- [6] G. Beliakov, *A method of introducing weights into OWA operators and other symmetric functions*, Springer International Publishing, Cham, 2017, pages 37-52. https://doi.org/10.1007/978-3-319-51052-1_3
- [7] O. Bozorg-Haddad, B. Zolghadr-Asli, H. A. Loáiciga, *A handbook on multi-attribute decision-making methods*, John Wiley and Sons, 2021. <https://doi.org/10.1002/9781119563501>

- [8] H. Brezis, *Functional analysis, Sobolev spaces and partial differential equations*, Springer, New York, NY, 2011. <https://doi.org/10.1007/978-0-387-70914-7>
- [9] O. Castillo, P. Melin, J. Kacprzyk, W. Pedrycz, *Type-2 fuzzy logic: Theory and applications*, In 2007 IEEE International Conference on Granular Computing (GRC 2007), pages 145-145, 2007. <https://doi.org/10.1007/978-3-540-76284-3>
- [10] J. Chachi, A. Chaji, *An OWA-based approach to quantile fuzzy regression*, Computers and Industrial Engineering, **159** (2021), 107498. <https://doi.org/10.1016/j.cie.2021.107498>
- [11] J. Chachi, A. Kazemifard, M. Jalalvand, *A multi-attribute assessment of fuzzy regression models*, Iranian Journal of Fuzzy Systems, **18** (2021), 131-148. <https://doi.org/10.22111/IJFS.2021.6181>
- [12] A. R. Chaji, *Analytic approach on maximum bayesian entropy ordered weighted averaging operators*, Computers and Industrial Engineering, **105** (2017), 260-264. <https://doi.org/10.1016/j.cie.2016.12.041>
- [13] A. R. Chaji, H. Fukuyama, R. K. Shiraz, *Selecting a model for generating OWA operator weights in magdm problems by maximum entropy membership function*, Computers and Industrial Engineering, **124** (2018), 370-378. <https://doi.org/10.1016/j.cie.2018.07.040>
- [14] S. Chakraborty, *Topsis and modified topsis: A comparative analysis*, Decision Analytics Journal, **2** (2022), 100021. <https://doi.org/10.1016/j.dajour.2021.100021>
- [15] S. Chakraborty, P. Chatterjee, P. P. Das, *Simultaneous evaluation of criteria and alternatives (seca) method*, In Multi-Criteria Decision-Making Methods in Manufacturing Environments, Apple Academic Press, 2024, pages 325-335. <https://doi.org/10.1201/9781003377030-30>
- [16] S. Dai, *Complex fuzzy ordered weighted distance measures*, Iranian Journal of Fuzzy Systems, **17** (2020), 107-114. <https://doi.org/10.1007/s40314-022-02061-4>
- [17] S. De La Rosa de Sáa, M. A. Gil, G. González-Rodríguez, M. T. López, M. A. Lubiano, *Fuzzy rating scale-based questionnaires and their statistical analysis*, IEEE Transactions on Fuzzy Systems, **23** (2015), 111-126. <https://doi.org/10.1109/TFUZZ.2014.2307895>
- [18] L. De Miguel, M. Sesma-Sara, M. Elkanó, M. Asiain, H. Bustince, *An algorithm for group decision making using n-dimensional fuzzy sets, admissible orders and OWA operators*, Information Fusion, **37** (2017), 126-131. <https://doi.org/10.1016/j.inffus.2017.01.007>
- [19] D. Diakoulaki, G. Mavrotas, L. Papayannakis, *Determining objective weights in multiple criteria problems: The critic method*, Computers and Operations Research, **22** (1995), 763-770. [https://doi.org/10.1016/0305-0548\(94\)00059-H](https://doi.org/10.1016/0305-0548(94)00059-H)
- [20] P. D'Urso, J. Chachi, *Owa fuzzy regression*, International Journal of Approximate Reasoning, **142** (2022), 430-450. <https://doi.org/10.1016/j.ijar.2021.12.009>
- [21] P. D'Urso, J. Chachi, A. Kazemifard, L. De Giovanni, *OWA-based multi-criteria decisionmaking based on fuzzy methods*, Annals of Operations Research, In press, 2024. <https://doi.org/10.1007/s10479-024-05926-5>
- [22] M. El Alaoui, *Fuzzy TOPSIS: Logic, approaches, and case studies*, CRC Press, 2021. <https://doi.org/10.1201/9781003168416>
- [23] J. García-Lapresta, L. Luis, B. Llamazares, T. Peña, *Generating OWA weights from individual assessments*, pages 135-147. Springer Berlin Heidelberg, Berlin, Heidelberg, 2011. https://doi.org/10.1007/978-3-642-17910-5_7
- [24] R. Ginevičius, *A new determining method for the criteria weights in multicriteria evaluation*, International Journal of Information Technology and Decision Making, **10** (2011), 1067-1095. <https://doi.org/10.1142/S0219622011004713>
- [25] I. Á. Harmati, R. Fullér, I. Felde, *On stability of maximal entropy OWA operator weights*, Fuzzy Sets and Systems, **448** (2022), 145-156. <https://doi.org/10.1016/j.fss.2022.01.003>

- [26] M. A. Hatefi, *Indifference threshold-based attribute ratio analysis: A method for assigning the weights to the attributes in multiple attribute decision making*, Applied Soft Computing, **74** (2019), 643-651. <https://doi.org/10.1016/j.asoc.2018.10.050>
- [27] M. A. Hatefi, *Braw: Block-wise rating the attribute weights in madm*, Computers and Industrial Engineering, **156** (2021), 107274. <https://doi.org/10.1016/j.cie.2021.107274>
- [28] M. A. Hatefi, *An improved rank order centroid method (iroc) for criteria weight estimation: An application in the engine/vehicle selection problem*, Informatica, **34** (2023), 249-270. <https://doi.org/10.15388/23-INFOR507>
- [29] M. A. Hatefi, *A typology scheme for the criteria weighting methods in madm*, International Journal of Information Technology and Decision Making, **22** (2023), 1439-1488. <https://doi.org/10.1142/S0219622022500985>
- [30] M. A. Hatefi, *A new method for weighting decision making attributes: An application in high-tech selection in oil and gas industry*, Soft Computing, **28** (2024), 281-303. <https://doi.org/10.1007/s00500-023-09282-7>
- [31] M. A. Hatefi, S. A. Razavi, V. Abiri, *A novel multi-attribute model to select appropriate weighting method in decision making, an empirical application in petroleum industry*, Group Decision and Negotiation, **32** (2023), 1351-1390. <https://doi.org/10.1007/s10726-023-09846-w>
- [32] W. He, B. Dutta, R. M. Rodríguez, A. A. Alzahrani, L. Martínez, *Induced OWA operator for group decision making dealing with extended comparative linguistic expressions with symbolic translation*, Mathematics, **9** (2021), 20. <https://doi.org/10.3390/math9010020>
- [33] F. Hosseinzadeh Lotfi, T. Allahviranloo, W. Pedrycz, M. Shahriari, H. Sharafi, S. Razipour Ghaleh Jough, *The criteria importance through inter-criteria correlation (CRITIC) in uncertainty environment*, Springer International Publishing, Cham, 2023, pages 309-324. https://doi.org/10.1007/978-3-031-44742-6_13
- [34] G. Ilieva, *Fuzzy group full consistency method for weight determination*, Cybernetics and Information Technologies, **20** (2020), 50-58. <https://doi.org/10.2478/cait-2020-0015>
- [35] C. Ji, X. Lu, W. Zhang, *Development of new operators for expert opinions aggregation: Average-induced ordered weighted averaging operators*, Journal of Ambient Intelligence and Humanized Computing, **36** (2021), 997-1014. <https://doi.org/10.1002/int.22328>
- [36] A. Kazemifard, *An extension of topsis model based on monotonic utility of criteria*, Journal of Advanced Mathematical Modeling, **10** (2020), 196-214. <https://doi.org/10.22055/JAMM.2020.27384.1647>
- [37] A. Kazemifard, J. Chachi, *MADM approach to analyse the performance of fuzzy regression models*, Journal of Ambient Intelligence and Humanized Computing, **13** (2020), 4019-4031. <https://doi.org/10.1007/s12652-021-03394-4>
- [38] A. Krylovas, E. K. Zavadskas, N. Kosareva, S. Dadelo, *New kemira method for determining criteria priority and weights in solving mcdm problem*, International Journal of Information Technology and Decision Making, **13** (2014), 1119-1133. <https://doi.org/10.1142/S0219622014500825>
- [39] S. Kubler, J. Robert, W. Derigent, A. Voisin, Y. Le Traon, *A state-of-the-art survey and testbed of fuzzy ahp (fahp) applications*, Expert Systems with Applications, **65** (2016), 398-422. <https://doi.org/10.1016/j.eswa.2016.08.064>
- [40] R. Likert, *A technique for the measurement of attitudes*, Archives of Psychology, **140** (1932), 1-55. <https://books.google.com/books?id=9rotAAAAYAAJ>
- [41] B. Liu, *Uncertainty theory*, 5th edn, Uncertainty Theory Laboratory, Tsinghua University, China, 2024. <https://doi.org/10.1007/978-3-540-73165-8>
- [42] Y. Liu, Z. Sun, H. Liang, Y. Dong, *Ranking range model in multiple attribute decision making: A comparison of selected methods*, Computers and Industrial Engineering, **155** (2021), 107180. <https://doi.org/10.1016/j.cie.2021.107180>
- [43] J. M. Mendel, *Type-1 fuzzy sets and fuzzy logic*, In Explainable Uncertain Rule-Based Fuzzy Systems, Springer, 2024, pages 17-73. https://doi.org/10.1007/978-3-031-35378-9_2

- [44] S. Naz, A. Shafiq, S. A. Butt, *Cilos-waspas approach based on schweizer-sklar power operators for evaluating cosmetic brands in a group decision-making environment*, Granular Computing, **9** (2024), 1-37. <https://doi.org/10.1007/s41066-024-00481-7>
- [45] W. A. Oliveira, D. J. Fiorotto, X. Song, D. F. Jones, *An extended goal programming model for the multiobjective integrated lot-sizing and cutting stock problem*, European Journal of Operational Research, **295** (2021), 996-1007. <https://doi.org/10.1016/j.ejor.2021.03.049>
- [46] D. Pamučar, Ž. Stević, S. Sremac, *A new model for determining weight coefficients of criteria in mcdm models: Full consistency method (fucom)*, Symmetry, **10** (2018), 393. <https://doi.org/10.3390/sym10090393>
- [47] V. Pandey, Komal, H. Dincer, *A review on topsis method and its extensions for different applications with recent development*, Soft Computing, **27** (2023), 18011-18039. <https://doi.org/10.1007/s00500-023-09011-0>
- [48] V. Podvezko, E. K. Zavadskas, A. Podvezko, *An extension of the new objective weight assessment methods cilos and idocriw to fuzzy mcdm*, Economic Computation and Economic Cybernetics Studies and Research, **54** (2022), 59-75. <https://doi.org/10.24818/18423264/54.2.20.04>
- [49] Y. Qin, Q. Qi, P. Shi, S. Lou, P. J. Scott, X. Jiang, *Multi-attribute decision-making methods in additive manufacturing: The state of the art*, Processes, **11** (2023), 497. <https://doi.org/10.3390/pr11020497>
- [50] R Core Team, *R: A language and environment for statistical computing*, R Foundation for Statistical Computing, Vienna, Austria, 2022. <https://www.R-project.org>
- [51] P. Rani, S. M. Chen, A. R. Mishra, *Multiple attribute decision making based on mairca, standard deviation-based method, and pythagorean fuzzy sets*, Information Sciences, **644** (2023), 119274. <https://doi.org/10.1016/j.ins.2023.119274>
- [52] R. Rao, *Bharat: A simple and effective multi-criteria decision-making method that does not need fuzzy logic, part-1: Multi-attribute decision-making applications in the industrial environment*, International Journal of Industrial Engineering Computations, **15** (2024), 13-40. <https://doi.org/10.5267/j.ijiec.2023.12.003>
- [53] R. Venkata Rao, *Introduction to multiple attribute decision-making (madm) methods*, Decision Making in the Manufacturing Environment: Using Graph Theory and Fuzzy Multiple Attribute Decision Making Methods, 2007, pages 27-41. https://doi.org/10.1007/978-1-84628-819-7_3
- [54] L. R. P. Roselli, A. T. de Almeida, *The use of the success-based decision rule to support the holistic evaluation process in fitradeoff*, International Transactions in Operational Research, **30** (2023), 1299-1319. <https://doi.org/10.1111/itor.12958>
- [55] H. Royden, P. Fitzpatrick, *Real analysis (4th ed.)*, Pearson, Pearson Modern Classics for Advanced Mathematics Series, 2017. ISBN13:9780131437470
- [56] C. E. Shannon, *A mathematical theory of communication*, Bell System Technical Journal, **27** (1948), 379-423 and 623-656. <https://doi.org/10.1002/j.1538-7305.1948.tb01338.x>
- [57] M. Soltanifar, *Improved kemeny median indicator ranks accordance method*, Asia-Pacific Journal of Operational Research, **40** (2003), 2250024. <https://doi.org/10.1142/S0217595922500245>
- [58] M. Soltanifar, A. Krylovas, N. Kosareva, *Voting-kemeny median indicator ranks accordance method for determining criteria priority and weights in solving multi-attribute decision-making problems*, Soft Computing, **27** (2023), 6613-6628. <https://doi.org/10.1007/s00500-022-07807-0>
- [59] V. Torra, *Hesitant fuzzy sets*, International Journal of Intelligent Systems, **25** (2010), 529-539. <https://doi.org/10.1002/int.20418>
- [60] W. Wang, J. M. Mendel, *Multiple attribute group decision making with linguistic variables and complete unknown weight information*, Iranian Journal of Fuzzy Systems, **16** (2019), 145-157. <https://doi.org/10.22111/IJFS.2019.4788>
- [61] L. Wasserman, *All of nonparametric statistics*, Springer Texts in Statistics, Springer New York, 2006. <https://doi.org/10.1007/0-387-30623-4>

- [62] L. Wilkinson, M. Friendly, *The history of the cluster heat map*, The American Statistician, **63** (2009), 179-184. <https://doi.org/10.1198/tas.2009.0033>
- [63] Z. Xu, *Uncertain multi-attribute decision making: Methods and applications*, Springer-Verlag, Berlin Heidelberg, 2015. <https://doi.org/10.1007/978-3-662-45640-8>
- [64] Z. Xu, S. Zhang, *Fuzzy multi-attribute decision-making: Theory, methods and applications*, Springer International Publishing, Cham, 2022, pages 621-658. https://doi.org/10.1007/978-3-030-96935-6_18
- [65] R. R. Yager, *On ordered weighted averaging aggregation operators in multicriteria decision making*, IEEE Transactions on Fuzzy Systems, Man and Cybernetics, **18** (1988), 183-190. <https://doi.org/10.1109/21.87068>
- [66] R. R. Yager, *On the fusion of multiple multi-criteria aggregation functions with focus on the fusion of OWA aggregations*, Knowledge-Based Systems, **191** (2020), 105216. <https://doi.org/10.1016/j.knosys.2019.105216>
- [67] G. L. Yang, J. B. Yang, D. L. Xu, M. Khoveyni, *A three-stage hybrid approach for weight assignment in madm*, Omega, **71** (2017), 93-105. <https://doi.org/10.1016/j.omega.2016.09.011>
- [68] G. Yari, A. R. Chaji, *Determination of ordered weighted averaging operator weights based on the m-entropy measures*, International Journal of Intelligent Systems, **27** (2012), 1020-1033. <https://doi.org/10.1002/int.21559>
- [69] G. Yari, A. R. Chaji, *Maximum bayesian entropy method for determining ordered weighted averaging operator weights*, Computers and Industrial Engineering, **63** (2012), 338-342. <https://doi.org/10.1016/j.cie.2012.03.010>
- [70] K. P. Yoon, C. L. Hwang, *Multiple attribute decision making: An introduction*, Sage publications, 1995. <https://doi.org/10.4135/9781412985161>
- [71] L. A. Zadeh, *Fuzzy sets*, Informtion Control, **8** (1965), 338-353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- [72] M. Zarghami, F. Szidarovszky, *Revising the OWA operator for multi criteria decision making problems under uncertainty*, European Journal of Operational Research, **198** (2009), 259-265. <https://doi.org/10.1016/j.ejor.2008.09.014>
- [73] E. K. Zavadskas, J. Antucheviciene, S. Kar, *Multi-objective and multi-attribute optimization for sustainable development decision aiding*, Sustainability, **11** (2019), 3069. <https://doi.org/10.3390/books978-3-03921-143-2>
- [74] E. K. Zavadskas, V. Podvezko, *Integrated determination of objective criteria weights in mcdm*, International Journal of Information Technology and Decision Making, **15** (2016), 267-283. <https://doi.org/10.1142/S0219622016500036>
- [75] F. Zhou, T. Y. Chen, *Multiple criteria group decision analysis using a Pythagorean fuzzy programming model for multidimensional analysis of preference based on novel distance measures*, Computers and Industrial Engineering, **148** (2020), 106670. <https://doi.org/10.1016/j.cie.2020.106670>
- [76] H. J. Zimmermann, *Fuzzy set theory and its applications*, 4th ed., Kluwer Nihoff, Boston, 2001. <https://doi.org/10.1007/978-94-010-0646-0>
- [77] M. Žižović, D. Pamučar, *New model for determining criteria weights: Level based weight assessment (lbwa) model*, Decision Making: Applications in Management and Engineering, **2** (2019), 126-137. <https://doi.org/10.31181/dmame1902102z>